Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/47-1.2.3.3-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [96]. This is test number [47].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (96)	0.00 (0)
Mathematica	95.83 (92)	4.17 (4)
Maple	51.04 (49)	48.96 (47)
Fricas	51.04 (49)	48.96 (47)
Mupad	51.04 (49)	48.96 (47)
Sympy	43.75 (42)	56.25 (54)
Giac	38.54 (37)	61.46 (59)
Maxima	17.71 (17)	82.29 (79)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

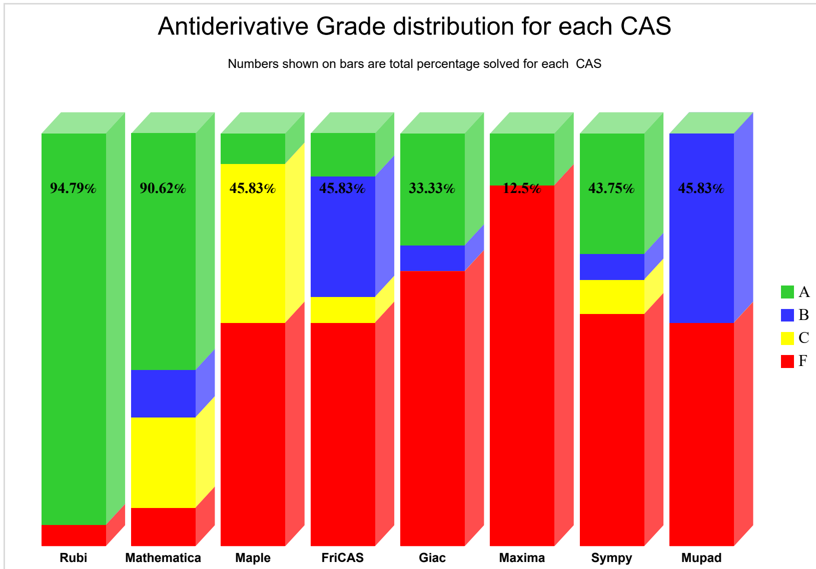
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

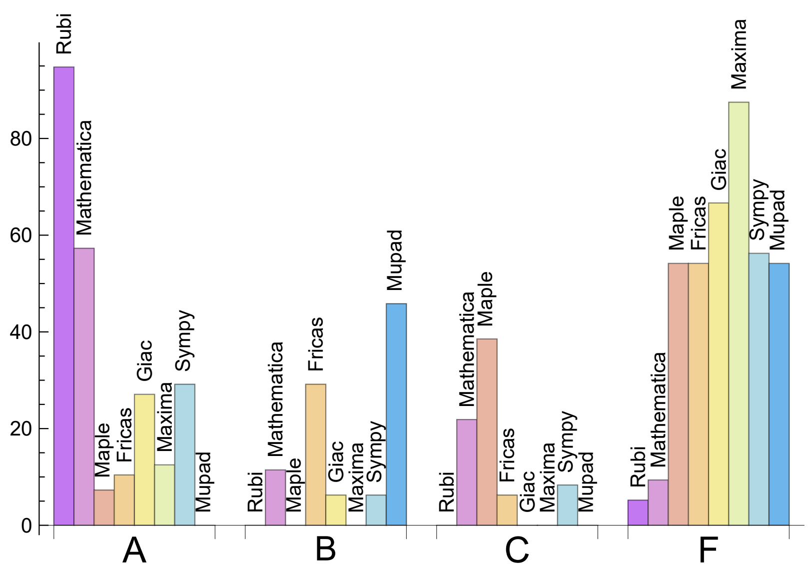
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.792	0.000	0.000	5.208
Mathematica	57.292	11.458	21.875	9.375
Sympy	29.167	6.250	8.333	56.250
Giac	27.083	6.250	0.000	66.667
Maxima	12.500	0.000	0.000	87.500
Fricas	10.417	29.167	6.250	54.167
Maple	7.292	0.000	38.542	54.167
Mupad	0.000	45.833	0.000	54.167

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	47	89.36	0.00	10.64
Maple	47	100.00	0.00	0.00
Mupad	47	0.00	100.00	0.00
Sympy	54	16.67	68.52	14.81
Giac	59	91.53	1.69	6.78
Maxima	79	98.73	0.00	1.27

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.19
Maxima	0.28
Giac	0.39
Rubi	0.40
Fricas	0.48
Mathematica	1.16
Mupad	7.00
Sympy	9.82

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	51.29	0.41	44.00	0.26
Maxima	128.29	1.10	72.00	1.08
Giac	331.35	1.75	147.00	0.94
Rubi	363.12	1.00	288.00	1.00
Sympy	435.00	2.78	110.50	0.48
Fricas	1192.80	3.44	349.00	2.21
Mathematica	2014.07	2.04	134.50	0.90
Mupad	2816.22	6.61	269.00	1.48

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

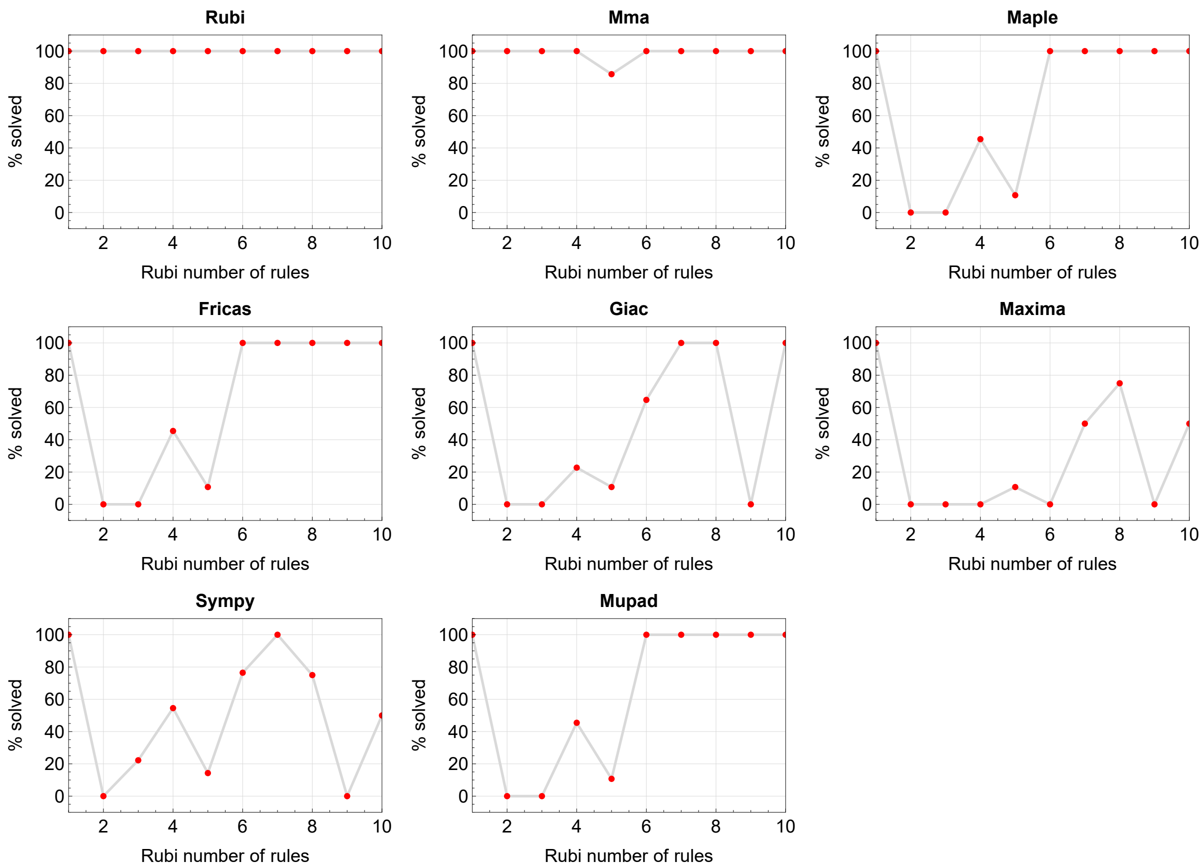


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

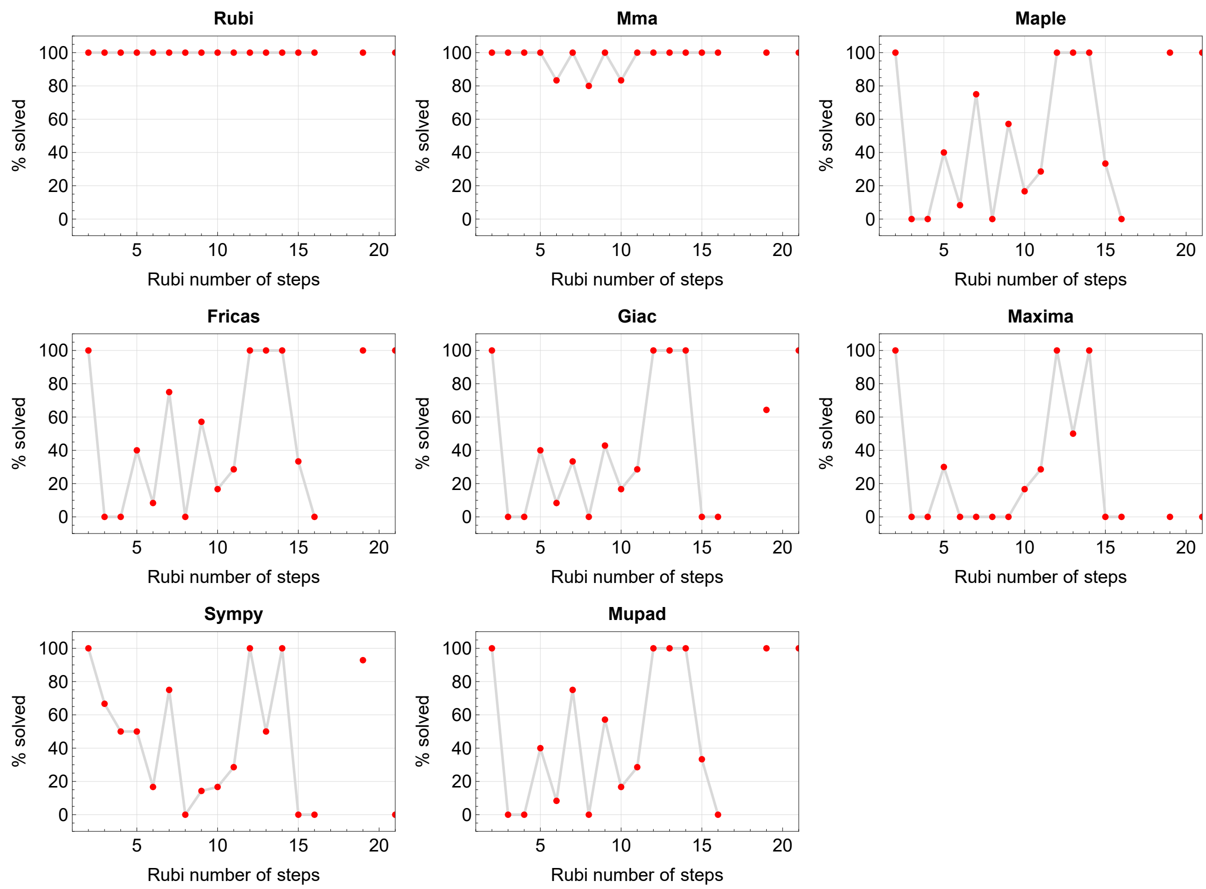


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

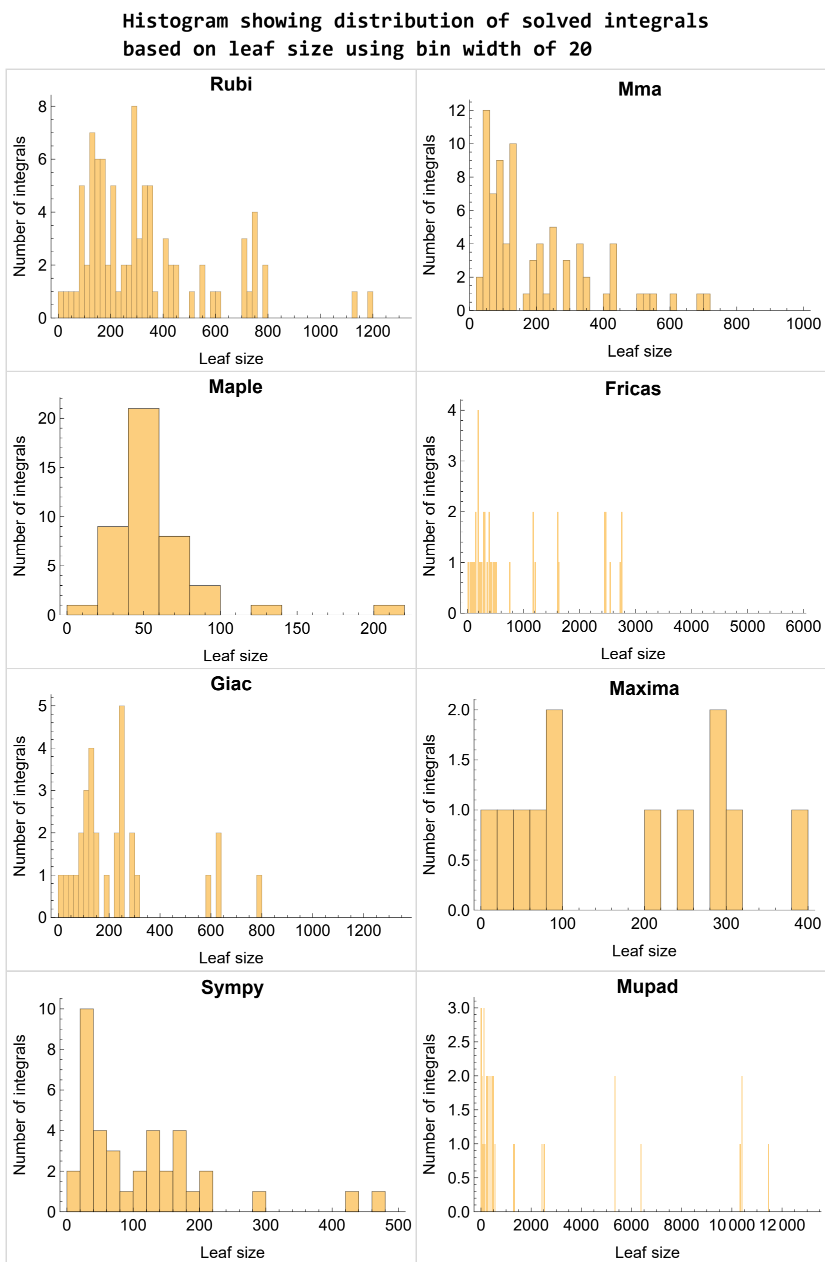


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

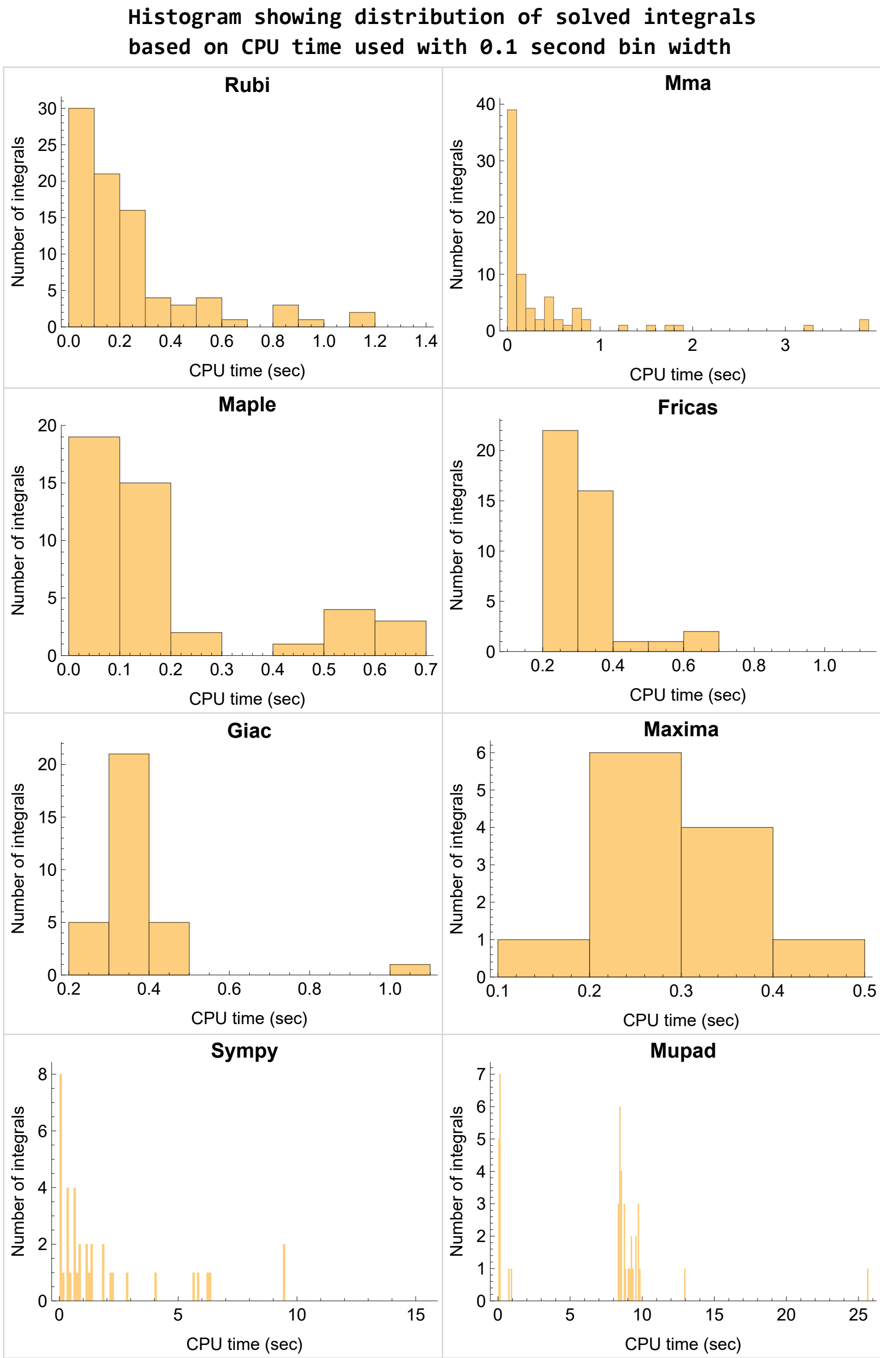


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

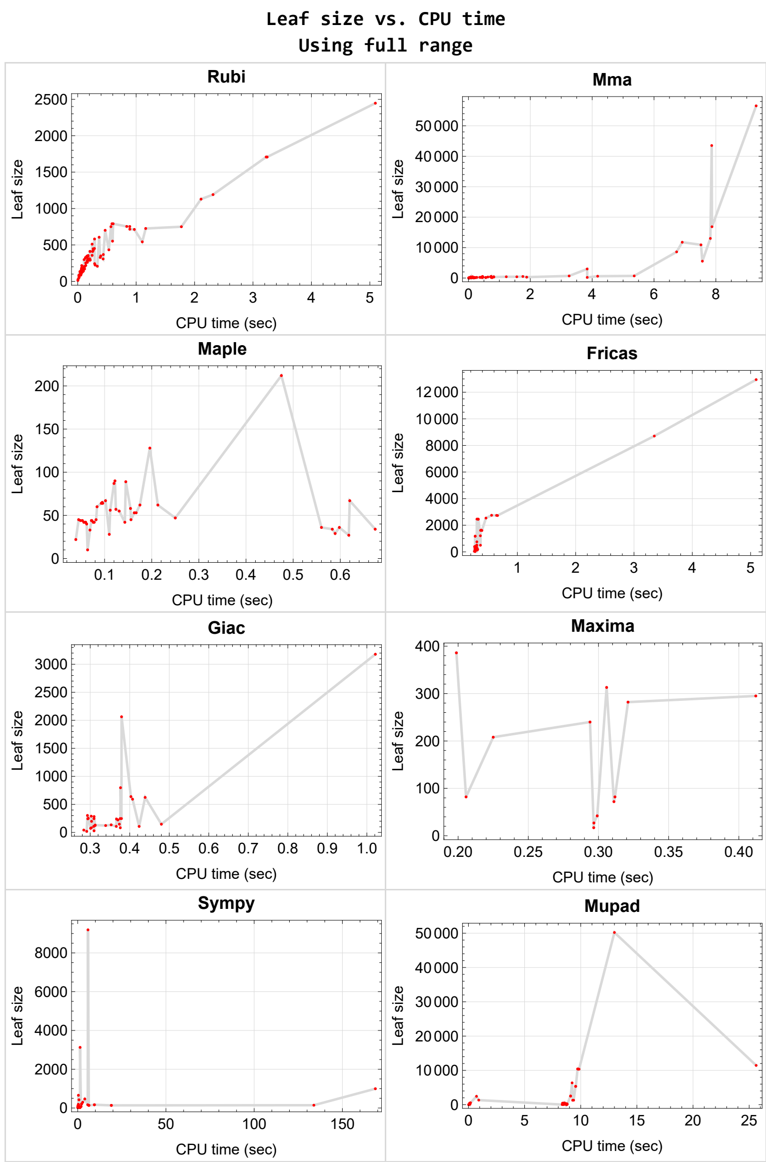


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{59, 90, 94, 95, 96\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{79, 83, 84, 86, 89\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

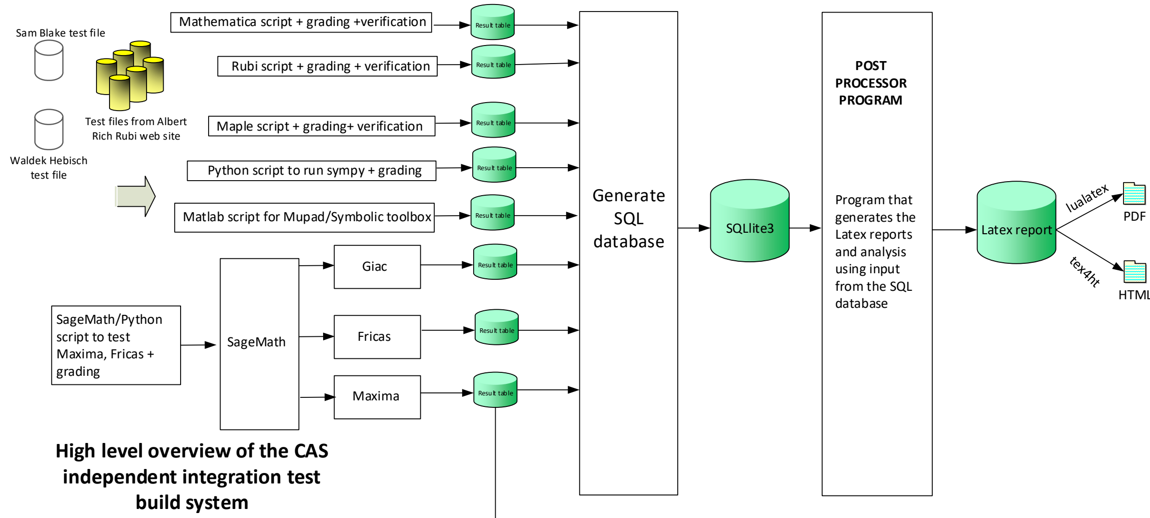
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 11, 13, 15, 16, 19, 22, 24, 26, 27, 30, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 85, 87, 88, 91, 92, 93 }

B grade { 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 89 }

C grade { 5, 6, 7, 8, 9, 10, 12, 14, 17, 18, 20, 21, 23, 25, 28, 29, 31, 32, 33, 39, 41 }

F normal fail { 58, 63, 64, 65 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 15, 26, 34, 35, 66, 67, 68 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41 }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 10, 14, 21, 23, 26, 31, 32, 33, 34, 35 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 17, 18, 19, 20, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 66, 67, 68 }

C grade { 11, 12, 13, 22, 24, 25 }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { 85, 86, 87, 88, 89 }

Maxima

A grade { 1, 2, 11, 15, 22, 26, 34, 36, 38, 66, 67, 68 }

B grade { }

C grade { }

F normal fail { 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { 35 }

Giac

A grade { 1, 2, 3, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 27, 30, 31, 32, 33, 34, 35, 36, 38, 40 }

B grade { 4, 26, 37, 66, 67, 68 }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 17, 18, 20, 28, 29, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93 }

F(-1) timedout fail { 41 }

F(-2) exception fail { 60, 61, 91, 92 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 66, 67, 68 }

C grade { }

F normal fail { }

F(-1) timedout fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 27, 28, 29, 30, 31, 36, 38 }

B grade { 26, 34, 35, 66, 67, 68 }

C grade { 12, 23, 42, 43, 44, 47, 50, 62 }

F normal fail { 48, 49, 58, 70, 71, 85, 86, 87, 89 }

F(-1) timedout fail { 3, 4, 37, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 64, 65, 69, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 88, 90, 91, 92, 93, 94, 95, 96 }

F(-2) exception fail { 32, 33, 45, 46, 63, 72, 73, 74 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	334	34	282	1631	165	283	1331
N.S.	1	1.00	1.10	0.11	0.92	5.35	0.54	0.93	4.36
time (sec)	N/A	0.177	0.072	0.674	0.321	0.380	9.414	0.310	0.916

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	337	36	313	1613	168	303	1293
N.S.	1	1.00	1.04	0.11	0.97	4.99	0.52	0.94	4.00
time (sec)	N/A	0.132	0.070	0.560	0.306	0.369	9.495	0.293	9.281

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	754	754	534	34	0	2749	0	593	2510
N.S.	1	1.00	0.71	0.05	0.00	3.65	0.00	0.79	3.33
time (sec)	N/A	0.839	0.456	0.583	0.000	0.558	0.000	0.407	9.088

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	425	36	0	2741	0	625	2438
N.S.	1	1.00	1.29	0.11	0.00	8.33	0.00	1.90	7.41
time (sec)	N/A	0.136	0.134	0.598	0.000	0.643	0.000	0.439	0.715

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	2461	136	0	10409
N.S.	1	1.00	0.08	0.07	0.00	3.11	0.17	0.00	13.16
time (sec)	N/A	0.586	0.035	0.167	0.000	0.322	19.014	0.000	9.717

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	2461	136	0	10411
N.S.	1	1.00	0.08	0.07	0.00	3.11	0.17	0.00	13.16
time (sec)	N/A	0.606	0.026	0.163	0.000	0.306	6.282	0.000	9.776

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	69	57	0	2453	136	0	10337
N.S.	1	1.00	0.20	0.16	0.00	7.03	0.39	0.00	29.62
time (sec)	N/A	0.394	0.033	0.124	0.000	0.331	19.107	0.000	9.776

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	69	55	0	2453	136	0	10343
N.S.	1	1.00	0.09	0.07	0.00	3.27	0.18	0.00	13.77
time (sec)	N/A	0.568	0.029	0.131	0.000	0.327	6.382	0.000	9.850

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	55	42	0	1177	75	0	5341
N.S.	1	1.00	0.13	0.10	0.00	2.86	0.18	0.00	13.00
time (sec)	N/A	0.209	0.037	0.143	0.000	0.276	1.807	0.000	9.560

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	55	42	0	389	24	239	459
N.S.	1	1.00	0.12	0.09	0.00	0.86	0.05	0.53	1.02
time (sec)	N/A	0.286	0.016	0.076	0.000	0.267	0.864	0.367	0.102

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	22	72	61	73	72	33
N.S.	1	1.00	0.75	0.26	0.85	0.72	0.86	0.85	0.39
time (sec)	N/A	0.031	0.018	0.039	0.311	0.273	0.067	0.301	8.345

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	135	89	0	209	190	108	95
N.S.	1	1.00	0.96	0.64	0.00	1.49	1.36	0.77	0.68
time (sec)	N/A	0.066	0.144	0.145	0.000	0.272	0.341	0.366	0.087

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	258	27	0	189	19	247	311
N.S.	1	1.00	0.74	0.08	0.00	0.54	0.05	0.71	0.90
time (sec)	N/A	0.157	0.135	0.618	0.000	0.320	1.148	0.376	8.752

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	55	42	0	389	20	245	145
N.S.	1	1.00	0.17	0.13	0.00	1.18	0.06	0.74	0.44
time (sec)	N/A	0.158	0.017	0.078	0.000	0.266	1.277	0.310	0.137

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	28	27	43	26	29	21
N.S.	1	1.00	1.15	1.04	1.00	1.59	0.96	1.07	0.78
time (sec)	N/A	0.005	0.012	0.110	0.297	0.262	0.066	0.310	0.026

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	131	56	0	317	49	147	269
N.S.	1	1.00	1.00	0.43	0.00	2.42	0.37	1.12	2.05
time (sec)	N/A	0.063	0.063	0.112	0.000	0.281	0.661	0.374	0.117

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	53	40	0	349	24	0	399
N.S.	1	1.00	0.34	0.25	0.00	2.22	0.15	0.00	2.54
time (sec)	N/A	0.068	0.013	0.062	0.000	0.289	0.087	0.000	8.420

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	55	42	0	509	24	0	483
N.S.	1	1.00	0.32	0.25	0.00	2.98	0.14	0.00	2.82
time (sec)	N/A	0.117	0.015	0.060	0.000	0.304	0.086	0.000	8.465

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	111	58	0	221	49	123	233
N.S.	1	1.00	0.95	0.50	0.00	1.89	0.42	1.05	1.99
time (sec)	N/A	0.047	0.051	0.155	0.000	0.291	0.659	0.339	8.430

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	511	57	44	0	1177	76	0	5341
N.S.	1	1.00	0.11	0.09	0.00	2.30	0.15	0.00	10.45
time (sec)	N/A	0.247	0.035	0.049	0.000	0.277	1.868	0.000	9.534

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	421	26	223	447
N.S.	1	1.00	0.14	0.11	0.00	1.02	0.06	0.54	1.09
time (sec)	N/A	0.212	0.014	0.073	0.000	0.279	0.876	0.370	8.373

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	33	82	97	82	82	44
N.S.	1	1.00	0.93	0.34	0.85	1.00	0.85	0.85	0.45
time (sec)	N/A	0.040	0.043	0.069	0.312	0.284	0.070	0.303	0.046

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	87	0	195	148	108	109
N.S.	1	1.00	0.92	0.62	0.00	1.39	1.06	0.77	0.78
time (sec)	N/A	0.064	0.114	0.120	0.000	0.279	0.301	0.309	0.113

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	257	29	0	193	20	247	312
N.S.	1	1.00	0.74	0.08	0.00	0.56	0.06	0.71	0.90
time (sec)	N/A	0.178	0.098	0.589	0.000	0.303	1.175	0.379	8.547

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	417	26	253	208
N.S.	1	1.00	0.16	0.12	0.00	1.17	0.07	0.71	0.59
time (sec)	N/A	0.178	0.021	0.072	0.000	0.299	1.303	0.295	8.443

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.003	0.005	0.064	0.297	0.274	0.062	0.291	0.014

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	129	64	0	285	51	147	269
N.S.	1	1.00	1.00	0.50	0.00	2.21	0.40	1.14	2.09
time (sec)	N/A	0.080	0.056	0.093	0.000	0.290	0.685	0.480	0.098

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	55	42	0	317	26	0	399
N.S.	1	1.00	0.33	0.25	0.00	1.92	0.16	0.00	2.42
time (sec)	N/A	0.074	0.013	0.059	0.000	0.273	0.086	0.000	0.108

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	57	44	0	477	26	0	483
N.S.	1	1.00	0.34	0.26	0.00	2.82	0.15	0.00	2.86
time (sec)	N/A	0.106	0.014	0.053	0.000	0.300	0.091	0.000	8.575

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	114	64	0	245	51	135	245
N.S.	1	1.00	0.91	0.51	0.00	1.96	0.41	1.08	1.96
time (sec)	N/A	0.055	0.039	0.096	0.000	0.276	0.672	0.353	8.513

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	71	47	0	153	163	107	133
N.S.	1	1.00	0.53	0.35	0.00	1.13	1.21	0.79	0.99
time (sec)	N/A	0.085	0.069	0.250	0.000	0.287	0.498	0.424	8.803

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	72	62	0	158	0	123	1
N.S.	1	1.00	0.44	0.38	0.00	0.96	0.00	0.75	0.01
time (sec)	N/A	0.061	0.040	0.213	0.000	0.286	0.000	0.313	8.700

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	89	62	0	181	0	131	1
N.S.	1	1.00	0.49	0.34	0.00	1.01	0.00	0.73	0.01
time (sec)	N/A	0.086	0.049	0.175	0.000	0.296	0.000	0.313	8.758

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	42	108	112	42	39
N.S.	1	1.00	1.00	0.86	0.86	2.20	2.29	0.86	0.80
time (sec)	N/A	0.021	0.019	0.056	0.299	0.286	0.136	0.284	8.496

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	90	0	291	423	83	127
N.S.	1	1.00	1.00	1.05	0.00	3.38	4.92	0.97	1.48
time (sec)	N/A	0.054	0.061	0.122	0.000	0.306	0.708	0.376	0.135

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	293	45	240	754	109	243	555
N.S.	1	1.00	1.16	0.18	0.95	2.98	0.43	0.96	2.19
time (sec)	N/A	0.137	0.073	0.045	0.294	0.305	0.348	0.294	0.186

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	251	65	0	2540	0	3179	6366
N.S.	1	1.00	1.21	0.31	0.00	12.21	0.00	15.28	30.61
time (sec)	N/A	0.336	0.119	0.095	0.000	0.461	0.000	1.022	9.231

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	346	45	295	1608	167	290	1308
N.S.	1	1.00	1.11	0.14	0.95	5.17	0.54	0.93	4.21
time (sec)	N/A	0.204	0.077	0.156	0.412	0.392	5.668	0.303	9.367

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	716	716	88	67	0	8707	0	0	11453
N.S.	1	1.00	0.12	0.09	0.00	12.16	0.00	0.00	16.00
time (sec)	N/A	0.892	0.036	0.620	0.000	3.349	0.000	0.000	25.624

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	753	551	45	0	2730	0	639	2520
N.S.	1	1.00	0.73	0.06	0.00	3.63	0.00	0.85	3.35
time (sec)	N/A	0.891	0.734	0.082	0.000	0.659	0.000	0.403	9.101

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	12946	0	0	50213
N.S.	1	1.00	0.20	0.15	0.00	29.90	0.00	0.00	115.97
time (sec)	N/A	0.533	0.060	0.102	0.000	5.095	0.000	0.000	13.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	127	0	0	0	466	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	3.30	0.00	0.00
time (sec)	N/A	0.098	0.562	0.000	0.000	0.000	4.012	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	296	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.77	0.00	0.00
time (sec)	N/A	0.062	0.221	0.000	0.000	0.000	2.824	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	209	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.52	0.00	0.00
time (sec)	N/A	0.019	0.070	0.000	0.000	0.000	2.159	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	131	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	186	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	0	0	214	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.64	0.00	0.00
time (sec)	N/A	0.020	0.089	0.000	0.000	0.000	2.254	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	188	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.167	0.384	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	136	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.210	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	83	0	0	0	994	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	7.42	0.00	0.00
time (sec)	N/A	0.038	0.078	0.000	0.000	0.000	168.683	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	333	333	227	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.266	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	298	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	424	424	188	0	0	0	0	0	0
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.738	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	272	136	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.445	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	83	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	582	582	346	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.424	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	701	701	426	0	0	0	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	0.716	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	0	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10
time (sec)	N/A	0.005	0.301	0.160	0.278	0.297	0.000	0.370	8.931

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	213	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.197	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	171	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	110	0	0	0	144	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	1.07	0.00	0.00
time (sec)	N/A	0.042	0.072	0.000	0.000	0.000	133.828	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	357	357	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	60	82	137	656	198	59
N.S.	1	1.00	0.92	0.97	1.32	2.21	10.58	3.19	0.95
time (sec)	N/A	0.025	0.181	0.084	0.206	0.313	0.391	0.303	8.392

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	128	208	495	3128	798	131
N.S.	1	1.00	0.93	0.97	1.58	3.75	23.70	6.05	0.99
time (sec)	N/A	0.068	0.765	0.196	0.225	0.366	1.342	0.377	8.490

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	205	212	386	1209	9190	2064	227
N.S.	1	1.00	0.94	0.97	1.77	5.55	42.16	9.47	1.04
time (sec)	N/A	0.138	3.841	0.475	0.199	0.366	5.814	0.379	8.548

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	295	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	1.883	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	216	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.560	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	134	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	200	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.432	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	368	368	327	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	0.821	0.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	552	552	509	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	1.759	0.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	750	750	5537	0	0	0	0	0	0
N.S.	1	1.00	7.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.774	7.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	543	543	2980	0	0	0	0	0	0
N.S.	1	1.00	5.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.105	3.839	0.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	328	603	0	0	0	0	0	0
N.S.	1	0.91	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	4.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	726	726	11767	0	0	0	0	0	0
N.S.	1	1.00	16.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.163	6.913	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1129	1129	16855	0	0	0	0	0	0
N.S.	1	1.00	14.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.111	7.875	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1707	1707	13018	0	0	0	0	0	0
N.S.	1	1.00	7.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.242	7.822	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1191	1191	10910	0	0	0	0	0	0
N.S.	1	1.00	9.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.316	7.518	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	713	713	8593	0	0	0	0	0	0
N.S.	1	1.00	12.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.969	6.731	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1708	1708	43535	0	0	0	0	0	0
N.S.	1	1.00	25.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.226	7.866	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	2446	2446	56566	0	0	0	0	0	0
N.S.	1	1.00	23.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.095	9.303	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	424	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	1.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	690	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	3.255	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	245	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.366	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	414	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	1.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	701	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	5.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.006	1.326	0.115	0.261	0.307	0.000	1.205	10.626

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	606	606	438	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.816	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	447	338	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.628	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	243	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.007	0.796	0.126	0.261	0.262	0.000	0.365	10.360

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	41	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.58	0.00	1.08	1.08
time (sec)	N/A	0.007	0.624	0.079	0.271	0.276	0.000	0.372	11.181

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	54	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	2.08	0.00	1.08	1.08
time (sec)	N/A	0.007	0.912	0.095	0.267	0.320	0.000	0.398	11.289

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [38] had the largest ratio of [.5881999999999945]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	8	1.00	17	0.471
2	A	13	7	1.00	18	0.389
3	A	19	6	1.00	17	0.353
4	A	13	10	1.00	18	0.556
5	A	19	6	1.00	26	0.231
6	A	19	6	1.00	26	0.231
7	A	7	4	1.00	27	0.148
8	A	19	6	1.00	27	0.222
9	A	19	6	1.00	18	0.333
10	A	19	7	1.00	18	0.389
11	A	10	7	1.00	18	0.389
12	A	19	6	1.00	16	0.375
13	A	19	6	1.00	13	0.462
14	A	19	6	1.00	18	0.333
15	A	5	5	1.00	18	0.278
16	A	7	4	1.00	18	0.222
17	A	7	4	1.00	18	0.222
18	A	7	4	1.00	18	0.222
19	A	7	4	1.00	18	0.222
20	A	19	6	1.00	20	0.300
21	A	19	7	1.00	20	0.350
22	A	11	8	1.00	20	0.400
23	A	19	6	1.00	18	0.333
24	A	19	6	1.00	15	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	19	6	1.00	20	0.300
26	A	5	5	1.00	20	0.250
27	A	7	4	1.00	20	0.200
28	A	7	4	1.00	20	0.200
29	A	7	4	1.00	20	0.200
30	A	7	4	1.00	20	0.200
31	A	9	6	1.00	25	0.240
32	A	9	6	1.00	26	0.231
33	A	9	6	1.00	33	0.182
34	A	5	5	1.00	17	0.294
35	A	6	6	1.00	22	0.273
36	A	11	8	1.00	17	0.471
37	A	5	4	1.00	22	0.182
38	A	14	10	1.00	17	0.588
39	A	15	9	1.00	22	0.409
40	A	21	8	1.00	17	0.471
41	A	9	6	1.00	22	0.273
42	A	5	4	1.00	21	0.190
43	A	5	4	1.00	21	0.190
44	A	3	3	1.00	19	0.158
45	A	6	4	1.00	21	0.190
46	A	7	4	1.00	21	0.190
47	A	3	3	1.00	20	0.150
48	A	9	5	1.00	21	0.238
49	A	7	5	1.00	21	0.238
50	A	4	4	1.00	19	0.210
51	A	10	5	1.00	21	0.238
52	A	11	5	1.00	21	0.238
53	A	11	5	1.00	21	0.238
54	A	8	5	1.00	21	0.238
55	A	5	4	1.00	19	0.210
56	A	15	5	1.00	21	0.238
57	A	16	5	1.00	21	0.238
58	A	6	5	1.00	23	0.217
59	N/A	0	0	1.00	21	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	10	5	1.00	21	0.238
61	A	8	5	1.00	21	0.238
62	A	6	5	1.00	19	0.263
63	A	6	5	1.00	21	0.238
64	A	8	5	1.00	21	0.238
65	A	10	5	1.00	21	0.238
66	A	2	1	1.00	22	0.045
67	A	2	1	1.00	24	0.042
68	A	2	1	1.00	24	0.042
69	A	5	3	1.00	26	0.115
70	A	5	3	1.00	26	0.115
71	A	3	2	1.00	24	0.083
72	A	6	3	1.00	26	0.115
73	A	7	3	1.00	26	0.115
74	A	8	3	1.00	26	0.115
75	A	9	4	1.00	26	0.154
76	A	9	5	1.00	26	0.192
77	A	4	3	0.91	24	0.125
78	A	10	4	1.00	26	0.154
79	A	11	4	1.00	26	0.154
80	A	11	4	1.00	26	0.154
81	A	11	5	1.00	26	0.192
82	A	5	3	1.00	24	0.125
83	A	15	4	1.00	26	0.154
84	A	16	4	1.00	26	0.154
85	A	6	5	1.00	26	0.192
86	A	6	5	1.00	26	0.192
87	A	6	5	1.00	26	0.192
88	A	6	5	1.00	26	0.192
89	A	6	5	1.00	26	0.192
90	N/A	0	0	1.00	26	0.000
91	A	10	5	1.00	26	0.192
92	A	8	5	1.00	26	0.192
93	A	6	5	1.00	24	0.208
94	N/A	0	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	N/A	0	0	1.00	26	0.000
96	N/A	0	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{d+ex^3}{a+cx^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 305

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} + \frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}} + \frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

```
[Out] 1/3*d*arctan(c^(1/6)*x/a^(1/6))/a^(5/6)/c^(1/6)-1/6*e*ln(a^(1/3)+c^(1/3)*x^
2)/a^(1/3)/c^(2/3)+1/6*arctan(2*c^(1/6)*x/a^(1/6)+3^(1/2))*(-e*3^(1/2)*a^(1
/2)+d*c^(1/2))/a^(5/6)/c^(2/3)+1/6*arctan(2*c^(1/6)*x/a^(1/6)-3^(1/2))*(e*3
^(1/2)*a^(1/2)+d*c^(1/2))/a^(5/6)/c^(2/3)-1/12*ln(a^(1/3)+c^(1/3)*x^2-a^(1/
6)*c^(1/6)*x*3^(1/2))*(-e*a^(1/2)+d*3^(1/2)*c^(1/2))/a^(5/6)/c^(2/3)+1/12*l
n(a^(1/3)+c^(1/3)*x^2+a^(1/6)*c^(1/6)*x*3^(1/2))*(e*a^(1/2)+d*3^(1/2)*c^(1/
2))/a^(5/6)/c^(2/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1430, 649, 209, 266, 648, 631, 210, 642}

$$\int \frac{d+ex^3}{a+cx^6}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)}{6a^{5/6}c^{2/3}} + \frac{\arctan\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+\sqrt{3}\right)\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)}{6a^{5/6}c^{2/3}} + \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}} + \frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$

```
[In] Int[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) - ((Sqrt[c]*d + Sqrt[3]
*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6*a^(5/6)*c^(2/3)) +
((Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(
6*a^(5/6)*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - (
(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c
^(1/3)*x^2])/(12*a^(5/6)*c^(2/3)) + ((Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\frac{2\sqrt[3]{c}d}{\sqrt[3]{a}}-\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\frac{2\sqrt[3]{c}d}{\sqrt[3]{a}}+\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{1+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\frac{\sqrt[3]{c}d}{\sqrt[3]{a}}-ex}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}}$$

$$= \frac{d\int \frac{1}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a} - \frac{e\int \frac{x}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}} - \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{-\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}x}{\sqrt[3]{a}}}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12a^{5/6}c^{2/3}}$$

$$+ \frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\int \frac{\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}x}{\sqrt[3]{a}}}{1+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12a^{5/6}c^{2/3}}$$

$$+ \frac{\left(d-\frac{\sqrt{3}\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{1+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12a} + \frac{\left(d+\frac{\sqrt{3}\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12a}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$

$$- \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

$$+ \frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

$$- \frac{\left(\sqrt{3}\sqrt{c}d-3\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{18a^{5/6}c^{2/3}}$$

$$+ \frac{\left(\sqrt{3}\sqrt{c}d+3\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{18a^{5/6}c^{2/3}}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}}$$

$$+ \frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$

$$- \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

$$+ \frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.10

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt[6]{a}\sqrt{c}d+\sqrt{3}a^{2/3}e\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} + \frac{\left(\sqrt[6]{a}\sqrt{c}d-\sqrt{3}a^{2/3}e\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{2/3}} - \frac{\left(-\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{2/3}}$$

```
[In] Integrate[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) + ((a^(1/6)*Sqrt[c]*d +
 Sqrt[3]*a^(2/3)*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x)/a^(1/6)])/(6*
a*c^(2/3)) + ((a^(1/6)*Sqrt[c]*d - Sqrt[3]*a^(2/3)*e)*ArcTan[(Sqrt[3]*a^(1/
6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/
(6*a^(1/3)*c^(2/3)) - ((Sqrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e)*Log[a^(1/3)
- Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(2/3)) - ((-(Sqrt[3]*a^
(1/6)*Sqrt[c]*d) - a^(2/3)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(
1/3)*x^2])/(12*a*c^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.11

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6c+a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{\_R^5}}{6c}$
default	$\frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}e}{12a} - \frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}d}{12a} + \frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)\sqrt{3}e}{6a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\text{arcta}}{}$

```
[In] int((e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/c*sum((_R^3*e+d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1631 vs. $2(207) = 414$.

Time = 0.38 (sec) , antiderivative size = 1631, normalized size of antiderivative = 5.35

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(-3) + 1)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e
^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2*a*c*
d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e^2 + sqrt(-3)*(a*c
^2*d^4 - 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqrt(-(c^2*d^6
 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(-(c^2*d^6 - 6*
a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/
3)) + 1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*
d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2
*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e^2 - sqrt(-3)
*(a*c^2*d^4 - 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e - a^4*c^2*e)*sqrt(-(c^
2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(-(c^2*d^6
 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2)
)^(1/3)) - 1/12*(sqrt(-3) + 1)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d
^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e^2 + sq
rt(-3)*(a*c^2*d^4 - 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqr
t(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt(-(
c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a
^2*c^2))^(1/3)) + 1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4
*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(
-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e
^2 - sqrt(-3)*(a*c^2*d^4 - 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e - a^4*c^2
*e)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*
sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*
e^3)/(a^2*c^2))^(1/3)) + 1/6*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a
^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5
- 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x - (a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^
2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a*c^2*d^4 - 3*a^2*c*d^2*e^2)*((a^2*c^2*sqrt
(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)
/(a^2*c^2))^(1/3)) + 1/6*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*
d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2
*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + (a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) - a*c^2*d^4 + 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt(-
(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(
a^2*c^2))^(1/3))
```

Sympy [A] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.54

$$\int \frac{d+ex^3}{a+cx^6}\,dx$$
$$=\text{RootSum}\left(46656t^6a^5c^4+t^3\cdot\left(432a^4c^2e^3-1296a^3c^3d^2e\right)+a^3e^6+3a^2cd^2e^4+3ac^2d^4e^2+c^3d^6,\left(t\mapsto t\log\right.\right.$$

```
[In] integrate((e*x**3+d)/(c*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*a**5*c**4 + _t**3*(432*a**4*c**2*e**3 - 1296*a**3*c**3*
d**2*e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6,
Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e - 6*_t*a**3*e**4 + 36*_t*a**
2*c*d**2*e**2 - 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c**2*d
**5))))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{d+ex^3}{a+cx^6}\,dx =& -\frac{e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}c^{\frac{2}{3}}}+\frac{d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{3\,a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\\ &+\frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d+a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}}\\ &-\frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d-a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}}\\ &-\frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e-a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\\ &+\frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e+a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\end{aligned}$$

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

```
[Out] -1/6*e*log(c^(1/3)*x^2 + a^(1/3))/(a^(1/3)*c^(2/3)) + 1/3*d*arctan(c^(1/3)*
x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/12*(sqrt(3)*a^
(1/6)*sqrt(c)*d + a^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x +
a^(1/3))/(a*c^(2/3)) - 1/12*(sqrt(3)*a^(1/6)*sqrt(c)*d - a^(2/3)*e)*log(c^(
```

```
1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) - 1/6*(sqrt(3)*
a^(5/6)*c^(1/6)*e - a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6
)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/6*(
sqrt(3)*a^(5/6)*c^(1/6)*e + a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3
)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3)))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.93

$$\int \frac{d+ex^3}{a+cx^6}\,dx = -\frac{e|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,(ac^5)^{\frac{1}{3}}} + \frac{(ac^5)^{\frac{1}{6}}\,d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} + \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d-\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d+\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d+(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d-(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

```
[Out] -1/6*e*abs(c)*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + 1/3*(a*c^5)^(1/6)*d*ar
ctan(x/(a/c)^(1/6))/(a*c) + 1/6*((a*c^5)^(1/6)*c^3*d - sqrt(3)*(a*c^5)^(2/3
)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) + 1/6*((a*c^5)
^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/
(a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d + (a*c^5)^(2/3)*e)
*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(3)*(a*
c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)
^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 1331, normalized size of antiderivative = 4.36

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \ln\left(a^3\,c^3\,\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{-a^5\,c^5}-3\,a^3\,c^3\,d^2\,e-3\,a\,d\,e^2\,\sqrt{-a^5\,c^5}}{a^5\,c^4}\right)^{1/3}+e\,x\,\sqrt{-a^5\,c^5}+a^2\,c^3\,d\,x\right)\,\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{-a^5\,c^5}-3\,a^3\,c^3\,d^2\,e-3\,a\,d\,e^2\,\sqrt{-a^5\,c^5}}{216\,a^5\,c^4}\right)^{1/3}+\ln\left(a^3\,c^3\,\left(-\frac{a^4\,c^2\,e^3-c\,d^3\,\sqrt{-a^5\,c^5}-3\,a^3\,c^3\,d^2\,e+3\,a\,d\,e^2\,\sqrt{-a^5\,c^5}}{a^5\,c^4}\right)^{1/3}-e\,x\,\sqrt{-a^5\,c^5}+a^2\,c^3\,d\,x\right)\,\Bigg(-$$

```
[In] int((d + e*x^3)/(a + c*x^6),x)
```

```
[Out] log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a
*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(-a^5*c^5)^(1/2) + a^2*c^3*
d*x)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3
*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4)
)^(1/3) - e*x*(-a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(-a^5*
c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(
1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*
e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(-a^5*c^5)^(1/2) +
 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e
- 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2
)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e -
3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(-a^5*c^5)^(1/2)
 - (a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a
*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*
e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2)
)/(a^5*c^4))^(1/3)*1i)/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e
^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))
/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2)
 - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(
-a^5*c^5)^(1/2) - 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) -
 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*
c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3
*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log(a^3
*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(-a^5*c^5)^(1/2) + 3^(1/2)*a^3*c
^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-
a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)
```

```
*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^
5*c^5)^(1/2))/(216*a^5*c^4))^(1/3)
```

3.2 $\int \frac{d+ex^3}{a-cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 323

$$\int \frac{d+ex^3}{a-cx^6}\,dx = -\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

```
[Out] 1/6*ln(a^(1/6)+c^(1/6)*x)*(d-e*a^(1/2)/c^(1/2))/a^(5/6)/c^(1/6)-1/12*ln(a^(
1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)*(d-e*a^(1/2)/c^(1/2))/a^(5/6)/c^(1/6)-1
/6*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)/a^(1/6)*3^(1/2))*(d-e*a^(1/2)/c^(1/2))/
a^(5/6)/c^(1/6)*3^(1/2)-1/6*ln(a^(1/6)-c^(1/6)*x)*(e*a^(1/2)+d*c^(1/2))/a^(
5/6)/c^(2/3)+1/12*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)*(e*a^(1/2)+d*c^
(1/2))/a^(5/6)/c^(2/3)+1/6*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)/a^(1/6)*3^(1/2)
)*(e*a^(1/2)+d*c^(1/2))/a^(5/6)/c^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1431, 206, 31, 648, 631, 210, 642}

$$\int \frac{d+ex^3}{a-cx^6}\,dx=\frac{\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{3}a^{5/6}c^{2/3}}-\frac{\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}}$$

```
[In] Int[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] -1/2*((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(a^(1/6) - 2*c^(1/6)*x)/(Sqrt[3]*a^(
1/6))])/(Sqrt[3]*a^(5/6)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(a^(1/6
) + 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/(2*Sqrt[3]*a^(5/6)*c^(2/3)) - ((Sqrt[c
]*d + Sqrt[a]*e)*Log[a^(1/6) - c^(1/6)*x])/(6*a^(5/6)*c^(2/3)) + ((d - (Sqr
t[a]*e)/Sqrt[c])*Log[a^(1/6) + c^(1/6)*x])/(6*a^(5/6)*c^(1/6)) - ((d - (Sqr
t[a]*e)/Sqrt[c])*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6
)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(
1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1431

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Dist[(d + e*q)/2, Int[1/(a + c*q*x^n), x], x] + Dist[(d -
 e*q)/2, Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{c}x^3}\,dx+\frac{1}{2}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^3}\,dx\\
&= \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x}\,dx}{6a^{2/3}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{2\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}{a^{2/3}-\sqrt{a}\sqrt[6]{c}x+\sqrt[3]{a}\sqrt[3]{c}x^2}\,dx}{6a^{2/3}}\\
&\quad+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}\,dx}{6a^{2/3}}+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{2\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x}{a^{2/3}+\sqrt{a}\sqrt[6]{c}x+\sqrt[3]{a}\sqrt[3]{c}x^2}\,dx}{6a^{2/3}}
\end{aligned}$$

$$= -\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}}$$
$$+ \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\int \frac{\sqrt{a}\sqrt[6]{c}+2\sqrt[3]{a}\sqrt[3]{c}x}{a^{2/3}+\sqrt{a}\sqrt[6]{c}x+\sqrt[3]{a}\sqrt[3]{c}x^2}\,dx}{12a^{5/6}c^{2/3}}$$
$$+ \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{a^{2/3}-\sqrt{a}\sqrt[6]{c}x+\sqrt[3]{a}\sqrt[3]{c}x^2}\,dx}{4\sqrt[3]{a}}$$
$$- \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{-\sqrt{a}\sqrt[6]{c}+2\sqrt[3]{a}\sqrt[3]{c}x}{a^{2/3}-\sqrt{a}\sqrt[6]{c}x+\sqrt[3]{a}\sqrt[3]{c}x^2}\,dx}{12a^{5/6}\sqrt[6]{c}} + \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{a^{2/3}+\sqrt{a}\sqrt[6]{c}x+\sqrt[3]{a}\sqrt[3]{c}x^2}\,dx}{4\sqrt[3]{a}}$$

$$= -\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}}$$
$$- \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}}$$
$$+ \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$
$$- \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{2a^{5/6}c^{2/3}}$$
$$+ \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{2a^{5/6}\sqrt[6]{c}}$$

$$= -\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}}$$
$$- \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}}$$
$$- \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}}$$
$$+ \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04

$$\int \frac{d+ex^3}{a-cx^6}\,dx$$

$$= \frac{-2\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)-2\sqrt{c}d\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{}$$

```
[In] Integrate[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] (-2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt
[3]] + 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))
/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/6)
- c^(1/6)*x] + 2*Sqrt[c]*d*Log[a^(1/6) + c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/
6) + c^(1/6)*x] - Sqrt[c]*d*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]
+ Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[c]*d*Log[
a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) + a^(1/6
)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6c-a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{\_R^5}}{6c}$
default	$-\frac{\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)e}{6c\left(\frac{a}{c}\right)^{\frac{1}{3}}}-\frac{\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)d}{6c\left(\frac{a}{c}\right)^{\frac{5}{6}}}+\frac{e\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{e\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{6a}+\frac{d\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln(}{}$

```
[In] int((e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/c*sum((_R^3*e+d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1613 vs. 2(223) = 446.

Time = 0.37 (sec) , antiderivative size = 1613, normalized size of antiderivative = 4.99

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
[Out] -1/12*(sqrt(-3) + 1)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e
^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*
d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 + sqrt(-3)*(a*c
^2*d^4 + 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a
*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3
)) + 1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d
^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*
a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 - sqrt(-3)*
(a*c^2*d^4 + 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e - a^4*c^2*e)*sqrt((c^2*
d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6 +
 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^
(1/3)) - 1/12*(sqrt(-3) + 1)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^
2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 +
 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 + sqrt(-
3)*(a*c^2*d^4 + 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqrt((c
^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt((c^2*d^6
 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2)
)^(1/3)) + 1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*
a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5
 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 - sqrt
(-3)*(a*c^2*d^4 + 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e - a^4*c^2*e)*sqrt(
(c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt((c^2*d
^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^
2))^(1/3)) + 1/6*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/
(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*d^3*
e^2 - 3*a^2*d*e^4)*x + (a^4*c^2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2
*e^4)/(a^5*c^3)) - a*c^2*d^4 - 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt((c^2*d^6 +
6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(
1/3)) + 1/6*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c
^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*d^3*e^2 -
3*a^2*d*e^4)*x - (a^4*c^2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/
(a^5*c^3)) + a*c^2*d^4 + 3*a^2*c*d^2*e^2)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d
^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))
```

Sympy [A] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.52

$$\int \frac{d+ex^3}{a-cx^6}\,dx = -\operatorname{RootSum}\left(46656t^6a^5c^4+t^3\left(-432a^4c^2e^3-1296a^3c^3d^2e\right)+a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6,\left(t\mapsto\right.\right.$$

```
[In] integrate((e*x**3+d)/(-c*x**6+a),x)
```

```
[Out] -RootSum(46656*_t**6*a**5*c**4 + _t**3*(-432*a**4*c**2*e**3 - 1296*a**3*c**
3*d**2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6
, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e + 6*_t*a**3*e**4 + 36*_t*a
**2*c*d**2*e**2 + 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 - 2*a*c*d**3*e**2 - c**2
*d**5))))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.97

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(x^2-x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqrt
(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) +
1/6*sqrt(3)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/sqrt
(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) +
1/12*(sqrt(c)*d + sqrt(a)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)
/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/12*(sqrt(c)*d - sq
rt(a)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sq
rt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 1/6*(sqrt(c)*d - sqrt(a)*e)*log(x + (sqr
t(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/6*(sqrt(c)*d +
 sqrt(a)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(
2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.94

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{e|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(-ac^5\right)^{\frac{1}{3}}} + \frac{\left(-ac^5\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}}c^3d-\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}}c^3d+\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}c^3d+\left(-ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}c^3d-\left(-ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

```
[Out] 1/6*e*abs(c)*log(x^2 + (-a/c)^(1/3))/(-a*c^5)^(1/3) + 1/3*(-a*c^5)^(1/6)*d*
arctan(x/(-a/c)^(1/6))/(a*c) + 1/6*((-a*c^5)^(1/6)*c^3*d - sqrt(3)*(-a*c^5)
^(2/3)*e)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/6*(
(-a*c^5)^(1/6)*c^3*d + sqrt(3)*(-a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(-a/
c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d + (-a*
c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4) - 1/
12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d - (-a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(-
a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.00

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \ln\left(a^3\,c^3\,{\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{a^5\,c^5}+3\,a^3\,c^3\,d^2\,e+3\,a\,d\,e^2\,\sqrt{a^5\,c^5}}{a^5\,c^4}\right)}^{1/3}+e\,x\,\sqrt{a^5\,c^5}+a^2\,c^3\,d\,x\right)\,{\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{a^5\,c^5}+3\,a^3\,c^3\,d^2\,e+3\,a\,d\,e^2\,\sqrt{a^5\,c^5}}{216\,a^5\,c^4}\right)}^{1/3}+\ln\left(a^3\,c^3\,{\left(-\frac{a^4\,c^2\,e^3-c\,d^3\,\sqrt{a^5\,c^5}+3\,a^3\,c^3\,d^2\,e-3\,a\,d\,e^2\,\sqrt{a^5\,c^5}}{a^5\,c^4}\right)}^{1/3}-e\,x\,\sqrt{a^5\,c^5}+a^2\,c^3\,d\,x\right)\,\left(-\frac{a^4\,c^2\,\cdots}{\cdots}\right.$$

```
[In] int((d + e*x^3)/(a - c*x^6),x)
```

```
[Out] log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*
d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x
)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5
*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5
*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)
 - e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2
) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log
(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e
^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(a^5*c^5)^(1/2) + 3^(1/2)*a^3*
c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a
^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*
(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c
^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(a^5*c^5)^(1/2) - (a^3*c^3*(-(a^4
*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1
/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i)
/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(
1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) +
log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*
d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(a^5*c^5)^(1/2) - 3^(1/2)*a
^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2
*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/
2)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^
5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^
5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3
) + 2*e*x*(a^5*c^5)^(1/2) + 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i
- 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1
/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3)
```

3.3 $\int \frac{d+ex^4}{a+cx^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 754

$$
\begin{aligned}
\int \frac{d+ex^4}{a+cx^8}\,dx = & -\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& +\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& +\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& -\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& +\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
& -\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
& -\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
& +\frac{\left(d+\sqrt{2}d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}
\end{aligned}
$$

```
[Out] -1/8*arctan((-2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(1
/2))*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))*(2-2^(1/2))^(1/2)/a^(7/8)/c^(5/8)+1
/8*arctan((2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(1/2)
)*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))*(2-2^(1/2))^(1/2)/a^(7/8)/c^(5/8)+1/4*
arctan((-2*c^(1/8)*x+a^(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*
(-e*a^(1/2)+d*(1-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)-1/4*
arctan((2*c^(1/8)*x+a^(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*(
-e*a^(1/2)+d*(1-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)+1/8*l
n(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*(-e*a^(1/2)+d*(1
-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/
4)*x^2+a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*(-e*a^(1/2)+d*(1-2^(1/2))*c^(1/
2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)+1/8*ln(a^(1/4)+c^(1/4)*x^2+a^(1/8)*
c^(1/8)*x*(2+2^(1/2))^(1/2))*(d+d*2^(1/2)-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8
)/(4+2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2+2^(1/
2))^(1/2))*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4+2*2^(1/2))
^(1/2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used

$= \{1429, 1183, 648, 632, 210, 642\}$

$$\int \frac{d+ex^4}{a+cx^8}\,dx = -\frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)}{8a^{7/8}c^{5/8}} + \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)}{8a^{7/8}c^{5/8}} + \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)}{8a^{7/8}c^{5/8}} - \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)}{8a^{7/8}c^{5/8}} + \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}} + \frac{\left(-\frac{\sqrt{a}e}{\sqrt{c}}+\sqrt{2}d+d\right)\log\left(\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}$$

```
[In] Int[(d + e*x^4)/(a + c*x^8),x]
```

```
[Out] -1/8*(Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[
2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(a^(7/8)*
c^(5/8)) + (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[
(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*
a^(7/8)*c^(5/8)) + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8
))])/(8*a^(7/8)*c^(5/8)) - (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]
]*a^(1/8))])/(8*a^(7/8)*c^(5/8)) + (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*L
og[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*
(2 - Sqrt[2])]*a^(7/8)*c^(5/8)) - (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Lo
g[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(
2 - Sqrt[2])]*a^(7/8)*c^(5/8)) - (((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log
[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2
```

```
 + Sqrt[2])]*a^(7/8)*c^(5/8)) + ((d + Sqrt[2]*d - (Sqrt[a]*e)/Sqrt[c])*Log[
a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2
+ Sqrt[2])]*a^(7/8)*c^(1/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q - (d - e*q^2)*x
^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Dist[1/(2*Sqrt[2]*c*q^3),
 Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n), x
], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &
& NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}+\left(-d+\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}+\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}d}{c^{3/8}}-\left(\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}}$$

$$+ \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}d}{c^{3/8}}+\left(\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}}$$

$$+ \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}d}{c^{3/8}}-\left(\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(-d+\frac{\sqrt{a}e}{\sqrt{c}}\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}}$$

$$+ \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}d}{c^{3/8}}+\left(\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(-d+\frac{\sqrt{a}e}{\sqrt{c}}\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}}$$

$$
\begin{aligned}
&= -\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}a^{3/4}c^{3/4}} \\
&- \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}a^{3/4}c^{3/4}} \\
&+ \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
&- \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
&+ \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}a^{3/4}c^{3/4}} \\
&+ \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}a^{3/4}c^{3/4}} \\
&- \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
&+ \frac{\left(d+\sqrt{2}d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}
\end{aligned}
$$

$$= \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$- \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$- \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$+ \frac{\left(d+\sqrt{2}d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}$$
$$+ \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\operatorname{Subst}\left(\int\frac{1}{-\frac{\left(2-\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx,x,-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$
$$+ \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\operatorname{Subst}\left(\int\frac{1}{-\frac{\left(2-\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx,x,\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$
$$- \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\operatorname{Subst}\left(\int\frac{1}{-\frac{\left(2+\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx,x,-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$
$$- \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\operatorname{Subst}\left(\int\frac{1}{-\frac{\left(2+\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx,x,\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

$$= -\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$+\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$-\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$+\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$-\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}}$$
$$+\frac{\left(d+\sqrt{2}d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^4}{a+cx^8}\,dx$$
$$=\frac{-2\sqrt[8]{a}\arctan\left(\cot\left(\frac{\pi}{8}\right)-\frac{\sqrt[8]{c}x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(\sqrt{a}e\cos\left(\frac{\pi}{8}\right)+\sqrt{c}d\sin\left(\frac{\pi}{8}\right)\right)+2\sqrt[8]{a}\arctan\left(\cot\left(\frac{\pi}{8}\right)+\frac{\sqrt[8]{c}x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)}{}$$

```
[In] Integrate[(d + e*x^4)/(a + c*x^8),x]
```

```
[Out] (-2*a^(1/8)*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Co
s[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + 2*a^(1/8)*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Cs
c[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) - a^(1/8)*Log
```

```
[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8
] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^
(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Lo
g[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[P
i/8]) + Sqrt[a]*e*Sin[Pi/8]) - a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8
)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) + 2*A
rcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/
8] - a^(5/8)*e*Sin[Pi/8]) + 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi
/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]))/(8*a*c^(5/8))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.05

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\left(\_R^4 e+d\right)\ln\left(x-\_R\right)}{\_R^7}}{8c}$	34
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\left(\_R^4 e+d\right)\ln\left(x-\_R\right)}{\_R^7}}{8c}$	34

```
[In] int((e*x^4+d)/(c*x^8+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*sum((_R^4*e+d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2749 vs. $2(514) = 1028$.

Time = 0.56 (sec) , antiderivative size = 2749, normalized size of antiderivative = 3.65

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4
*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3
*c^2))*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x + (a^
5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)
*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^
```

2)))) - 1/8*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e
^3)/(a^3*c^2)))*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)
*x - (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12
*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*
c*d*e^4)*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)
/(a^3*c^2)))) - 1/8*sqrt(-sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 +
38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e +
4*a*d*e^3)/(a^3*c^2)))*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a
^3*e^6)*x + (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e
^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d^3*e^2
- a^3*c*d*e^4)*sqrt(-sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*
a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*
d*e^3)/(a^3*c^2)))) + 1/8*sqrt(-sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6
*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^
3*e + 4*a*d*e^3)/(a^3*c^2)))*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e
^4 + a^3*e^6)*x - (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d
^3*e^2 - a^3*c*d*e^4)*sqrt(-sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2
+ 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e
+ 4*a*d*e^3)/(a^3*c^2)))) + 1/8*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^
2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e
- 4*a*d*e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2
*e^4 + a^3*e^6)*x + (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c
^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2
*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d
*e^3)/(a^3*c^2))^(1/4)) - 1/8*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2
+ 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e -
4*a*d*e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e
^4 + a^3*e^6)*x - (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d
^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e
^3)/(a^3*c^2))^(1/4)) - 1/8*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 3
8*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*
a*d*e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4
+ a^3*e^6)*x + (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^
4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d^3*
e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/
(a^3*c^2))^(1/4)) + 1/8*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^
2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*
e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^

```
3*e^6)*x - (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d^3*e^2
- a^3*c*d*e^4)*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4
*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3
*c^2))^(1/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**4+d)/(c*x**8+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \int \frac{ex^4+d}{cx^8+a}\,dx$$

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="maxima")
[Out] integrate((e*x^4 + d)/(c*x^8 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.79

$$\int \frac{d+ex^4}{a+cx^8}\,dx = -\frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} - \frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} + \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} + \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} - \frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a} + \frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a} + \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a} - \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*a
rctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)
))/a - 1/8*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(a/c)^(1
/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)
^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(a/c)^(5/8) + d*sqrt(-sqrt(2) + 2)*(a
/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)
*(a/c)^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(a/c)^(5/8) + d*sqrt(-sqrt(2) +
 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2
) + 2)*(a/c)^(1/8)))/a - 1/16*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) - d*sqrt(sq
rt(2) + 2)*(a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(
1/4))/a + 1/16*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(a/c
)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a + 1/16*
```

```
(e*sqrt(sqrt(2) + 2)*(a/c)^(5/8) + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^
2 + x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a - 1/16*(e*sqrt(sqrt(2
) + 2)*(a/c)^(5/8) + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqrt(-sq
rt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 2510, normalized size of antiderivative = 3.33

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^4)/(a + c*x^8),x)
[Out] (atan((c^3*d^6*x - a^3*e^6*x + a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*x
*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*((a
^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^
5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*
((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) - 2*a^2*c
^2*d^3*e^2*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^
4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4
) - 3*a^3*c*d*e^4*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4
*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5
))^(1/4)))*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^
4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4
))/4 - (atan((a^3*e^6*x - c^3*d^6*x - a*c^2*d^4*e^2*x + a^2*c*d^2*e^4*x + (
2*d*e*x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^
3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*
d^5*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*
e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^
5*c^3*e*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*
d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4)
- 2*a^2*c^2*d^3*e^2*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2)
+ 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*
c^5))^(1/4) - 3*a^3*c*d*e^4*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5
)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2
))/(a^7*c^5))^(1/4)))*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2
) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^
7*c^5))^(1/4))/4 - atan((c^3*d^6*x*1i - a^3*e^6*x*1i + a*c^2*d^4*e^2*x*1i -
 a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)
)*2i)/(a^3*c^2))/(a*c^3*d^5*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)
```

```
)/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c
^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1
/2))/(a^7*c^5))^(5/4) - 2*a^2*c^2*d^3*e^2*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*
d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-
a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4*((a^2*e^4*(-a^7*c^5)^(1/2)
 + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2
*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*
d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-
a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i + atan((a^3*e^6*x*1i - c^3*d^6*x*1
i - a*c^2*d^4*e^2*x*1i + a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^4*(-a^7*c^5)^(1
/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*
d^2*e^2*(-a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*(-(a^2*e^4*(-a^7*c^5)^(
1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c
*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a^2*e^4*(-a^7*c^
5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6
*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) - 2*a^2*c^2*d^3*e^2*(-(a^2*
e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4
*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e -
 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*(-(a^
2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5
*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i
```

3.4 $\int \frac{d+ex^4}{a-cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 329

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

```
[Out] 1/8*arctan(-1+c^(1/8)*x*2^(1/2)/a^(1/8))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1
/8)*2^(1/2)+1/8*arctan(1+c^(1/8)*x*2^(1/2)/a^(1/8))*(d-e*a^(1/2)/c^(1/2))/a
^(7/8)/c^(1/8)*2^(1/2)-1/16*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*2^(1/2
))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8)*2^(1/2)+1/16*ln(a^(1/4)+c^(1/4)*x^
2+a^(1/8)*c^(1/8)*x*2^(1/2))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8)*2^(1/2)+
1/4*arctan(c^(1/8)*x/a^(1/8))*(e*a^(1/2)+d*c^(1/2))/a^(7/8)/c^(5/8)+1/4*arc
tanh(c^(1/8)*x/a^(1/8))*(e*a^(1/2)+d*c^(1/2))/a^(7/8)/c^(5/8)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1431, 218, 214, 211, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{d+ex^4}{a-cx^8}\,dx = {} & \frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{4a^{7/8}c^{5/8}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}} \\ & + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{4a^{7/8}c^{5/8}} \\ & - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}} \\ & + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}\end{aligned}$$

```
[In] Int[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*c^(5/8)) -
 ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 - (Sqrt[2]*c^(1/8)*x)/a^(1/8)])/(4*Sqr
t[2]*a^(7/8)*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 + (Sqrt[2]*c^(1
/8)*x)/a^(1/8)])/(4*Sqrt[2]*a^(7/8)*c^(1/8)) + ((Sqrt[c]*d + Sqrt[a]*e)*Arc
Tanh[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*c^(5/8)) - ((d - (Sqrt[a]*e)/Sqrt[c])
*Log[a^(1/4) - Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)
*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8
)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)*c^(1/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1431

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Dist[(d + e*q)/2, Int[1/(a + c*q*x^n), x], x] + Dist[(d -
 e*q)/2, Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{c}x^4}\,dx+\frac{1}{2}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^4}\,dx$$

$$= \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{a+\sqrt{a}\sqrt{c}x^4}\,dx}{4\sqrt[4]{a}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt[4]{a}+\sqrt[4]{c}x^2}{a+\sqrt{a}\sqrt{c}x^4}\,dx}{4\sqrt[4]{a}}$$
$$+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[4]{a}-\sqrt[4]{c}x^2}\,dx}{4a^{3/4}}+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[4]{a}+\sqrt[4]{c}x^2}\,dx}{4a^{3/4}}$$

$$= \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}$$
$$+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8a^{3/4}\sqrt[4]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8a^{3/4}\sqrt[4]{c}}$$
$$-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\frac{\sqrt{2}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{-\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}-x^2}\,dx}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\frac{\sqrt{2}\sqrt[8]{a}}{\sqrt[8]{c}}-2x}{-\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}-x^2}\,dx}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

$$= \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}$$
$$-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$
$$+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$
$$+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}$$
$$-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

$$= \frac{(\sqrt{c}d+\sqrt{a}e)\tan^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}$$
$$+ \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{(\sqrt{c}d+\sqrt{a}e)\tanh^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}$$
$$- \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$
$$+ \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.29

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \frac{\left(\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4ac^{5/8}}$$
$$- \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\arctan\left(\frac{-\sqrt{2}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2}\sqrt[8]{a}}\right)}{4\sqrt{2}ac^{5/8}}$$
$$- \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\arctan\left(\frac{\sqrt{2}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2}\sqrt[8]{a}}\right)}{4\sqrt{2}ac^{5/8}}$$
$$- \frac{\left(\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\log\left(\sqrt[8]{a}-\sqrt[8]{c}x\right)}{8ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\sqrt{c}d-a^{5/8}e\right)\log\left(\sqrt[8]{a}+\sqrt[8]{c}x\right)}{8ac^{5/8}}$$
$$+ \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\log\left(\sqrt[4]{a}-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}ac^{5/8}}$$
$$- \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\log\left(\sqrt[4]{a}+\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}ac^{5/8}}$$

```
[In] Integrate[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a*c^(5/8))
 - ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*ArcTan[(-(Sqrt[2]*a^(1/8)) + 2*c^(1/
8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sqrt[2]*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) +
a^(5/8)*e)*ArcTan[(Sqrt[2]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sq
rt[2]*a*c^(5/8)) - ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*Log[a^(1/8) - c^(1/8)*x
])/(8*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) - a^(5/8)*e)*Log[a^(1/8) + c^(1/8
)*x])/(8*a*c^(5/8)) + ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) - Sqr
t[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a*c^(5/8)) - ((-(a^(1/8)*
Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x
^2])/(8*Sqrt[2]*a*c^(5/8))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}-a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c}$	36
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}-a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c}$	36

```
[In] int((e*x^4+d)/(-c*x^8+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c*sum((_R^4*e+d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2741 vs. 2(220) = 440.

Time = 0.64 (sec) , antiderivative size = 2741, normalized size of antiderivative = 8.33

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e
^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c
^2)))*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x + (a^5
*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2
*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*s
qrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 +
12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))
) - 1/8*sqrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a
^3*c^2)))*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x -
(a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c
*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^
4)*sqrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^
2)))) - 1/8*sqrt(-sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*
c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^
3)/(a^3*c^2)))*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)
```

```
*x + (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 + a^3*c
*d*e^4)*sqrt(-sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*
d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(
a^3*c^2)))) + 1/8*sqrt(-sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 3
8*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*
a*d*e^3)/(a^3*c^2)))*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^
3*e^6)*x - (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 +
 a^3*c*d*e^4)*sqrt(-sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^
2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*
e^3)/(a^3*c^2)))) + 1/8*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e
^3)/(a^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^
3*e^6)*x + (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 -
 a^3*c*d*e^4)*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e
^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c
^2))^(1/4)) - 1/8*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a
^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)
*x - (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c
*d*e^4)*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 1
2*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(
1/4)) - 1/8*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^
2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x +
(a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c
*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^
4)*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^
3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)
) + 1/8*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 +
12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^
(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x - (a^5
*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2
*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(
-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*
d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**4+d)/(-c*x**8+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \int -\frac{ex^4+d}{cx^8-a}\,dx$$

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="maxima")
[Out] -integrate((e*x^4 + d)/(c*x^8 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(220) = 440$.

Time = 0.44 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.90

$$\int \frac{d+ex^4}{a-cx^8}\,dx$$
$$= -\frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$- \frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$+ \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$+ \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$- \frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$
$$+ \frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$
$$+ \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$
$$- \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*(e*sqrt(-sqrt(2) + 2)*(-a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))
*arctan((2*x + sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(-a/c)^(
1/8)))/a - 1/8*(e*sqrt(-sqrt(2) + 2)*(-a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(-a
/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2
)*(-a/c)^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8) + d*sqrt(-sqrt(2
) + 2)*(-a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(-s
qrt(2) + 2)*(-a/c)^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8) + d*sq
rt(-sqrt(2) + 2)*(-a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(-a/c)^(1/8)
)/(sqrt(-sqrt(2) + 2)*(-a/c)^(1/8)))/a - 1/16*(e*sqrt(-sqrt(2) + 2)*(-a/c)^
(5/8) - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2) + 2)*(-a
/c)^(1/8) + (-a/c)^(1/4))/a + 1/16*(e*sqrt(-sqrt(2) + 2)*(-a/c)^(5/8) - d*s
qrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(-a/c)^(1/8) +
 (-a/c)^(1/4))/a + 1/16*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8) + d*sqrt(-sqrt(2)
 + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1
/4))/a - 1/16*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8) + d*sqrt(-sqrt(2) + 2)*(-a/
c)^(1/8))*log(x^2 - x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1/4))/a
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 2438, normalized size of antiderivative = 7.41

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^4)/(a - c*x^8),x)
```

```
[Out] (atan((a^3*e^6*x + c^3*d^6*x - a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*x
*(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a
^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*((a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^
2*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*
d*e^4*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e
 + 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^
2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c
^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4))/4 - (atan((a*c^
2*d^4*e^2*x - c^3*d^6*x - a^3*e^6*x + a^2*c*d^2*e^4*x + (2*d*e*x*(a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*(-(a^2*e^4*(a^7*c
^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 + 6
*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a^2*e^4*(a^7*
c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 +
6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^2*(-(a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4*(-
(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^
5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*(-(a^2*e^4*
(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e
^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4))/4 - atan((a^3*e^6*x*1
i + c^3*d^6*x*1i - a*c^2*d^4*e^2*x*1i - a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^
4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d
*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*((a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^2*((a
^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*
c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4
*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*
```

```
a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^2*e^4
*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*
e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i + atan((a*c^2
*d^4*e^2*x*1i - c^3*d^6*x*1i - a^3*e^6*x*1i + a^2*c*d^2*e^4*x*1i + (d*e*x*(
a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5
*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*(-(a^
2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a
^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*
c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3
*e^2*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e
 - 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^
3*c*d*e^4*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*
d^3*e - 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))
*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4
*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i
```

3.5 $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 791

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} - \frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} + \frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} - \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} - \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}$$

```
[Out] -1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2))/(2*d^(
1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(
1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/
2))/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(
2*d*e-b)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-(2*d*
e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2)+1/8*ln
(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(
2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2
```

```
)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)
)/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)
+(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1
/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)-1/8*ln(d^(1/2
)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/
2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e
^(1/2)+(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}$$

```
[In] Int[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt
[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + S
qrt[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] - 2*
```

```
Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*
Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d
]*Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[
Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sq
rt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d
]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqr
t[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b
 + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{-b+2de}x^2}{e}+x^4}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{-b+2de}x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}$$

$$+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}$$

$$+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}$$

$$-\frac{\int \frac{-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}$$

$$-\frac{\int \frac{-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} - \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}{e}-x^2}\,dx, x, -\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}{e}-x^2}\,dx, x, \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}{e}-x^2}\,dx, x, -\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}{e}-x^2}\,dx, x, \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}$$
$$- \frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}$$
$$+ \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}$$
$$- \frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}$$
$$+ \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.08

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \frac{1}{4}\mathrm{RootSum}\left[d^2+b\#1^4 + e^2\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1^3+2e^2\#1^7}\&\right]$$

```
[In] Integrate[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
[Out] RootSum[d^2 + b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b
*#1^3 + 2*e^2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(e^{2}\_Z^{8}+\_Z^{4}b+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	53
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(e^{2}\_Z^{8}+\_Z^{4}b+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	53

```
[In] int((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^2+_Z^4*b+d
^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. 2(581) = 1162.

Time = 0.32 (sec) , antiderivative size = 2461, normalized size of antiderivative = 3.11

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e -
b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*
b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^
2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) +
b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b
)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b
*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b
^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4
)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e
^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b
^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^
2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)
) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(-sqrt(1/2)*sqrt(-((4
*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2
 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x
 + 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*
```

```
e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(-sqrt(1/2)*sqrt(-((4
*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2
 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) - 1/4*
sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/
(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d
^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*
sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*
sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/
(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d
^3*e + b^2*d^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*
d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4))
- b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2
+ 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*
d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^
2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) -
b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e
^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b
^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2
*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 +
12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2
 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2
*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(-sq
rt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^
3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^
2*d^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(-sqr
t(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3
 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2
*d^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt
(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d
^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3
*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b
^3*d^4)) + b)*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(
-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^
4*e^2 + 4*b*d^3*e + b^2*d^2))))
```

Sympy [A] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.17

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$$

$$= \text{RootSum}\left(t^8 \cdot \left(65536b^4d^2 + 524288b^3d^3e + 1572864b^2d^4e^2 + 2097152bd^5e^3 + 1048576d^6e^4\right) + t^4 \cdot \left(256b^3\right.\right.$$

```
[In] integrate((e*x**4+d)/(e**2*x**8+b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 + 524288*b**3*d**3*e + 1572864*b**2*d**4*e**
2 + 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(256*b**3 + 1024*b**2*
d*e + 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*d**2
 + 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 + 4*_t*b + 4*_t*d*e)/e)))
```

Maxima [F]

$$\int \frac{d + ex^4}{d^2 + bx^4 + e^2x^8} \, dx = \int \frac{ex^4 + d}{e^2x^8 + bx^4 + d^2} \, dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d + ex^4}{d^2 + bx^4 + e^2x^8} \, dx = \int \frac{ex^4 + d}{e^2x^8 + bx^4 + d^2} \, dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 10409, normalized size of antiderivative = 13.16

$$\int \frac{d + ex^4}{d^2 + bx^4 + e^2x^8} \, dx = \text{Too large to display}$$

```
[In] int((d + e*x^4)/(b*x^4 + d^2 + e^2*x^8),x)
```

```
[Out] 2*atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e
^12) + (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e
)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)
))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b
^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 -
 65536*b^2*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2
*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 +
 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d
^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 19660
```

8*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)
^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3
*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 + 256*b*d^6*e
^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d
*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*
d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 - 4*b^4*
d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 + ((b - 2*d*e)*(b +
2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b
^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 327
68*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 +
20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((
b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d
^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*
b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^
13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*
e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2
)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4
*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2
*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*
e^2)))^(1/4))/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^
2*d^4*e^12) + (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*
b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d
^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 -
 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6
*e^12 - 65536*b^2*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) +
4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^
5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 409
6*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11
+ 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b +
 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*
b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 + 256*
b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(
b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 +
 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^1
3 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 + ((b - 2*
d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*
e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d
^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (
-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(
b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^
9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*
b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2

+ 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 +
64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e
^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*1i))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*
b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*
e^3 + 24*b^2*d^4*e^2)))^(1/4) - atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 +
24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(
1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^
14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d
^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((b - 2*d*e)*
(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4
+ 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 -
262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^1
0 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*(-(b^3
 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d
^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 25
6*d^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 +
((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2
+ 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x
*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(
b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^
4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(
(x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9
 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2
*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b
^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^
4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4
096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*
e^12 - 196608*b^2*d^8*e^13))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4
*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5
*e^3 + 24*b^2*d^4*e^2)))^(3/4) + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3
*e^10 + 64*b^3*d^4*e^11))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*
d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^
3 + 24*b^2*d^4*e^2)))^(1/4)*1i)/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^
3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2)
+ 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*
d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 +
1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^
11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((b - 2*d*e)*(b +
2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b
^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 26214
4*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 4
9152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*(-(b^3 + ((

b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7
*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 + ((b -
2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) - (x*(32*b*d
^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 + ((
b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(6553
6*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240
*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^1
3) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*
d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 1
96608*b^2*d^8*e^13))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e
^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(3/4) + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 +
64*b^3*d^4*e^11))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
+ 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(1/4)))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*
e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 +
24*b^2*d^4*e^2)))^(1/4)*2i + atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*
b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2
) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^
(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^
14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d
^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 -
32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10
+ 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 -
((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2
+ 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d
^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 - ((b
- 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 1
6*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x*(3
2*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3
- ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d
^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-
(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b
^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*
(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9
- 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b
^2*d^8*e^13) + x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 20
48*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^

12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*
d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^
3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^
10 - 64*b^3*d^4*e^11))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2
*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 +
 24*b^2*d^4*e^2)))^(1/4)*1i)/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d
^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4
*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5
*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2)
 + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b
*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 +
4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^
11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 - 32768
*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20
480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b -
 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^
14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 - ((b - 2*
d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) - (x*(32*b*d^5*
e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 - ((b -
 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - (
(b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*
d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152
*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e
^13) + x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d
^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 655
36*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
+ 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*
b^3*d^4*e^11))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4
*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*
d^4*e^2)))^(1/4)))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*
b^2*d^4*e^2)))^(1/4)*2i - 2*atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b
^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2)
 + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b
*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(
1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^1
4 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^
6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i + x*(65536*d^9*e^15
- 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^

10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3
- ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^
2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i +
256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*(-(b
^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4
*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) +
(x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (
-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(
b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^
4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196
608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*
e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^
3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2
) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16
*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(
1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4))/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10
 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5
)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e
 + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*
e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d
^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*
d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152
*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i + x*(65536*d^
9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5
*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*
(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*
(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4
)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1
i)*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
1/4)*1i - (x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4
*e^12) - (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d
e)/(512(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^
2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b
^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^
4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4
096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*
e^12 - 196608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 102
4*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11
+ 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*

```
e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d
^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6
*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*(-(b^3 - ((b - 2*d*e)*(b + 2
*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^
3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i))*(-(b^3 - ((b - 2*d*e)
*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4
 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
```

3.6 $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 791

$$\begin{aligned}\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx =& -\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}-\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} \\ &+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} \\ &-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} \\ &+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} \\ &-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} \\ &+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\end{aligned}$$

```
[Out] -1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/(2*d^(
1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(
```

```
1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/
2))/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(
2*d*e-f)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-(2*d*
e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)+1/8*ln
(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(
2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2
)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)
)/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)
+(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1
/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)-1/8*ln(d^(1/2
)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/
2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e
^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}$$

```
[In] Int[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[
2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqr
t[2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] - 2*Sqrt
[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]
*Sqrt[e] - Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e
- f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*
Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]
]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sq
rt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e]
- Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqr
t[2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + Log[
Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqr
t[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
```

```
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}x^2}{e}+x^4}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} - \frac{\int \frac{-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} - \frac{\int \frac{-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}$$

$$= -\frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de - f}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de - f}}}$$

$$+ \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de - f}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de - f}}}$$

$$- \frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de - f}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de - f}}}$$

$$+ \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de - f}}x + \sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de - f}}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}{e} - x^2}\,dx, x, -\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}} + 2x\right)}{4\sqrt{d}\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}{e} - x^2}\,dx, x, \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}} + 2x\right)}{4\sqrt{d}\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}{e} - x^2}\,dx, x, -\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}} + 2x\right)}{4\sqrt{d}\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}{e} - x^2}\,dx, x, \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}} + 2x\right)}{4\sqrt{d}\sqrt{e}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}$$
$$- \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}$$
$$+ \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}$$
$$- \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}$$
$$+ \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.08

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \frac{1}{4}\text{RootSum}\Big[d^2+f\#1^4 + e^2\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{f\#1^3+2e^2\#1^7}\&\Big]$$

```
[In] Integrate[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 + f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(f
*#1^3 + 2*e^2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}f}\right)}{4}$	53
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}f}\right)}{4}$	53

```
[In] int((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^2+_Z^4*f+d
^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. 2(581) = 1162.

Time = 0.31 (sec) , antiderivative size = 2461, normalized size of antiderivative = 3.11

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e -
f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*
d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2*f^
2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) +
f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f
)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d
^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d
^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3
)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e
^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d
^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^
2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)
) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(-sqrt(1/2)*sqrt(-((4
*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f
 + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x
 + 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*
```

```
e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(-sqrt(1/2)*sqrt(-((4
*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f
 + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))) - 1/4*
sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/
(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3
*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*
sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*
sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/
(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3
*e*f + d^2*f^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*
f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3))
- f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2
+ 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*
e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^
2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) -
f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e
^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d
^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2
*(2*d*e + (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 +
12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2
 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5
*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(-sq
rt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^
3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^
2*f^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(-sqr
t(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3
 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2
*f^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt
(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d
^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 + 4*d^3*e
*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d
^4*f^3)) + f)*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(
-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^
4*e^2 + 4*d^3*e*f + d^2*f^2))))
```

Sympy [A] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.17

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$$

$$= \operatorname{RootSum}\left(t^8\cdot\left(1048576d^6e^4+2097152d^5e^3f+1572864d^4e^2f^2+524288d^3ef^3+65536d^2f^4\right)+t^4\cdot\left(1024d\right.\right.$$

```
[In] integrate((e*x**4+d)/(e**2*x**8+f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 + 2097152*d**5*e**3*f + 1572864*d**4*e**2*
f**2 + 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(1024*d**2*e**2*f + 10
24*d*e*f**2 + 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*e**2
 + 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e + 4*_t*f)/e)))
```

Maxima [F]

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8+fx^4+d^2}\,dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8+fx^4+d^2}\,dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 10411, normalized size of antiderivative = 13.16

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^4)/(f*x^4 + d^2 + e^2*x^8),x)
```

```
[Out] 2*atan((((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f
^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^
2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048
*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3
 - 65536*d^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2
*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f
 + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3
*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196
608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*
```

e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e
*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 + 256*d^6*e
^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^
10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d
*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*
e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + ((-(f^3 + ((f - 2*d*e)*(f
+ 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8
*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 3
2768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5
+ 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144
*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 4915
2*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13
*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*
f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f
^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e
^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*
e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e
f^2)/(512(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4))/(((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f +
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7
- 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e
^12*f^3 - 65536*d^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2)
+ 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^
5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4
096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^
4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f
+ 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 +
8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 + 25
6*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4
*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(
f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 +
8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - ((-(f^3 + ((f -
2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^
9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4
*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) +
(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*
f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 19660
8*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*
f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24

*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4
+ 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3
 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2
*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 2
4*d^4*e^2*f^2)))^(1/4)*1i))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*
d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^
3*f + 24*d^4*e^2*f^2)))^(1/4) - atan((((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)
^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3
+ 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^
14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5
*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 + ((f - 2*d*e
)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15
- 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*
f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*(-(f
^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*
d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) -
256*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*
d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i + ((
-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(
16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)
((x(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f
^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d
^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 -
 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^
12*f^3 - 196608*d^8*e^13*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) +
 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5
*e^3*f + 24*d^4*e^2*f^2)))^(3/4) + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e
^10*f^4 + 64*d^4*e^11*f^3) - x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^1
1*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^
2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*
f + 24*d^4*e^2*f^2)))^(1/4)*1i)/(((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2
) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*
d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f
+ 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11
*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 + ((f - 2*d*e)*(f
+ 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8
*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262
144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 +
 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*(-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e

^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d
^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*d^5*e
^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f -
 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 +
d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) - ((-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65
536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 102
40*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*
f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2
)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)
))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^
4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 -
 196608*d^8*e^13*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e
^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(3/4) + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4
+ 64*d^4*e^11*f^3) - x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 -
48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f
+ 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d
^4*e^2*f^2)))^(1/4)))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^
2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*2i - atan((((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1
/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5
)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3
 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^
14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6
*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2) + x*(65536*d^9*e^15
- 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f
^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3
- ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6
*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256
*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*d^5
*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f
 - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - ((-(f
^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*
d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((
(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*
(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4
)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f
^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608
*d^8*e^13*f^2) - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 -
2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12
*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*

```
d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^
3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10
*f^4 - 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f
^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e
^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(1/4)*1i)/(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) +
 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5
*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/
2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32
*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f
+ 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11
*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2) + x*(65536*d^9*e^15 - 327
68*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 +
20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f
 - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*
e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*d^5*e^13
*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*
d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2
*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + ((-(f^3 - ((f
 - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 -
 ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*
e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(26214
4*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 491
52*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^1
3*f^2) - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3
*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 6
5536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*
f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24
*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 6
4*d^4*e^11*f^3) + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*
d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f
 + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*
d^4*e^2*f^2)))^(1/4)*2i - 2*atan((((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/
2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32
*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)
^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3
+ 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^1
4*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*
e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i + x*(65536*d^9*e^1
5 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10
*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^
```

3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d
^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i
+ 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i + x
*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f
^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*
d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) -
((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*
e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 1
96608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^
8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*
d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1
/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f -
16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 +
24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(
1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 +
32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4))/(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)
^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f
^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*
d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3
*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*
d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 491
52*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i + x*(65536*
d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d
^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2)
)*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(51
2*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3
/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)
1i + x(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2
))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
1/4)*1i + ((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e
f^2)/(512(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 -
4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^
12*f^3 - 196608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1
024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^
4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*
d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3

```
*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6
*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f - 4*d^2*
e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2
*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^
3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i))*(-(f^3 - ((f - 2*d*e)
*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4
 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)
```

3.7 $\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 349

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} - \frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

```
[Out] -1/2*arctan(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2))*e^(
1/2)*2^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2)-1/2*
arctanh(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2))*e^(1/2)
*2^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2)-1/2*arct
an(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2))*e^(1/2)*2^(1
/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2)-1/2*arctanh(x
*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2))*e^(1/2)*2^(1/2)/
(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1433, 1107, 213, 209}

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} - \frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

```
[In] Int[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] -((Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e
]]])/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]])) - (
Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]]
)/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]) - (Sqrt
[e]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]])/(
Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]) - (Sqrt[e]
*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]])/(Sqr
t[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{b+2de}x^2}{e}+x^4}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{b+2de}x^2}{e}+x^4}\,dx}{2e}\\
&= \frac{\int \frac{1}{-\frac{\sqrt{b-2de}}{2e}-\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}}-\frac{\int \frac{1}{\frac{\sqrt{b-2de}}{2e}-\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}}\\
&\quad+\frac{\int \frac{1}{-\frac{\sqrt{b-2de}}{2e}+\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}}-\frac{\int \frac{1}{\frac{\sqrt{b-2de}}{2e}+\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}}\\
&= -\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\\
&\quad-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.20

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx=\frac{1}{4}\text{RootSum}\Big[d^2-b\#1^4+e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{-b\#1^3+2e^2\#1^7}\&\Big]$$

```
[In] Integrate[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
[Out] RootSum[d^2 - b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(-
(b*#1^3) + 2*e^2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^2\_Z^8-\_Z^4b+d^2\right)}\frac{\left(-\_R^4e-d\right)\ln\left(x-\_R\right)}{-2\_R^7e^2+\_R^3b}\right)}{4}$	57
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^2\_Z^8-\_Z^4b+d^2\right)}\frac{\left(-\_R^4e-d\right)\ln\left(x-\_R\right)}{-2\_R^7e^2+\_R^3b}\right)}{4}$	57

```
[In] int((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((-_R^4*e-d)/(-2*_R^7*e^2+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^2-_Z^4*b
+d^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2453 vs. 2(261) = 522.

Time = 0.33 (sec) , antiderivative size = 2453, normalized size of antiderivative = 7.03

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b
)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b
*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2
)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b
)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/
(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d
^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*
d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4))
+ b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2
- 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*
d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^
2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) +
b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(-sqrt(1/2)*sqrt(((4*d^4*
e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*
b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x + 1/
2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 -
```

```
 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b
^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(-
sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*
e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e +
b^2*d^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(
2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(-s
qrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e
^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b
^2*d^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sq
rt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4
*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 - 4*b*d
^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
 b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqr
t(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*
d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 -
4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^
5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d
*e - (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*
d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4
*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5
*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(-sqrt(1
/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 -
 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d
^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e
 + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(-sqrt(1
/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 -
 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d
^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(
-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^
4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 - 4*b*d^3*
e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^
3*d^4)) - b)*sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(
-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^
4*e^2 - 4*b*d^3*e + b^2*d^2))))
```

Sympy [A] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.39

$$\int \frac{d + ex^4}{d^2 - bx^4 + e^2x^8}\,dx$$

$$= \operatorname{RootSum}\left(t^8 \cdot \left(65536b^4d^2 - 524288b^3d^3e + 1572864b^2d^4e^2 - 2097152bd^5e^3 + 1048576d^6e^4\right) + t^4\left(-256b^3 -\right.\right.$$

```
[In] integrate((e*x**4+d)/(e**2*x**8-b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 - 524288*b**3*d**3*e + 1572864*b**2*d**4*e**
2 - 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(-256*b**3 + 1024*b**2
*d*e - 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*d**
2 - 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 - 4*_t*b + 4*_t*d*e)/e)))
```

Maxima [F]

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8-bx^4+d^2}\,dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8-bx^4+d^2}\,dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 10337, normalized size of antiderivative = 29.62

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x)
```

```
[Out] 2*atan(((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e
^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)
/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2))
)^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^
6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 -
65536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e
^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3
*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*
```

b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))
^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
- 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 - 256*b*d^6*e^13
+ 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e
))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*
e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 + 4*b^4*d^2*
e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d
*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^
3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*
d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480
*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*
d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^
15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^
5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i
)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512
*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/
4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*
1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
1/4))/((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^
12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6
*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 6
5536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^
2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24
*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*
e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b
^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^
(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e -
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 - 256*b*d^6*e^13
+ 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e)
)^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
- 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^13 + 4*b^4*d^
2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2
*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*
d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*
b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 204
80*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b -
2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*
e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*
d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*
1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(5

```
12*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11
)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*1i))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2
*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*
e^2)))^(1/4) - atan(((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 +
 48*b^2*d^4*e^12) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2
 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*
b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*
e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^
3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2
) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 -
 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e
^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*((b^3 + ((b - 2*d*e)^5*(b
 + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*
b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d
^6*e^13 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 + ((b - 2*d*e)^5*(b + 2
*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*
d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x*(32*b*d^5*e^13 + 4*b
^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 + ((b - 2*d*e)^5*(
b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8
*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 3
2768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10
 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) - ((b^3 + (
(b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 +
 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*
d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152
*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e
^13))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(3/4) + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)
)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512
*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/
4)*1i)/((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e
^12) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)
/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2))
)^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^
6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 -
65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e
^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3
*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*
b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1
```

/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 3
2*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^13 + 16*
b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2)
+ 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*
d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) - (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24
*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2
) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14
- 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*
e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b
 + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*
b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 2621
44*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 +
49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*((b^3 + ((
b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) + 256*d^7
*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*((b^3 + ((b -
2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)))*((b^3 + ((b
 - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 1
6*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*2i - atan(
((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) +
((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(
b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(51
2*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1
/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4
*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 1966
08*b^2*d^8*e^13) + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8
- 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^
6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4
*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5
*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^13 + 16*b^4*d^3
*e^10 + 64*b^3*d^4*e^11))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d
^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3
 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3
*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) +
4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^
5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2)
 + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b
*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 -
4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^
11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 + 32768
*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20
480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b -

2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^1
4 - 256*b*d^6*e^13 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 - ((b - 2*d*
e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e
^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i)/((x*(32*b*d^5
*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 - ((b -
 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((
b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d
^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*
b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^
13) + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^
3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 6553
6*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 -
4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2
*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^13 + 16*b^4*d^3*e^10 + 64*b^
3*d^4*e^11))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^
2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4
e^2)))^(1/4) - (x(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b
^2*d^4*e^12) - ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*
b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d
^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 -
 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^
2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8
 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*
d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 10
24*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11
 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*
d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d
^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^
13 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e)
)^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
 - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)))*((b^3 - ((b - 2*d*e)^5*(b + 2*d
*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^
3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*2i - 2*atan(((x*(32*b*d^5*e^13
 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 - ((b - 2*d*
e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e
^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2
*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^
6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e
^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d
^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1
i + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*
e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*

b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4))/((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4

```
- 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 +
256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e
)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^
4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i))*((b^3 - ((b -
 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
```

3.8 $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 751

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}-\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}$$
$$+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}$$

```
[Out] -1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2))/(2*d^(
1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(
```

```
1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/
2))/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(
2*d*e+f)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-(2*d*
e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2)+1/8*ln
(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(
2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2
)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)
)/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)
+(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1
/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)-1/8*ln(d^(1/2
)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/
2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e
^(1/2)+(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$
\begin{aligned}
\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = & -\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} \\
& +\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} \\
& -\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} \\
& +\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} \\
& -\frac{\log\left(-x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}} \\
& +\frac{\log\left(x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}
\end{aligned}
$$

```
[In] Int[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[
2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqr
t[2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] - 2*Sqrt
[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]
*Sqrt[e] - Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e
+ f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*
Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]
]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sq
rt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e]
- Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqr
t[2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + Log[
Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqr
t[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
```

```
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}x^2}{e}+x^4}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}$$
$$+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}$$
$$-\frac{\int \frac{-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$-\frac{\int \frac{-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}+2x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} - \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}{e}-x^2}\,dx, x, -\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}{e}-x^2}\,dx, x, \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}{e}-x^2}\,dx, x, -\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}{e}-x^2}\,dx, x, \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}+2x\right)}{4\sqrt{d}\sqrt{e}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$- \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$+ \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$
$$- \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}$$
$$+ \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.09

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = \frac{1}{4}\text{RootSum}\left[d^2-f\#1^4 + e^2\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{-f\#1^3+2e^2\#1^7}\&\right]$$

```
[In] Integrate[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
[Out] RootSum[d^2 - f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(-
(f*#1^3) + 2*e^2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(e^{2}\_Z^{8}-f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}-\_R^{3}f}\right)}{4}$	55
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(e^{2}\_Z^{8}-f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}-\_R^{3}f}\right)}{4}$	55

```
[In] int((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^2-_Z^4*f+d
^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2453 vs. $2(541) = 1082$.

Time = 0.33 (sec) , antiderivative size = 2453, normalized size of antiderivative = 3.27

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f
)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d
^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2
)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f
)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/
(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3
*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*
f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3))
+ f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2
- 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*
e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^
2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) +
f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(-sqrt(1/2)*sqrt(((4*d^4*
e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*
d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x + 1/
2*(2*d*e + (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 -
```

```
 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e
^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d
^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(-
sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*
e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f +
d^2*f^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(
2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(-s
qrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e
^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d
^2*f^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sq
rt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4
*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 - 4*d^3
*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 -
 d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqr
t(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*
d^4*e^2 - 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 -
4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*
f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d
*e - (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^
6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4
*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f
^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(-sqrt(1
/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 -
 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f
^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e
 + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(-sqrt(1
/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 -
 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f
^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(
-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^
4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 - 4*d^3*e*
f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^
4*f^3)) - f)*sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(
-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^
4*e^2 - 4*d^3*e*f + d^2*f^2))))
```

Sympy [A] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.18

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$$

$$= \text{RootSum}\left(t^8\cdot\left(1048576d^6e^4-2097152d^5e^3f+1572864d^4e^2f^2-524288d^3ef^3+65536d^2f^4\right)+t^4\left(-102\right.\right.$$

```
[In] integrate((e*x**4+d)/(e**2*x**8-f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 - 2097152*d**5*e**3*f + 1572864*d**4*e**2*
f**2 - 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(-1024*d**2*e**2*f + 1
024*d*e*f**2 - 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*e**
2 - 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e - 4*_t*f)/e)))
```

Maxima [F]

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx=\int \frac{ex^4+d}{e^2x^8-fx^4+d^2}\,dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx=\int \frac{ex^4+d}{e^2x^8-fx^4+d^2}\,dx$$

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 10343, normalized size of antiderivative = 13.77

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x)
```

```
[Out] 2*atan(((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^
2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2
)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*
d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3
- 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e
^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e
^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 19660
8*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e
```

```
))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^
3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 - 256*d^6*e^13
*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*
f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e
))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^
3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + (((f^3 + ((f - 2*d*e)^5*(f + 2
*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*
e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*
d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 204
80*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*
e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*
e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*
1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3
)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^
2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
1/4))/((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2
)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)
))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d
^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 -
 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^
2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^
8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608
*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 - 256*d^6*e^13*
f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f
^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - (((f^3 + ((f - 2*d*e)^5*(f +
 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^
3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 3276
8*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 2
0480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f
- 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
 d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^1
0*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^
5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2
)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
```

^(3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f
^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*
f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
^(1/4)*1i))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e
f^2)/(512(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4) - atan(((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2
*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 2
4*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^
8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*
d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1
/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f
 - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^1
1*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*((f^3 + ((f - 2*d*e)^5*
(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 -
8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d
^6*e^13*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e
^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2
*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*
e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i + (((f^3 + ((f - 2*d*e)^5
*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 -
 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 +
 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^
5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) - ((f^3 +
 ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*
e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(26214
4*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 491
52*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^1
3*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2
)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)
))^(3/4) + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^
3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2)
)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*1i)/((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^
2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2
)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*
d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3
- 65536*d^7*e^13*f^2) + ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e
^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e
^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 19660
8*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^
(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 -

32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^13*f + 1
6*d^3*e^10*f^4 + 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*
d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2)
+ 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^
5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) - (((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1
/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*
f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^
11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*
(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 -
8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 26
2144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5
+ 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*((f^3 +
((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) + 256*d
^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) + x*(32*d^5*e
^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)))*((f^3 + ((f
 - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*2i - atan(
((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(
512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^
(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e
^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 19
6608*d^8*e^13*f^2) + x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^
7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*
e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) +
 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5
*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^13*f + 16*d^3*e
^10*f^4 + 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^1
1*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2
*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f
 + 24*d^4*e^2*f^2)))^(1/4)*1i - (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2)
+ 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^
5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/
2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32
*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f
- 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11
*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2) - x*(65536*d^9*e^15 + 327
68*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 +
20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f
 - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +

```
d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e
^14 - 256*d^6*e^13*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3) - x*(32*d^5*e^13*
f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*
e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f
^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i)/((((f^3 - ((f
 - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 -
((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144
*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 4915
2*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13
*f^2) + x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*
e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65
536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f
- 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d
^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^13*f + 16*d^3*e^10*f^4 + 64*
d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^
4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*
e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2
*f^2)))^(1/4) + (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f -
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f
 - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*
d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f
^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^
7*e^12*f^3 - 196608*d^8*e^13*f^2) - x*(65536*d^9*e^15 + 32768*d^8*e^14*f -
1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f
^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f +
2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3
*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^
13*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3) - x*(32*d^5*e^13*f + 4*d^2*e^10*f
^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)))*((f^3 - ((f - 2*d*e)^5*(f + 2*d
*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*
f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*2i - 2*atan(((((f^3 - ((f - 2*
d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2
*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f -
 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10
*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5
*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)
*1i + x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^
9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 6553
6*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f -
```

4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64
*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 4
8*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f -
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(1/4) - (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*
f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24
*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e
^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e
^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 19660
8*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 + 32768*d^8*e^
14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5
*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^
5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4
- 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 +
256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f +
4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^
5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4
- 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4))/((((f^3 - ((f - 2*d
e)^5(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*
f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*
e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*
e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*
1i + x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9
*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536
*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*
d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48
*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)*1i + (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^
2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2
*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f
+ 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3
*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196
608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 + 32768*d^8*
e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d
^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e
)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14

```
+ 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f
 + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e
)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i))*((f^3 - ((f -
 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)
```

3.9 $\int \frac{1+x^4}{1+bx^4+x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 411

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2-b}}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}} + \frac{\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}} + \frac{\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}}$$

```
[Out] -1/4*arctan((-2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1
/2))^(1/2)+1/4*arctan((2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))/(2
-(2-b)^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(
1/2)+1/8*ln(1+x^2+x*(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2)-1/4*arctan
((-2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)+
1/4*arctan((2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2
))^(1/2)-1/8*ln(1+x^2-x*(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)+1/8*ln
(1+x^2+x*(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}} - \frac{\arctan\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}} + \frac{\arctan\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}} + \frac{\log\left(\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}} + \frac{\log\left(\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}$$

```
[In] Int[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]]/Sqrt[2 + S
qrt[2 - b]] - ArcTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]]/(
4*Sqrt[2 - Sqrt[2 - b]]) + ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sq
rt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]]) + ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x
)/Sqrt[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 - Sqrt[
2 - b]]*x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) + Log[1 + Sqrt[2 - Sqrt[2 - b]]*
x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x + x^2]
/(8*Sqrt[2 + Sqrt[2 - b]]) + Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2]/(8*Sqrt
[2 + Sqrt[2 - b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{2-b}x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2-b}x^2+x^4}\,dx \\
&= \frac{\int \frac{\sqrt{2-\sqrt{2-b}}-x}{1-\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}}+\frac{\int \frac{\sqrt{2-\sqrt{2-b}}+x}{1+\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}}+\frac{\int \frac{\sqrt{2+\sqrt{2-b}}-x}{1-\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2-b}}}+\frac{\int \frac{\sqrt{2+\sqrt{2-b}}+x}{1+\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2-b}}} \\
&= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{2-b}}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{2-b}}x+x^2}\,dx \\
&\quad +\frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{2-b}}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2+\sqrt{2-b}}x+x^2}\,dx \\
&\quad -\frac{\int \frac{-\sqrt{2-\sqrt{2-b}}+2x}{1-\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2-\sqrt{2-b}}}+\frac{\int \frac{\sqrt{2-\sqrt{2-b}}+2x}{1+\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2-\sqrt{2-b}}} \\
&\quad -\frac{\int \frac{-\sqrt{2+\sqrt{2-b}}+2x}{1-\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2+\sqrt{2-b}}}+\frac{\int \frac{\sqrt{2+\sqrt{2-b}}+2x}{1+\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2+\sqrt{2-b}}}
\end{aligned}
$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}}$$
$$-\frac{\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}}+\frac{\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}}$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2-\sqrt{2-b}-x^2}\,dx,x,-\sqrt{2-\sqrt{2-b}}+2x\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2-\sqrt{2-b}-x^2}\,dx,x,\sqrt{2-\sqrt{2-b}}+2x\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2+\sqrt{2-b}-x^2}\,dx,x,-\sqrt{2+\sqrt{2-b}}+2x\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2+\sqrt{2-b}-x^2}\,dx,x,\sqrt{2+\sqrt{2-b}}+2x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}$$
$$-\frac{\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}}$$
$$-\frac{\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}}+\frac{\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.13

$$\int\frac{1+x^4}{1+bx^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1+b\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{b\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] RootSum[1 + b*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(b*#1^3 + 2*
#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+\_Z^{4}b+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+\_R^{3}b}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+\_Z^{4}b+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+\_R^{3}b}\right)}{4}$	42

```
[In] int((x^4+1)/(x^8+b*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1177 vs. 2(321) = 642.

Time = 0.28 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.86

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b
 + 8)) + b)/(b^2 + 4*b + 4)))*log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 +
6*b^2 + 12*b + 8)) - b - 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b -
 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(sqrt(1
/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2
 + 4*b + 4)))*log(-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b +
8)) - b - 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^
2 + 12*b + 8)) + b)/(b^2 + 4*b + 4))) + x) - 1/4*sqrt(-sqrt(1/2)*sqrt(-((b^
2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*
log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2)*sq
rt(-sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8))
 + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(-sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*s
qrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*log(-1/2*((b^2
 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2)*sqrt(-sqrt(1/2)
*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 +
4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^
3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*log(1/2*((b^2 + 4*b + 4)*sqrt
```

```
((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b + 2)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b
 + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x) -
1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b +
 8)) - b)/(b^2 + 4*b + 4)))*log(-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6
*b^2 + 12*b + 8)) + b + 2)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2
)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(-sqrt(1/
2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 +
 4*b + 4)))*log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8))
 + b + 2)*sqrt(-sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 +
 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x) - 1/4*sqrt(-sqrt(1/2)*sqrt(((b^2 +
4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*log(
-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b + 2)*sqrt(
-sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b
)/(b^2 + 4*b + 4))) + x)
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.18

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx$$
$$= \text{RootSum}\left(t^8\cdot\left(65536b^4+524288b^3+1572864b^2+2097152b+1048576\right)+t^4\cdot\left(256b^3+1024b^2+1024b\right)\right.$$

```
[In] integrate((x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] RootSum(_t**8*(65536*b**4 + 524288*b**3 + 1572864*b**2 + 2097152*b + 104857
6) + _t**4*(256*b**3 + 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_t**
5*b**2 + 4096*_t**5*b + 4096*_t**5 + 4*_t*b + 4*_t + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \int \frac{x^4+1}{x^8+bx^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \int \frac{x^4+1}{x^8+bx^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 5341, normalized size of antiderivative = 13.00

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \text{Too large to display}$$

```
[In] int((x^4 + 1)/(b*x^4 + x^8 + 1),x)
```

```
[Out] - atan((((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*
b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^
2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608
*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) +
 x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 10
24*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32
*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) +
x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*
b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i - ((-(4*b + (
(b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 +
 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b
 + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 -
 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) - x*(32768*b + 65536
*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-
(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) - x*(32*b - 48*b^2 +
24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32
*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((b - 2)*(b + 2)^5)^
(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4
*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*
b^5 + 4096*b^6 - 4096*b^7 - 262144) + x*(32768*b + 65536*b^2 - 32768*b^3 -
20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b
+ 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4
) - 256*b + 64*b^3 - 16*b^4 + 256) + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-
(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
```

```
 + b^4 + 16)))^(1/4) + ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(
512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^
5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(26
2144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*
b^7 - 262144) - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5
+ 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2
+ b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 1
6*b^4 + 256) - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2
)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)))
*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*
b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) +
4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4
*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*
b^5 + 4096*b^6 - 4096*b^7 - 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3
 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*
(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(
3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(
-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^
3 + b^4 + 16)))^(1/4) - ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/
(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)
*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^
(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^
6 - 4096*b^7 - 262144)*1i - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4
+ 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1
/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*
b^3 + 16*b^4 - 256)*1i + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b -
 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)
))^(1/4))/(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)*(b + 2)^5)^(
1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144
*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7
- 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 +
 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 +
 b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*b^4
- 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5
)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i +
 ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8
*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b
^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 19660
8*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144)*
1i - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6
- 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512
*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i
+ x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) +
```

4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4) - atan((((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i - ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4) + ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +

```
 b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i - x*(32*b - 48*b^2 + 24*b
^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4) - ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) +
 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(
4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
+ b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152
*b^5 + 4096*b^6 - 4096*b^7 - 262144)*1i - x*(32768*b + 65536*b^2 - 32768*b^
3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)
*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^
(3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*
(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b
^3 + b^4 + 16)))^(1/4))/(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)
/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2
)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))
^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b
^6 - 4096*b^7 - 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4
 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(
1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64
*b^3 + 16*b^4 - 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b
- 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16
)))^(1/4)*1i + ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*
b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2)*(b + 2)^
5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(26
2144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*
b^7 - 262144)*1i - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b
^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b
^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*
b^4 - 256)*1i + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b +
2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*
1i))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2
+ 8*b^3 + b^4 + 16)))^(1/4)
```

3.10 $\int \frac{1+x^4}{1+3x^4+x^8} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 451

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] 1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1/
2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(
1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/
4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(
3-5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4)
)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/
4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))
```

```
^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)
*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1434, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{1+x^4}{1+3x^4+x^8}\,dx = &-\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} \\ &+\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}} \\ &-\frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} \\ &+\frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}} \\ &-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} \\ &+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} \\ &-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} \\ &+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}\end{aligned}$$

```
[In] Int[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2^(
3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(
1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (
2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)
*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/
4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sq
rt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sq
rt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt
[5]) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))
^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1434

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2
 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$=\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}}$$

$$+ \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}}$$

$$- \frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\,2^{3/4}\sqrt{5}}$$

$$- \frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\,2^{3/4}\sqrt{5}}$$

$$- \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$= -\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$-\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$+\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}$$

$$- \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$- \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$+ \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$+ \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.12

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^3 + 2*
#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	42

```
[In] int((x^4+1)/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.86

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right) - \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right) - \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right) + \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right)$$

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(sqrt(2)*sqr
t(sqrt(5) - 3))*(sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqr
```

```
t(5) - 3))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*(sqrt(5) + 5) + 20
*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(-sqr
t(2)*sqrt(sqrt(5) - 3))*(sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)
*sqrt(sqrt(5) - 3))*log(-sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*(sqrt(5)
 + 5) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqrt(10)
*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqr
t(sqrt(2)*sqrt(-sqrt(5) - 3))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3)
)*(sqrt(5) - 5) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*l
og(sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*(sqrt(5) - 5) + 20*x) + 1/40*
sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*log(-sqrt(10)*sqrt(-sqrt(2)*sqrt
(-sqrt(5) - 3))*(sqrt(5) - 5) + 20*x)
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(25600t^5+16t+x\right)\right)\right)$$

```
[In] integrate((x**4+1)/(x**8+3*x**4+1),x)
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(25600*_t**5 + 1
6*_t + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \int \frac{x^4+1}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.53

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right)$$

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) + 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) - 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*
log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) + 1/40*sqrt(5*sqrt(5) + 5)
*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*
log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.02

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{3\,2^{3/4}\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{3\,2^{3/4}\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20}$$

```
[In] int((x^4 + 1)/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(-
5^(1/2) - 3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (3*2^(3/4)*5
^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^(1/
2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/20 - (2^(3/4)*5^
(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(2*2^(1/2)*(- 5^(1/2) -
3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)*x*(-
5^(1/2) - 3)^(1/4)*3i)/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^(1/2)*5^(1/2
)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*
atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) -
2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^
(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1
/2))))*(5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2)
- 3)^(1/4)*7i)/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2)
 - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(2*2^(1/2)*(5
^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1
/4)*1i)/20
```

3.11 $\int \frac{1+x^4}{1+2x^4+x^8} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{1+x^4}{1+2x^4+x^8} \, dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {28, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1+x^4}{1+2x^4+x^8} \, dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\arctan\left(\sqrt{2}x+1\right)}{2\sqrt{2}} - \frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}$$

```
[In] Int[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{1+x^4}\,dx \\
&= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} \\
&= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{2\sqrt{2}} \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \frac{-2\arctan\left(1-\sqrt{2}x\right)+2\arctan\left(1+\sqrt{2}x\right)-\log\left(1-\sqrt{2}x+x^2\right)+\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

```
[In] Integrate[(1 + x^4)/(1 + 2*x^4 + x^8),x]
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+1\right)} \frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$	22
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}$	52

```
[In] int((x^4+1)/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{1+x^4}{1+2x^4+x^8}\,dx = &\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i+1)\,\sqrt{2}\right)\\ &-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i-1)\,\sqrt{2}\right)\\ &+\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i-1)\,\sqrt{2}\right)\\ &-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i+1)\,\sqrt{2}\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] (1/8*I + 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*lo
g(2*x - (I - 1)*sqrt(2)) + (1/8*I - 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2))
 - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{1+x^4}{1+2x^4+x^8}\,dx = &-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}\\ &+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}\end{aligned}$$

```
[In] integrate((x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{8}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{8}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int((x^4 + 1)/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.12 $\int \frac{1+x^4}{1+x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 140

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln(x^2+
x+1)-1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1433, 1108, 648, 632, 210, 642}

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) - \frac{1}{8}\log\left(x^2-x+1\right) + \frac{1}{8}\log\left(x^2+x+1\right) - \frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}} + \frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

```
[In] Int[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[Sqrt[3] - 2*x]/4 + ArcTan[(
1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/
8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqr
t[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= \frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx+\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx\\
&\quad+\frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx-\frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{8\sqrt{3}}\\
&= -\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}\\
&\quad-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&\quad-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)\\
&\quad-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{1+x^4}{1+x^4+x^8}\,dx &= \frac{1}{48}\left(4i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)\right.\\
&\quad-4i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+4\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)\\
&\quad\left.+4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-6\log\left(1-x+x^2\right)+6\log\left(1+x+x^2\right)\right)
\end{aligned}
$$

```
[In] Integrate[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x + x^2] +
6*Log[1 + x + x^2])/48
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-3\_R^3+\_R+x\right)\right)}{4}-\frac{\ln(4x^2-4x+4)}{8}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(4x^2+4x+4)}{8}+$
default	$-\frac{\ln(x^2-x+1)}{8}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}$

```
[In] int((x^4+1)/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(-3*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/8*ln(4*x^2-4*x+4)
+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/8*ln(4*x^2+4*x+4)+1/12*arctan(1
/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{1+x^4}{1+x^4+x^8}\,dx =& -\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(i\sqrt{3}-3\right)+12x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(-i\sqrt{3}+3\right)+12x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\left(i\sqrt{3}+3\right)\sqrt{-i\sqrt{3}-1}+12x\right)\\ &-\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{-i\sqrt{3}-1}\left(-i\sqrt{3}-3\right)+12x\right)\\ &+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\ &+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
[Out] -1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*sqrt(3
) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3
) - 1)*(-I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(sqr
t(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) - 1/24*sqrt(6)*sqrt(-I*sq
rt(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x) + 1/12
*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2
*x - 1)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.36

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}+9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1+9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}+9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1+9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(9216t^5+8t+x\right)\right)\right)$$

```
[In] integrate((x**4+1)/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 + 9216*(-1/8 - sqrt(3)*I/24)*
*5) + (-1/8 + sqrt(3)*I/24)*log(x - 1 + 9216*(-1/8 + sqrt(3)*I/24)**5 + sqr
t(3)*I/3) + (1/8 - sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 + 9216*(1/8 - sqrt
(3)*I/24)**5) + (1/8 + sqrt(3)*I/24)*log(x + 1 + 9216*(1/8 + sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(92
16*_t**5 + 8*_t + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \int \frac{x^4+1}{x^8+x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1) -
 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{4}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2\,x-\sqrt{3}\right) + \frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^2
 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3)) +
 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int((x^4 + 1)/(x^4 + x^8 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 + 1/4) + atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 + 1i/4) + atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.13 $\int \frac{1+x^4}{1+x^8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 347

$$\begin{aligned}\int \frac{1+x^4}{1+x^8}\,dx =& -\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)\\ &-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)\\ &-\frac{\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}}\\ &-\frac{\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}}\end{aligned}$$

```
[Out] -1/8*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)
+1/8*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-
1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2^(1/
2))^(1/2))/(2-2^(1/2))^(1/2)-1/8*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2)
)^(1/2))*(4+2*2^(1/2))^(1/2)+1/8*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))
^(1/2))*(4+2*2^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2)+1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1427, 1108, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{1+x^4}{1+x^8}\,dx =& -\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)\\ &-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)\\ &-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)}{8\sqrt{2-\sqrt{2}}}\\ &-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)}{8\sqrt{2+\sqrt{2}}}\end{aligned}$$

```
[In] Int[(1 + x^4)/(1 + x^8),x]
```

```
[Out] -1/4*(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[
2]]]) - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sq
rt[2]]])/4 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2
 + Sqrt[2]]])/4 + (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/S
qrt[2 - Sqrt[2]]])/4 - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[
2]]) + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) - Log[1 - S
qrt[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]]) + Log[1 + Sqrt[2 + Sqrt[2]]
*x + x^2]/(8*Sqrt[2 + Sqrt[2]])
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 632</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1427

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[2*d*e, 2]}, Dist[e^2/(2*c), Int[1/(d + q*x^(n/2) + e*x^n), x], x] +
Dist[e^2/(2*c), Int[1/(d - q*x^(n/2) + e*x^n), x], x]] /; FreeQ[{a, c, d, e
}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{2}x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2}x^2+x^4}\,dx \\
&= \frac{\int \frac{\sqrt{2-\sqrt{2}}-x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2-\sqrt{2}}+x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2+\sqrt{2}}-x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\sqrt{2+\sqrt{2}}+x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}} \\
&= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx \\
&\quad +\frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx \\
&\quad -\frac{\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{8\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{8\sqrt{2-\sqrt{2}}}-\frac{\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{8\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{8\sqrt{2+\sqrt{2}}}
\end{aligned}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}} - \frac{1}{4}\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,-\sqrt{2-\sqrt{2}}+2x\right) - \frac{1}{4}\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,\sqrt{2-\sqrt{2}}+2x\right) - \frac{1}{4}\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,-\sqrt{2+\sqrt{2}}+2x\right) - \frac{1}{4}\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,\sqrt{2+\sqrt{2}}+2x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.74

$$\int\frac{1+x^4}{1+x^8}\,dx = \frac{1}{8}\Bigg(2\arctan\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right) + \log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right) + \log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\left(x-\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) - \log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) + \log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)\Bigg)$$

```
[In] Integrate[(1 + x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[x*S
ec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]
]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 - 2*x*Cos[Pi/8]]*(-Cos[Pi/8] + Sin[
Pi/8]) + 2*ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + 2*Ar
cTan[(x + Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x
*Sin[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(Cos[Pi/
8] + Sin[Pi/8]))/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	27
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	27
meijerg	Expression too large to display	566

```
[In] int((x^4+1)/(x^8+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.54

$$\begin{aligned}\int \frac{1+x^4}{1+x^8}\,dx &= \frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(8\sqrt{2}\left((-1)^{\frac{5}{8}}+(-1)^{\frac{1}{8}}\right)+16x\right)\\ &\quad-\frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left((-1)^{\frac{5}{8}}+(-1)^{\frac{1}{8}}\right)+16x\right)\\ &\quad-\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(i(-1)^{\frac{5}{8}}+i(-1)^{\frac{1}{8}}\right)+16x\right)\\ &\quad+\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(-i(-1)^{\frac{5}{8}}-i(-1)^{\frac{1}{8}}\right)+16x\right)\\ &\quad-\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32x+(16i+16)(-1)^{\frac{5}{8}}-(16i+16)(-1)^{\frac{1}{8}}\right)\\ &\quad+\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32x-(16i-16)(-1)^{\frac{5}{8}}+(16i-16)(-1)^{\frac{1}{8}}\right)\\ &\quad-\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32x+(16i-16)(-1)^{\frac{5}{8}}-(16i-16)(-1)^{\frac{1}{8}}\right)\\ &\quad+\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32x-(16i+16)(-1)^{\frac{5}{8}}+(16i+16)(-1)^{\frac{1}{8}}\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(-1)^(1/8)*log(8*sqrt(2)*((-1)^(5/8) + (-1)^(1/8)) + 16*x) - 1/
8*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*((-1)^(5/8) + (-1)^(1/8)) + 16*x) - 1/8
*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(I*(-1)^(5/8) + I*(-1)^(1/8)) + 16*x)
+ 1/8*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(-I*(-1)^(5/8) - I*(-1)^(1/8)) +
16*x) - (1/8*I + 1/8)*(-1)^(1/8)*log(32*x + (16*I + 16)*(-1)^(5/8) - (16*I
+ 16)*(-1)^(1/8)) + (1/8*I - 1/8)*(-1)^(1/8)*log(32*x - (16*I - 16)*(-1)^(5
/8) + (16*I - 16)*(-1)^(1/8)) - (1/8*I - 1/8)*(-1)^(1/8)*log(32*x + (16*I -
 16)*(-1)^(5/8) - (16*I - 16)*(-1)^(1/8)) + (1/8*I + 1/8)*(-1)^(1/8)*log(32
*x - (16*I + 16)*(-1)^(5/8) + (16*I + 16)*(-1)^(1/8))
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.05

$$\int \frac{1+x^4}{1+x^8}\,dx = \operatorname{RootSum}\left(1048576t^8+1,\left(t\mapsto t\log\left(4096t^5+4t+x\right)\right)\right)$$

```
[In] integrate((x**4+1)/(x**8+1),x)
```

```
[Out] RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 + 4*_t + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1+x^8}\,dx = \int \frac{x^4+1}{x^8+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{1+x^4}{1+x^8}\,dx =& \frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) \\ &+ \frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) \\ &+ \frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) \\ &+ \frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) \\ &+ \frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right) \\ &- \frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right) \\ &+ \frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) \\ &- \frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) + 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2
) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sq
rt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt
(-sqrt(2) + 2)) + 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) +
 1) - 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*s
qrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(2*sqrt(2
) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.90

$$\int \frac{1+x^4}{1+x^8}\,dx = -\ln\left(\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)+256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)+\mathrm{atan}\left(\frac{x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}+\frac{x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{2}\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)\right.$$
$$-\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{3}{4}+\frac{1}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\,(1-\mathrm{i})\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}$$
$$+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{4}-\frac{3}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\,(1+1\mathrm{i})-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}$$
$$+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{2}-\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{4}+\frac{3}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

```
[In] int((x^4 + 1)/(x^8 + 1),x)
```

```
[Out] atan((x*(2^(1/2) - 2)^(1/2)*1i)/2 + (x*(2^(1/2) + 2)^(1/2)*1i)/2 + (2^(1/2)
*x*(2^(1/2) - 2)^(1/2)*1i)/2 - (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*((2^(1
/2)*(2^(1/2) - 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - log((
(- 2*2^(1/2) - 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 16384*(
- 2*2^(1/2) - 4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) + 256)*((- 2*2^(1/2)
- 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16) - (atan(x*(2^(1/2) + 2)^(3/2)*(1
- 1i/2) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 - 1i) - 2
)*(2^(1/2) + 2)^(1/2)*1i)/8 + (atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i) - 2^(1
/2)*x*(2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2^(1/2) +
2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1/2 + 1i) + 2^(1/2)*(
2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1/2) + 2)^(1
/2)/16)*1i
```

3.14 $\int \frac{1+x^4}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 331

$$\begin{aligned}\int \frac{1+x^4}{1-x^4+x^8}\,dx = &-\frac{1}{4}\sqrt{2-\sqrt{3}}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\ &-\frac{1}{4}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{2-\sqrt{3}}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\ &-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}\\ &-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}\end{aligned}$$

```
[Out] -1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/8*ln
(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))-1/4*arctan((-
2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^
(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
(1/2*6^(1/2)-1/2*2^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(1/2*6^
(1/2)+1/2*2^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))/(1/2*6^(1/2)+1
/2*2^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1
/2)))*(1/2*6^(1/2)+1/2*2^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1
/2*6^(1/2)-1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1433, 1108, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{1+x^4}{1-x^4+x^8}\,dx =& -\frac{1}{4}\sqrt{2-\sqrt{3}}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\ &-\frac{1}{4}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\\ &-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{2-\sqrt{3}}}\\ &-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{2+\sqrt{3}}}\end{aligned}$$

```
[In] Int[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]
) - (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])
/4 + (Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]
)/4 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
])/4 - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) + Log[1 + S
qrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) - Log[1 - Sqrt[2 + Sqrt[3]]
*x + x^2]/(8*Sqrt[2 + Sqrt[3]]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqr
t[2 + Sqrt[3]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{3}x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{3}x^2+x^4}\,dx \\
&= \frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{2+\sqrt{3}}} \\
&= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx \\
&\quad +\frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&\quad -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{2-\sqrt{3}}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{2+\sqrt{3}}}
\end{aligned}
$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}$$
$$-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx,x,-\sqrt{2-\sqrt{3}}+2x\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx,x,\sqrt{2-\sqrt{3}}+2x\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx,x,-\sqrt{2+\sqrt{3}}+2x\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx,x,\sqrt{2+\sqrt{3}}+2x\right)$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{2-\sqrt{3}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{2-\sqrt{3}}}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}$$
$$-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.17

$$\int\frac{1+x^4}{1-x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#1^
7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	42

```
[In] int((x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.18

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)+4x\right)$$

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt(-3) + 1))*log(sqrt(2)*sqrt(sqrt(2)*sqrt(
sqrt(-3) + 1))*(sqrt(-3) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt(-
```

```
3) + 1))*log(-sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt(-3) + 1))*(sqrt(-3) + 1) + 4*x
) + 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(sqrt(-3) + 1))*log(sqrt(2)*sqrt(-sqrt(2)
*sqrt(sqrt(-3) + 1))*(sqrt(-3) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt
(sqrt(-3) + 1))*log(-sqrt(2)*sqrt(-sqrt(2)*sqrt(sqrt(-3) + 1))*(sqrt(-3) +
1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(-sqrt(-3) + 1))*log(sqrt(2)*sqrt(
sqrt(2)*sqrt(-sqrt(-3) + 1))*(sqrt(-3) - 1) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(
2)*sqrt(-sqrt(-3) + 1))*log(-sqrt(2)*sqrt(sqrt(2)*sqrt(-sqrt(-3) + 1))*(sqr
t(-3) - 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt(-3) + 1))*log(sqrt
(2)*sqrt(-sqrt(2)*sqrt(-sqrt(-3) + 1))*(sqrt(-3) - 1) + 4*x) + 1/8*sqrt(2)*
sqrt(-sqrt(2)*sqrt(-sqrt(-3) + 1))*log(-sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt(-3
) + 1))*(sqrt(-3) - 1) + 4*x)
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.06

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \operatorname{RootSum}\left(65536t^8-256t^4+1,\left(t\mapsto t\log\left(1024t^5+x\right)\right)\right)$$

```
[In] integrate((x**4+1)/(x**8-x**4+1),x)
[Out] RootSum(65536*_t**8 - 256*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.74

$$\begin{aligned}\int \frac{1+x^4}{1-x^4+x^8}\,dx =& \frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &-\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(6) - sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2)
)) + 1/8*(sqrt(6) - sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sq
rt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6)
 - sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sq
rt(6) - sqrt(2))) + 1/16*(sqrt(6) - sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/16*(sqrt(6) - sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) +
 1) + 1/16*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1
/16*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.44

$$\int \frac{1+x^4}{1-x^4+x^8} \, dx = -\text{atan}\left(\frac{\sqrt{6}\,x\,(27-27\text{i})}{27\,\sqrt{3}-81\text{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}+\frac{1}{8}\text{i}\right)+\sqrt{6}\,\left(-\frac{1}{8}+\frac{1}{8}\text{i}\right)\right) - \text{atan}\left(\frac{\sqrt{6}\,x\,(27+27\text{i})}{27\,\sqrt{3}-81\text{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}-\frac{1}{8}\text{i}\right)+\sqrt{6}\,\left(\frac{1}{8}+\frac{1}{8}\text{i}\right)\right) - \text{atan}\left(\frac{\sqrt{6}\,x\,(27-27\text{i})}{27\,\sqrt{3}+81\text{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}+\frac{1}{8}\text{i}\right)+\sqrt{6}\,\left(\frac{1}{8}-\frac{1}{8}\text{i}\right)\right) - \text{atan}\left(\frac{\sqrt{6}\,x\,(27+27\text{i})}{27\,\sqrt{3}+81\text{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}-\frac{1}{8}\text{i}\right)+\sqrt{6}\,\left(-\frac{1}{8}-\frac{1}{8}\text{i}\right)\right)$$

```
[In] int((x^4 + 1)/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((6^(1/2)*x*(27 - 27i))/(27*3^(1/2) - 81i))*(2^(1/2)*(1/8 + 1i/8) - 6
^(1/2)*(1/8 - 1i/8)) - atan((6^(1/2)*x*(27 + 27i))/(27*3^(1/2) - 81i))*(2^(
1/2)*(1/8 - 1i/8) + 6^(1/2)*(1/8 + 1i/8)) - atan((6^(1/2)*x*(27 - 27i))/(27
*3^(1/2) + 81i))*(2^(1/2)*(1/8 + 1i/8) + 6^(1/2)*(1/8 - 1i/8)) - atan((6^(1
/2)*x*(27 + 27i))/(27*3^(1/2) + 81i))*(2^(1/2)*(1/8 - 1i/8) - 6^(1/2)*(1/8
+ 1i/8))
```

3.15 $\int \frac{1+x^4}{1-2x^4+x^8} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1+x^4}{1-2x^4+x^8} \, dx = \frac{x}{2(1-x^4)} + \frac{\arctan(x)}{4} + \frac{\operatorname{arctanh}(x)}{4}$$

```
[Out] 1/2*x/(-x^4+1)+1/4*arctan(x)+1/4*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {28, 393, 218, 212, 209}

$$\int \frac{1+x^4}{1-2x^4+x^8} \, dx = \frac{\arctan(x)}{4} + \frac{\operatorname{arctanh}(x)}{4} + \frac{x}{2(1-x^4)}$$

```
[In] Int[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 - x^4)) + ArcTan[x]/4 + ArcTanh[x]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x^4}{(-1+x^4)^2}\,dx\\ &= \frac{x}{2(1-x^4)} - \frac{1}{2}\int \frac{1}{-1+x^4}\,dx\\ &= \frac{x}{2(1-x^4)} + \frac{1}{4}\int \frac{1}{1-x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{2(1-x^4)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = \frac{1}{8}\left(-\frac{4x}{-1+x^4} + 2\arctan(x) - \log(1-x) + \log(1+x)\right)$$

```
[In] Integrate[(1 + x^4)/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x)/(-1 + x^4) + 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/8
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{2(x^4-1)}+\frac{\arctan(x)}{4}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}$	28
default	$-\frac{1}{8(x+1)}+\frac{\ln(x+1)}{8}+\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{1}{8(x-1)}-\frac{\ln(x-1)}{8}$	42
parallelrisch	$-\frac{i\ln(x-i)x^4-i\ln(x+i)x^4+\ln(x-1)x^4-\ln(x+1)x^4-i\ln(x-i)+i\ln(x+i)-\ln(x-1)+\ln(x+1)+4x}{8(x^4-1)}$	79

```
[In] int((x^4+1)/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^4-1)+1/4*arctan(x)-1/8*ln(x-1)+1/8*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = \frac{2\,(x^4-1)\arctan{(x)}+(x^4-1)\log{(x+1)}-(x^4-1)\log{(x-1)}-4\,x}{8\,(x^4-1)}$$

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/8*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1) -
4*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = -\frac{x}{2x^4-2}-\frac{\log{(x-1)}}{8}+\frac{\log{(x+1)}}{8}+\frac{\operatorname{atan}{(x)}}{4}$$

```
[In] integrate((x**4+1)/(x**8-2*x**4+1),x)
[Out] -x/(2*x**4 - 2) - log(x - 1)/8 + log(x + 1)/8 + atan(x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = -\frac{x}{2\,(x^4-1)} + \frac{1}{4}\arctan\left(x\right) + \frac{1}{8}\log\left(x+1\right) - \frac{1}{8}\log\left(x-1\right)$$

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = -\frac{x}{2\,(x^4-1)} + \frac{1}{4}\arctan\left(x\right) + \frac{1}{8}\log\left(|x+1|\right) - \frac{1}{8}\log\left(|x-1|\right)$$

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1)
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = \frac{\mathrm{atan}(x)}{4} + \frac{\mathrm{atanh}(x)}{4} - \frac{x}{2\,\left(x^4-1\right)}$$

```
[In] int((x^4 + 1)/(x^8 - 2*x^4 + 1),x)
[Out] atan(x)/4 + atanh(x)/4 - x/(2*(x^4 - 1))
```

3.16 $\int \frac{1+x^4}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}} - \frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

```
[Out] arctan(x*2^(1/2)/(5^(1/2)-1)^(1/2))/(-2+2*5^(1/2))^(1/2)+arctanh(x*2^(1/2)/
(5^(1/2)-1)^(1/2))/(-2+2*5^(1/2))^(1/2)-arctan(x*2^(1/2)/(5^(1/2)+1)^(1/2))
/(2+2*5^(1/2))^(1/2)-arctanh(x*2^(1/2)/(5^(1/2)+1)^(1/2))/(2+2*5^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1433, 1107, 209, 213}

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}} - \frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

```
[In] Int[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1
+ Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/Sq
rt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5]
)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{5}x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{5}x^2+x^4}\,dx\\ &= \frac{1}{2}\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx - \frac{1}{2}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx + \frac{1}{2}\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx - \frac{1}{2}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}} - \frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

```
[In] Integrate[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1
+ Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/Sq
rt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5]
)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2-1\right)}\_R\ln\left(\_R^3-\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+\_Z^2-1\right)}\_R\ln\left(-\_R^3-\_R+x\right)\right)}{4}$	56
default	$-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}+\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{\sqrt{2\sqrt{5}-2}}-\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{\sqrt{2\sqrt{5}-2}}$	96

```
[In] int((x^4+1)/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(_R^3-_R+x),_R=RootOf(_Z^4-_Z^2-1))+1/4*sum(_R*ln(-_R^3-_R+x),
_R=RootOf(_Z^4+_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(95) = 190.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.42

$$
\begin{aligned}
\int \frac{1+x^4}{1-3x^4+x^8}\,dx =& \frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)\\
&-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)\\
&-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)\\
&+\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)\\
&-\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}+1}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+1}+4x\right)\\
&+\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}+1}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+1}+4x\right)\\
&+\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}+4x\right)\\
&-\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}+4x\right)
\end{aligned}
$$

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5)
+ 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log(-(sqrt(5)*sqrt(2) - sqrt(2)
)*sqrt(sqrt(5) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5) - 1)*log((sqrt(5)*sqr
t(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(5) - 1)*lo
g(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*x) - 1/8*sqrt(2)*sqrt(
-sqrt(5) + 1)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 1) + 4*x) + 1
/8*sqrt(2)*sqrt(-sqrt(5) + 1)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5
) + 1) + 4*x) + 1/8*sqrt(2)*sqrt(-sqrt(5) - 1)*log((sqrt(5)*sqrt(2) - sqrt(
2))*sqrt(-sqrt(5) - 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(5) - 1)*log(-(sqrt(5
)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 1) + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(256t^4-16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right) + \operatorname{RootSum}\left(256t^4+16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right)$$

```
[In] integrate((x**4+1)/(x**8-3*x**4+1),x)
[Out] RootSum(256*_t**4 - 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t + x))
) + RootSum(256*_t**4 + 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t +
 x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{1+x^4}{1-3x^4+x^8}\,dx = &-\frac{1}{4}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) \\ &+\frac{1}{4}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) \\ &-\frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) \\ &+\frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) \\ &+\frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) \\ &-\frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2*sqrt(5) - 2)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/4*sqrt(2*sqr
t(5) + 2)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/8*sqrt(2*sqrt(5) - 2)*log(a
bs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt
(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x + sqrt(1/2*sqrt(5
) - 1/2))) - 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.05

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}-1}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}+1}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}+1}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{5}}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{1-\sqrt{5}}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{-\sqrt{5}-1}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{4}$$

```
[In] int((x^4 + 1)/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(1 - 5^(1/2))^(1/2)*1875i)/(2*(875*5^(1/2) - 1875)
) - (2^(1/2)*5^(1/2)*x*(1 - 5^(1/2))^(1/2)*875i)/(2*(875*5^(1/2) - 1875)))*
(1 - 5^(1/2))^(1/2)*1i)/4 - (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1)^(1/2)*18
75i)/(2*(875*5^(1/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(5^(1/2) + 1)^(1/2)*875i
)/(2*(875*5^(1/2) + 1875)))*(5^(1/2) + 1)^(1/2)*1i)/4 - (2^(1/2)*atan((2^(1
/2)*x*(5^(1/2) - 1)^(1/2)*1875i)/(2*(875*5^(1/2) - 1875)) - (2^(1/2)*5^(1/2
)*x*(5^(1/2) - 1)^(1/2)*875i)/(2*(875*5^(1/2) - 1875)))*(5^(1/2) - 1)^(1/2)
*1i)/4 + (2^(1/2)*atan((2^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1875i)/(2*(875*5^(1
/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*875i)/(2*(875*5^(1/
2) + 1875)))*(- 5^(1/2) - 1)^(1/2)*1i)/4
```

3.17 $\int \frac{1+x^4}{1-4x^4+x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}}-\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

```
[Out] 1/4*arctan(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(3^(1/2)-1)^(1/2)+1/4*arcta
nh(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(3^(1/2)-1)^(1/2)-1/4*arctan(2^(1/4
)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(1+3^(1/2))^(1/2)-1/4*arctanh(2^(1/4)*x/(1+3
^(1/2))^(1/2))*2^(3/4)/(1+3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1433, 1107, 209, 213}

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

```
[In] Int[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[3]]) - Arc
Tan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 + Sqrt[3]]) + ArcTanh[
(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[3]]) - ArcTanh[(2
^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 + Sqrt[3]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{6}x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{6}x^2+x^4}\,dx\\
&= \frac{\int \frac{1}{-\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{2}}+\frac{\int \frac{1}{\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{2}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \frac{1}{8}\text{RootSum}\left[1-4\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

```
[In] Integrate[(1 + x^4)/(1 - 4*x^4 + x^8),x]
[Out] RootSum[1 - 4*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-2*#1^3 + #
1^7) & ]/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}}{8}$	40
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}}{8}$	40

```
[In] int((x^4+1)/(x^8-4*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*sum((_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(101) = 202$.

Time = 0.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.22

$$\begin{aligned}
\int \frac{1+x^4}{1-4x^4+x^8}\,dx &= \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{\sqrt{3}+2}}+2x\right) \\
&- \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(-\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{\sqrt{3}+2}}+2x\right) \\
&- \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+2x\right) \\
&+ \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(-\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+2x\right) \\
&+ \frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right) \\
&- \frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right) \\
&- \frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right) \\
&+ \frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)
\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*sqrt(-sqrt(sqrt(3) + 2))*log((sqrt(3)*sqrt(2) - sqrt(2))*sqrt(-
sqrt(sqrt(3) + 2)) + 2*x) - 1/8*sqrt(2)*sqrt(-sqrt(sqrt(3) + 2))*log(-(sqrt
(3)*sqrt(2) - sqrt(2))*sqrt(-sqrt(sqrt(3) + 2)) + 2*x) - 1/8*sqrt(2)*sqrt(-
sqrt(-sqrt(3) + 2))*log((sqrt(3)*sqrt(2) + sqrt(2))*sqrt(-sqrt(-sqrt(3) + 2
)) + 2*x) + 1/8*sqrt(2)*sqrt(-sqrt(-sqrt(3) + 2))*log(-(sqrt(3)*sqrt(2) + s
qrt(2))*sqrt(-sqrt(-sqrt(3) + 2)) + 2*x) + 1/8*sqrt(2)*(sqrt(3) + 2)^(1/4)*
log((sqrt(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1/4) + 2*x) - 1/8*sqrt(2)*(s
qrt(3) + 2)^(1/4)*log(-(sqrt(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1/4) + 2*
x) - 1/8*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log((sqrt(3)*sqrt(2) + sqrt(2))*(-sqr
t(3) + 2)^(1/4) + 2*x) + 1/8*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log(-(sqrt(3)*sqr
t(2) + sqrt(2))*(-sqrt(3) + 2)^(1/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.15

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \operatorname{RootSum}\left(1048576t^8-4096t^4+1,\left(t\mapsto t\log\left(4096t^5-12t+x\right)\right)\right)$$

```
[In] integrate((x**4+1)/(x**8-4*x**4+1),x)
[Out] RootSum(1048576*_t**8 - 4096*_t**4 + 1, Lambda(_t, _t*log(4096*_t**5 - 12*_
t + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-4\,x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-4\,x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
[Out] integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.54

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\,\sqrt{2}\,x\,\left(\sqrt{3}+2\right)^{1/4}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{3024\,\sqrt{2}\,\sqrt{3}\,x\,\left(\sqrt{3}+2\right)^{1/4}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}\right)\,\left(\sqrt{3}+2\right)^{1/4}}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\left(2-\sqrt{3}\right)^{1/4}\,5184\mathrm{i}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}-\frac{\sqrt{2}\,\sqrt{3}\,x\,\left(2-\sqrt{3}\right)^{1/4}\,3024\mathrm{i}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}\right)\,\left(2-\sqrt{3}\right)^{1/4}\,1\mathrm{i}}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\,\sqrt{2}\,x\,\left(2-\sqrt{3}\right)^{1/4}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}-\frac{3024\,\sqrt{2}\,\sqrt{3}\,x\,\left(2-\sqrt{3}\right)^{1/4}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}\right)\,\left(2-\sqrt{3}\right)^{1/4}}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\left(\sqrt{3}+2\right)^{1/4}\,5184\mathrm{i}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{\sqrt{2}\,\sqrt{3}\,x\,\left(\sqrt{3}+2\right)^{1/4}\,3024\mathrm{i}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}\right)\,\left(\sqrt{3}+2\right)^{1/4}\,1\mathrm{i}}{4}$$

```
[In] int((x^4 + 1)/(x^8 - 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(2 - 3^(1/2))^(1/4)*5184i)/(2160*3^(1/2)*(2 - 3^(1
/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (2^(1/2)*3^(1/2)*x*(2 - 3^(1/2))^(
1/4)*3024i)/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)))*
(2 - 3^(1/2))^(1/4)*1i)/4 - (2^(1/2)*atan((5184*2^(1/2)*x*(2 - 3^(1/2))^(1/
4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (3024*2
^(1/2)*3^(1/2)*x*(2 - 3^(1/2))^(1/4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3
888*(2 - 3^(1/2))^(1/2)))*(2 - 3^(1/2))^(1/4))/4 + (2^(1/2)*atan((5184*2^(1
/2)*x*(3^(1/2) + 2)^(1/4))/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2
) + 2)^(1/2)) + (3024*2^(1/2)*3^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3888*(3^(1/2)
 + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)))*(3^(1/2) + 2)^(1/4))/4 - (
2^(1/2)*atan((2^(1/2)*x*(3^(1/2) + 2)^(1/4)*5184i)/(3888*(3^(1/2) + 2)^(1/2
) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)) + (2^(1/2)*3^(1/2)*x*(3^(1/2) + 2)^(1
/4)*3024i)/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)))*(
3^(1/2) + 2)^(1/4)*1i)/4
```

3.18 $\int \frac{1+x^4}{1-5x^4+x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 171

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}} - \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

```
[Out] arctan(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-6*3^(1/2)+6*7^(1/2))^(1/2)+arct
anh(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-6*3^(1/2)+6*7^(1/2))^(1/2)-arctan(
x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(6*3^(1/2)+6*7^(1/2))^(1/2)-arctanh(x*2^
(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(6*3^(1/2)+6*7^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1433, 1107, 209, 213}

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}} - \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

```
[In] Int[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7])] - ArcTa
n[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt
[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7])] - ArcTanh[Sqrt[2/(
Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt[7])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{7}x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{7}x^2+x^4}\,dx\\
&= \frac{\int \frac{1}{-\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{1}{\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}+\frac{\int \frac{1}{-\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{1}{\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}\\
&= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.32

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1-5\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-5\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] RootSum[1 - 5*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-5*#1^3 + 2
*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	42

```
[In] int((x^4+1)/(x^8-5*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(123) = 246$.

Time = 0.30 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.98

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx$$
$$= \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12x\right)$$

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*s
qrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/24*
sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt(
3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*sqrt
(6)*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sqrt(3) -
 3*sqrt(6))*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/24*sqrt(6)
*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt(3) - 3
*sqrt(6))*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/24*sqrt(6)*s
qrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sqrt(3) + 3*sq
rt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*sqrt(6)*sqrt
(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt(3) + 3*sqrt
(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/24*sqrt(6)*sqrt(-
```

```
sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sqrt(3) + 3*sqrt(6
))*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*sqrt(6)*sqrt(-s
qrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt(3) + 3*sqrt(6
))*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.14

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \text{RootSum}\left(5308416t^8-11520t^4+1,\left(t\mapsto t\log\left(9216t^5-16t+x\right)\right)\right)$$

```
[In] integrate((x**4+1)/(x**8-5*x**4+1),x)
[Out] RootSum(5308416*_t**8 - 11520*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 16*
_t + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-5\,x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-5\,x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.82

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx$$
$$= \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\left(5-\sqrt{21}\right)^{1/4}}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{12}$$
$$-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\left(5-\sqrt{21}\right)^{1/4}12005\mathrm{i}}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}7889\mathrm{i}}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{12}$$
$$+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\left(\sqrt{21}+5\right)^{1/4}}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{12}$$
$$-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\left(\sqrt{21}+5\right)^{1/4}12005\mathrm{i}}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}7889\mathrm{i}}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{12}$$

```
[In] int((x^4 + 1)/(x^8 - 5*x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((12005*2^(3/4)*3^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(48
02*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2
))) - (7889*2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(6*(4802*2^(1/
2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))*(5
- 21^(1/2))^(1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(5 - 21^(1
/2))^(1/4)*12005i)/(2*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^
(1/2)*(5 - 21^(1/2))^(1/2))) - (2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(
1/4)*7889i)/(6*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(
5 - 21^(1/2))^(1/2))))*(5 - 21^(1/2))^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan(
(12005*2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(4802*2^(1/2)*(21^(1/2) +
 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (7889*2^(3/4)*3^
(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4))/(6*(4802*2^(1/2)*(21^(1/2) + 5)^(1/2
) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4))/12
- (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/4)*12005i)/(2*
(4802*2^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(
1/2))) + (2^(3/4)*3^(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4)*7889i)/(6*(4802*2
^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))
*(21^(1/2) + 5)^(1/4)*1i)/12
```

3.19 $\int \frac{1+x^4}{1-6x^4+x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

```
[Out] 1/4*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/4*arctanh(x/(2^(1/2)-1)
^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
-1/4*arctanh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1433, 1107, 209, 213}

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

```
[In] Int[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sqrt[1 + Sqr
t[2]]]/(4*Sqrt[1 + Sqrt[2]]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + S
qrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-2\sqrt{2}x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+2\sqrt{2}x^2+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{-1-\sqrt{2}+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-\sqrt{2}+x^2}\,dx\\
&\quad +\frac{1}{4}\int \frac{1}{-1+\sqrt{2}+x^2}\,dx-\frac{1}{4}\int \frac{1}{1+\sqrt{2}+x^2}\,dx\\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx = \frac{1}{4}\left(\sqrt{1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right) - \sqrt{-1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) + \sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right) - \sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)\right)$$

```
[In] Integrate[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] - Sqrt[-1 + Sqrt[2]]*ArcTan
[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] - S
qrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+2\_Z^2-1\right)}\_R\ln\left(-\_R^3-2\_R+x\right)\right)}{8}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-2\_Z^2-1\right)}\_R\ln\left(\_R^3-2\_R+x\right)\right)}{8}$	58
default	$\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$	78

```
[In] int((x^4+1)/(x^8-6*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(_R*ln(-_R^3-2*_R+x),_R=RootOf(_Z^4+2*_Z^2-1))+1/8*sum(_R*ln(_R^3-2*
_R+x),_R=RootOf(_Z^4-2*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(77) = 154.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.89

$$\begin{aligned}\int \frac{1+x^4}{1-6x^4+x^8}\,dx =& -\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)\\&+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)\\&+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)\\&-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)\\&-\frac{1}{8}\sqrt{-\sqrt{2}+1}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}+1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}-1}\log\left(\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right)\\&-\frac{1}{8}\sqrt{-\sqrt{2}-1}\log\left(-\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(sqrt(2) - 1)*log((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt(
sqrt(2) - 1)*log(-(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt(sqrt(2) +
 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) - 1/8*sqrt(sqrt(2) + 1)*log(-s
qrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) - 1/8*sqrt(-sqrt(2) + 1)*log((sqrt(2) +
 1)*sqrt(-sqrt(2) + 1) + x) + 1/8*sqrt(-sqrt(2) + 1)*log(-(sqrt(2) + 1)*sqr
t(-sqrt(2) + 1) + x) + 1/8*sqrt(-sqrt(2) - 1)*log((sqrt(2) - 1)*sqrt(-sqrt(
2) - 1) + x) - 1/8*sqrt(-sqrt(2) - 1)*log(-(sqrt(2) - 1)*sqrt(-sqrt(2) - 1)
 + x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\begin{aligned}\int \frac{1+x^4}{1-6x^4+x^8}\,dx =& \operatorname{RootSum}\left(4096t^4-128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right)\\&+\operatorname{RootSum}\left(4096t^4+128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right)\end{aligned}$$

```
[In] integrate((x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] RootSum(4096*_t**4 - 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5 - 20*_t +
 x))) + RootSum(4096*_t**4 + 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5 -
 20*_t + x)))
```

Maxima [F]

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-6\,x^4+1}\,dx$$

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 6*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{1+x^4}{1-6x^4+x^8}\,dx = &-\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)\\ &-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right)\\ &+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right)\\ &+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right)\\ &-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(sqrt(2) - 1)*arctan(x/sqrt(sqrt(2) + 1)) + 1/4*sqrt(sqrt(2) + 1)*
arctan(x/sqrt(sqrt(2) - 1)) - 1/8*sqrt(sqrt(2) - 1)*log(abs(x + sqrt(sqrt(2
) + 1))) + 1/8*sqrt(sqrt(2) - 1)*log(abs(x - sqrt(sqrt(2) + 1))) + 1/8*sqrt
(sqrt(2) + 1)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/8*sqrt(sqrt(2) + 1)*log(a
bs(x - sqrt(sqrt(2) - 1)))
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.99

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-1}\,49152\mathrm{i}}{34816\sqrt{2}-49152}-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}-1}\,34816\mathrm{i}}{34816\sqrt{2}-49152}\right)\sqrt{\sqrt{2}-1}\,1\mathrm{i}}{4} - \frac{\operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}+1}\,49152\mathrm{i}}{34816\sqrt{2}+49152}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+1}\,34816\mathrm{i}}{34816\sqrt{2}+49152}\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{4} + \frac{\operatorname{atan}\left(\frac{x\sqrt{1-\sqrt{2}}\,49152\mathrm{i}}{34816\sqrt{2}-49152}-\frac{\sqrt{2}\,x\sqrt{1-\sqrt{2}}\,34816\mathrm{i}}{34816\sqrt{2}-49152}\right)\sqrt{1-\sqrt{2}}\,1\mathrm{i}}{4} + \frac{\operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-1}\,49152\mathrm{i}}{34816\sqrt{2}+49152}+\frac{\sqrt{2}\,x\sqrt{-\sqrt{2}-1}\,34816\mathrm{i}}{34816\sqrt{2}+49152}\right)\sqrt{-\sqrt{2}-1}\,1\mathrm{i}}{4}$$

```
[In] int((x^4 + 1)/(x^8 - 6*x^4 + 1),x)
```

```
[Out] (atan((x*(1 - 2^(1/2))^(1/2)*49152i)/(34816*2^(1/2) - 49152) - (2^(1/2)*x*(
1 - 2^(1/2))^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(1 - 2^(1/2))^(1/2)*1i)
/4 - (atan((x*(2^(1/2) + 1)^(1/2)*49152i)/(34816*2^(1/2) + 49152) + (2^(1/2
)*x*(2^(1/2) + 1)^(1/2)*34816i)/(34816*2^(1/2) + 49152))*(2^(1/2) + 1)^(1/2
)*1i)/4 - (atan((x*(2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2) - 49152) - (2
^(1/2)*x*(2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(2^(1/2) - 1)
^(1/2)*1i)/4 + (atan((x*(- 2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2) + 4915
2) + (2^(1/2)*x*(- 2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) + 49152))*(- 2
^(1/2) - 1)^(1/2)*1i)/4
```

3.20 $\int \frac{1-x^4}{1+bx^4+x^8} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 511

$$
\begin{aligned}
\int \frac{1-x^4}{1+bx^4+x^8} \, dx = & -\frac{\sqrt{2+b} \arctan\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}} + \frac{\sqrt{2+b} \arctan\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}} \\
& + \frac{\sqrt{2+b} \arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}} - \frac{\sqrt{2+b} \arctan\left(\frac{\sqrt{2+\sqrt{2-b}}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}} \\
& + \frac{\sqrt{2-\sqrt{2-b}} \log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}} \\
& - \frac{\sqrt{2-\sqrt{2-b}} \log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}} \\
& - \frac{\sqrt{2+\sqrt{2-b}} \log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}} \\
& + \frac{\sqrt{2+\sqrt{2-b}} \log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}
\end{aligned}
$$

```
[Out] -1/4*arctan((-2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))*(2+b)^(1/2)
/(2-b)^(1/2)/(2-(2-b)^(1/2))^(1/2)+1/4*arctan((2*x+(2-(2-b)^(1/2))^(1/2))/(
2+(2-b)^(1/2))^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(2-(2-b)^(1/2))^(1/2)+1/8*ln(
1+x^2-x*(2-(2-b)^(1/2))^(1/2))*(2-(2-b)^(1/2))^(1/2)/(2-b)^(1/2)-1/8*ln(1+x
^2+x*(2-(2-b)^(1/2))^(1/2))*(2-(2-b)^(1/2))^(1/2)/(2-b)^(1/2)+1/4*arctan((-
2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(
2+(2-b)^(1/2))^(1/2)-1/4*arctan((2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))
^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(2+(2-b)^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+(2-
b)^(1/2))^(1/2))*(2+(2-b)^(1/2))^(1/2)/(2-b)^(1/2)+1/8*ln(1+x^2+x*(2+(2-b)^
(1/2))^(1/2))*(2+(2-b)^(1/2))^(1/2)/(2-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1435, 1183, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{1-x^4}{1+bx^4+x^8}\,dx = &-\frac{\sqrt{b+2}\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}+\frac{\sqrt{b+2}\arctan\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\sqrt{2-b}}\\&+\frac{\sqrt{b+2}\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}-\frac{\sqrt{b+2}\arctan\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\sqrt{2-b}}\\&+\frac{\sqrt{2-\sqrt{2-b}}\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-b}}\\&-\frac{\sqrt{2-\sqrt{2-b}}\log\left(\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-b}}\\&-\frac{\sqrt{\sqrt{2-b}+2}\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{2-b}}\\&+\frac{\sqrt{\sqrt{2-b}+2}\log\left(\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{2-b}}\end{aligned}$$

```
[In] Int[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -1/4*(Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]
]])/(Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sqr
t[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 -
b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b
]]])/(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) - (Sqrt[2 + b]*ArcTan[(Sqrt[2 +
Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2
 - b]) + (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2])/(8*
Sqrt[2 - b]) - (Sqrt[2 - Sqrt[2 - b]]*Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x^2
])/(8*Sqrt[2 - b]) - (Sqrt[2 + Sqrt[2 - b]]*Log[1 - Sqrt[2 + Sqrt[2 - b]]*x
 + x^2])/(8*Sqrt[2 - b]) + (Sqrt[2 + Sqrt[2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 -
 b]]*x + x^2])/(8*Sqrt[2 - b])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{\sqrt{2-b}+2x^2}{-1-\sqrt{2-b}x^2-x^4}\,dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x^2}{-1+\sqrt{2-b}x^2-x^4}\,dx}{2\sqrt{2-b}} \\
&= \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\sqrt{2-b}-\left(-2+\sqrt{2-b}\right)x}{1-\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}} + \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\sqrt{2-b}+\left(-2+\sqrt{2-b}\right)x}{1+\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}} \\
&\quad + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}\sqrt{2-b}-\left(2+\sqrt{2-b}\right)x}{1-\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}\sqrt{2-b}+\left(2+\sqrt{2-b}\right)x}{1+\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\left(\frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1-\sqrt{2+\sqrt{2-b}}x+x^2}\,dx\right)\\
&\quad -\frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1+\sqrt{2+\sqrt{2-b}}x+x^2}\,dx\\
&\quad +\frac{1}{8}\left(1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1-\sqrt{2-\sqrt{2-b}}x+x^2}\,dx\\
&\quad +\frac{1}{8}\left(1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1+\sqrt{2-\sqrt{2-b}}x+x^2}\,dx\\
&\quad +\frac{\sqrt{2-\sqrt{2-b}}\int \frac{-\sqrt{2-\sqrt{2-b}}+2x}{1-\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2-b}}-\frac{\sqrt{2-\sqrt{2-b}}\int \frac{\sqrt{2-\sqrt{2-b}}+2x}{1+\sqrt{2-\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2-b}}\\
&\quad -\frac{\sqrt{2+\sqrt{2-b}}\int \frac{-\sqrt{2+\sqrt{2-b}}+2x}{1-\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2-b}}+\frac{\sqrt{2+\sqrt{2-b}}\int \frac{\sqrt{2+\sqrt{2-b}}+2x}{1+\sqrt{2+\sqrt{2-b}}x+x^2}\,dx}{8\sqrt{2-b}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{2-\sqrt{2-b}}\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\\
&\quad -\frac{\sqrt{2-\sqrt{2-b}}\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\\
&\quad -\frac{\sqrt{2+\sqrt{2-b}}\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\\
&\quad +\frac{\sqrt{2+\sqrt{2-b}}\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\\
&\quad +\frac{1}{4}\left(-1-\frac{2}{\sqrt{2-b}}\right)\mathrm{Subst}\left(\int \frac{1}{-2-\sqrt{2-b}-x^2}\,dx,x,-\sqrt{2-\sqrt{2-b}}+2x\right)\\
&\quad +\frac{1}{4}\left(-1-\frac{2}{\sqrt{2-b}}\right)\mathrm{Subst}\left(\int \frac{1}{-2-\sqrt{2-b}-x^2}\,dx,x,\sqrt{2-\sqrt{2-b}}+2x\right)\\
&\quad -\frac{1}{4}\left(1-\frac{2}{\sqrt{2-b}}\right)\mathrm{Subst}\left(\int \frac{1}{-2+\sqrt{2-b}-x^2}\,dx,x,-\sqrt{2+\sqrt{2-b}}+2x\right)\\
&\quad -\frac{1}{4}\left(1-\frac{2}{\sqrt{2-b}}\right)\mathrm{Subst}\left(\int \frac{1}{-2+\sqrt{2-b}-x^2}\,dx,x,\sqrt{2+\sqrt{2-b}}+2x\right)
\end{aligned}
$$

$$= -\frac{\sqrt{2+\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-b}} + \frac{\sqrt{2-\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-b}}$$
$$+ \frac{\sqrt{2+\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-b}} - \frac{\sqrt{2-\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-b}}$$
$$+ \frac{\sqrt{2-\sqrt{2-b}}\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}$$
$$- \frac{\sqrt{2-\sqrt{2-b}}\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}$$
$$- \frac{\sqrt{2+\sqrt{2-b}}\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}$$
$$+ \frac{\sqrt{2+\sqrt{2-b}}\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.11

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1+b\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{b\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 - x^4)/(1 + b*x^4 + x^8),x]
[Out] -1/4*RootSum[1 + b*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(b*#1^
3 + 2*#1^7) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+\_Z^4 b+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+\_R^3 b}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+\_Z^4 b+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+\_R^3 b}\right)}{4}$	44

```
[In] int((-x^4+1)/(x^8+b*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1177 vs. 2(397) = 794.

Time = 0.28 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.30

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b
- 8)) + b)/(b^2 - 4*b + 4)))*log(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6
*b^2 + 12*b - 8)) - b + 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b +
2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/
2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2
- 4*b + 4)))*log(-1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8
)) - b + 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2
 + 12*b - 8)) + b)/(b^2 - 4*b + 4))) + x) + 1/4*sqrt(-sqrt(1/2)*sqrt(-((b^2
 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*l
og(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b + 2)*sqr
t(-sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8))
+ b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(-sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sq
rt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*log(-1/2*((b^2
- 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b + 2)*sqrt(-sqrt(1/2)*
sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4
*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3
 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*log(1/2*((b^2 - 4*b + 4)*sqrt(
(b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b
+ 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4))) + x) + 1
/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b -
8)) - b)/(b^2 - 4*b + 4)))*log(-1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*
b^2 + 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)
/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(-sqrt(1/2
)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 -
4*b + 4)))*log(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8))
+ b - 2)*sqrt(-sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 +
12*b - 8)) - b)/(b^2 - 4*b + 4))) + x) + 1/4*sqrt(-sqrt(1/2)*sqrt(((b^2 - 4
*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*log(-
1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b - 2)*sqrt(-
sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)
/(b^2 - 4*b + 4))) + x)
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.15

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = -\operatorname{RootSum}\left(t^8\cdot\left(65536b^4-524288b^3+1572864b^2-2097152b+1048576\right)+t^4\cdot\left(256b^3-1024b^2+1024\right.\right.$$

```
[In] integrate((-x**4+1)/(x**8+b*x**4+1),x)
[Out] -RootSum(_t**8*(65536*b**4 - 524288*b**3 + 1572864*b**2 - 2097152*b + 10485
76) + _t**4*(256*b**3 - 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_t*
*5*b**2 - 4096*_t**5*b + 4096*_t**5 + 4*_t*b - 4*_t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = \int -\frac{x^4-1}{x^8+bx^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = \int -\frac{x^4-1}{x^8+bx^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
[Out] integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 5341, normalized size of antiderivative = 10.45

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = \text{Too large to display}$$

```
[In] int(-(x^4 - 1)/(b*x^4 + x^8 + 1),x)
[Out] - atan((((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 3
2*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2
```

```
) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b
- 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 2
62144) + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*
b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/
(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) -
x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*
b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i - ((-(4*b + (
(b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 1966
08*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) - x*(32768*b
 - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65
536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b
 - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) + x*(32*b + 48*b^2 +
24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24
*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((b - 2)^5*(b + 2))^
(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b
 + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b -
 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4
+ 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) + x*(32768*b - 65536*b^2 - 3276
8*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b
- 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16
)))^(3/4) - 64*b^3 - 16*b^4 + 256) - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-
(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3
 + b^4 + 16)))^(1/4) + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(
512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^
5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(
1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6
 - 4096*b^7 + 262144) - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10
240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2)
- 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 1
6*b^4 + 256) + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b +
 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)))
*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*
b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b + ((b - 2)^5*(b + 2))^(1/2) -
4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b
 - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 1
6)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 40
96*b^6 - 4096*b^7 + 262144)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480
*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2
))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i
- 256*b + 64*b^3 + 16*b^4 - 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(
-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^
3 + b^4 + 16)))^(1/4) - ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/
(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b +
```

2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(2
62144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096
*b^7 + 262144)*1i - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*
b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*
b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*
b^3 + 16*b^4 - 256)*1i - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b -
 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)
))^(1/4))/(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
- 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4
b^2 + b^3)/(512(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196
608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144
)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^
6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*b^3 + 16*b^4
- 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2)
)^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i +
 ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8
b^3 + b^4 + 16)))^(1/4)(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3
)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 -
196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144)*1i - x*(
32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b
^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*b^3 + 16*b^4 - 256)*1i
- x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) -
4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i))*(-(4*b +
((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4
+ 16)))^(1/4) - atan((((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2)^5
*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1
/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6
- 4096*b^7 + 262144) + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 102
40*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) -
 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16
b^4 + 256) - x(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b +
2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i
 - ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b -
 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4
b^2 + b^3)/(512(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196
608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144
) - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 -
 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*
(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) + x*(32
*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 +
 b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b - ((b -
2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16))

```
)^(1/4)*(256*b + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(2
4*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^
3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) + x*(32768*b - 65
536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))
*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*
b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) - x*(32*b + 48*b^2 + 24*b^
3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
- 32*b - 8*b^3 + b^4 + 16)))^(1/4) + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) -
4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4
*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*
b^5 - 4096*b^6 - 4096*b^7 + 262144) - x*(32768*b - 65536*b^2 - 32768*b^3 +
20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(
b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4
) - 64*b^3 - 16*b^4 + 256) + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - (
(b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
 16)))^(1/4)))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b
^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*2i - 2*atan(((((-(4*b - ((b - 2)^5*(b
+ 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*
(((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8
*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 +
49152*b^5 - 4096*b^6 - 4096*b^7 + 262144)*1i + x*(32768*b - 65536*b^2 - 327
68*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b
 - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 1
6)))^(3/4)*1i - 256*b + 64*b^3 + 16*b^4 - 256)*1i + x*(32*b + 48*b^2 + 24*b
^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
 - 32*b - 8*b^3 + b^4 + 16)))^(1/4) - ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) -
 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((
b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4
096*b^6 - 4096*b^7 + 262144)*1i - x*(32768*b - 65536*b^2 - 32768*b^3 + 2048
0*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b +
2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i
 - 256*b + 64*b^3 + 16*b^4 - 256)*1i - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*
(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b
^3 + b^4 + 16)))^(1/4))/(((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)
/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)^5*(b +
 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(
262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 409
6*b^7 + 262144)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240
*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4
*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64
*b^3 + 16*b^4 - 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b
- 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16
)))^(1/4)*1i + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*
```

```
b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)^5*(b + 2))^(1/2)
 - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b -
 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 26
2144)*1i - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 204
8*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3
)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*b^3 + 16*
b^4 - 256)*1i - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b
+ 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*
1i))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b
- 8*b^3 + b^4 + 16)))^(1/4)
```

3.21 $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 411

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx=-\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}}$$

```
[Out] -1/4*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)-1/4*a
rctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)+1/8*ln(2*x^2
-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/4)*2^(1/4)-1/8*ln(
2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/4)*2^(1/4)+1/
4*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)+1/4*arct
an(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)-1/8*ln(2*x^2-2*
2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)+1/8*ln(2*x
^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1434, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}} - \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}} - \frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}}$$

```
[In] Int[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/2^(3
/4) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*
2^(3/4)) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]
)/(2*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(2*2^(3/4)) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*
```

```
(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqr
t[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)) + ((
3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1434

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2
 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\left(-1-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}}\,dx$$

$$+\frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}}\,dx$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}}\,dx$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}}\,dx$$

$$-\frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\;2^{3/4}}$$

$$-\frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\;2^{3/4}}$$

$$+\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$
\begin{aligned}
&= -\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}} \\
&\quad+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}} \\
&\quad+\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt[4]{2\left(3+\sqrt{5}\right)}} \\
&\quad-\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt[4]{2\left(3+\sqrt{5}\right)}} \\
&\quad+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}} \\
&\quad-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3+\sqrt{5}\right)}}
\end{aligned}
$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}}$$
$$+ \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$
$$- \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)} - 2\sqrt[4]{2\left(3-\sqrt{5}\right)}x + 2x^2\right)}{4\ 2^{3/4}}$$
$$+ \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)} + 2\sqrt[4]{2\left(3-\sqrt{5}\right)}x + 2x^2\right)}{4\ 2^{3/4}}$$
$$+ \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)} - 2\sqrt[4]{2\left(3+\sqrt{5}\right)}x + 2x^2\right)}{4\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$
$$- \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)} + 2\sqrt[4]{2\left(3+\sqrt{5}\right)}x + 2x^2\right)}{4\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.14

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 + 3*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	44

```
[In] int((-x^4+1)/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.02

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right)$$

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log((sqrt(5)*sqrt(2) + sqrt(2)
)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(sq
rt(5) - 3))*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(2)*sqrt(sqrt(5) - 3)
) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log((sqrt(5)*sqrt(2
) + sqrt(2))*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqrt(-sq
rt(2)*sqrt(sqrt(5) - 3))*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(2)*sqr
t(sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log((
```

```
sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3)) + 4*x) - 1/8*sq
rt(2)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqr
t(sqrt(2)*sqrt(-sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt
(5) - 3))*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))
 + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*log(-(sqrt(5)*sqrt(
2) - sqrt(2))*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3)) + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\operatorname{RootSum}\left(65536t^8+768t^4+1,\left(t\mapsto t\log\left(1024t^5+8t+x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8+3*x**4+1),x)
[Out] -RootSum(65536*_t**8 + 768*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + 8*_t +
 x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.54

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \frac{1}{16}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{\sqrt{5}-1} + \frac{1}{16}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{\sqrt{5}-1} - \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x+\sqrt{\sqrt{5}+1}\right)^2+2500\,x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x-\sqrt{\sqrt{5}+1}\right)^2+2500\,x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x+\sqrt{\sqrt{5}-1}\right)^2+1156\,x^2\right) - \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x-\sqrt{\sqrt{5}-1}\right)^2+1156\,x^2\right)$$

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi
+ 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi + 4*arct
an(x*sqrt(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) + 1/16*(pi + 4*arctan(-x*sqr
t(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) - 1/8*sqrt(sqrt(5) - 1)*log(2500*(x
+ sqrt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) - 1)*log(2500*(x - sq
rt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) + 1)*log(1156*(x + sqrt(s
qrt(5) - 1))^2 + 1156*x^2) - 1/8*sqrt(sqrt(5) + 1)*log(1156*(x - sqrt(sqrt(
5) - 1))^2 + 1156*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.09

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \frac{2^{3/4}\,\mathrm{atan}\left(\frac{1875\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}-\frac{875\,2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}\right)\,\left(\sqrt{5}-3\right)^{1/4}}{4} - \frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,1875\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,875\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}\right)\,\left(\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{4} + \frac{2^{3/4}\,\mathrm{atan}\left(\frac{1875\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}+\frac{875\,2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}\right)\,\left(-\sqrt{5}-3\right)^{1/4}}{4} - \frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,1875\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,875\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}\right)\,\left(-\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{4}$$

```
[In] int(-(x^4 - 1)/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*atan((1875*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2)
 - 3)^(1/2) - 250*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (875*2^(3/4)*5^(1
/2)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(1/2
)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan((2^
(3/4)*x*(5^(1/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 25
0*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^
(1/4)*875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(1/2)*5^(1/2)*(5^(1
/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4)*1i)/4 + (2^(3/4)*atan((1875*2^(3/4)*x
*(- 5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)
*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (875*2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(
1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(1/2)*(- 5^(1/2
) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan((2^(3/4)*x*(- 5^(1
/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5
^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*8
75i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(1/2)*(- 5^(1/2)
 - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/4
```

3.22 $\int \frac{1-x^4}{1+2x^4+x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{x}{2(1+x^4)} - \frac{\arctan\left(1-\sqrt{2}x\right)}{4\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{4\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}}$$

```
[Out] 1/2*x/(x^4+1)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*arctan(1+x*2^(1/2))*2^(1
/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {28, 393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{4\sqrt{2}} + \frac{\arctan\left(\sqrt{2}x+1\right)}{4\sqrt{2}} + \frac{x}{2(x^4+1)} - \frac{\log\left(x^2-\sqrt{2}x+1\right)}{8\sqrt{2}} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[In] Int[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]
/(4*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + Log[1 + Sqrt[2]*x + x
^2]/(8*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-x^4}{(1+x^4)^2}\,dx \\
&= \frac{x}{2(1+x^4)} + \frac{1}{2}\int \frac{1}{1+x^4}\,dx \\
&= \frac{x}{2(1+x^4)} + \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{x}{2(1+x^4)} + \frac{1}{8}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx \\
&\quad - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{8\sqrt{2}} \\
&= \frac{x}{2(1+x^4)} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{4\sqrt{2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{4\sqrt{2}} \\
&= \frac{x}{2(1+x^4)} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{4\sqrt{2}} \\
&\quad - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$
\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\left(\frac{8x}{1+x^4} - 2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right) - \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)
$$

```
[In] Integrate[(1 - x^4)/(1 + 2*x^4 + x^8),x]
[Out] ((8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
x^2])/16
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x}{2x^4+2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{8}$	33
default	$\frac{x}{2x^4+2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{16}$	63

```
[In] int((-x^4+1)/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x/(x^4+1)+1/8*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{\sqrt{2}((i+1)\,x^4+i+1)\log\left(2\,x+(i+1)\,\sqrt{2}\right)+\sqrt{2}(-(i-1)\,x^4-i+1)\log\left(2\,x-(i-1)\,\sqrt{2}\right)+\sqrt{2}((i\cdots}{16\,(x^4+1)\cdots}$$

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/16*(sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x + (I + 1)*sqrt(2)) + sqrt(2)*(-
(I - 1)*x^4 - I + 1)*log(2*x - (I - 1)*sqrt(2)) + sqrt(2)*((I - 1)*x^4 + I
- 1)*log(2*x + (I - 1)*sqrt(2)) + sqrt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x -
(I + 1)*sqrt(2)) + 8*x)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{x}{2x^4+2}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{16}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{16} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{8}$$

```
[In] integrate((-x**4+1)/(x**8+2*x**4+1),x)
[Out] x/(2*x**4 + 2) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 + sqrt(2)*log(x**2 +
sqrt(2)*x + 1)/16 + sqrt(2)*atan(sqrt(2)*x - 1)/8 + sqrt(2)*atan(sqrt(2)*x
+ 1)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2\left(x^4+1\right)}$$

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2\left(x^4+1\right)}$$

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{x}{2\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

```
[In] int(-(x^4 - 1)/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/8 - 1i/8) + x/(2*(x^4 + 1))
```

3.23 $\int \frac{1-x^4}{1+x^4+x^8} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int \frac{1-x^4}{1+x^4+x^8} \, dx = -\frac{1}{4}\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) -\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right) -\frac{1}{8}\sqrt{3}\log\left(1-\sqrt{3}x+x^2\right)+\frac{1}{8}\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)$$

```
[Out] -1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^2
+x+1)-1/4*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/4*arctan(1/3*(1+2*x)*3^(1/2
))*3^(1/2)-1/8*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/8*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1435, 1183, 648, 632, 210, 642}

$$\int \frac{1-x^4}{1+x^4+x^8} \, dx = -\frac{1}{4}\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right) -\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right) -\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)$$

```
[In] Int[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]]) + ArcTan[Sqrt[3] - 2*x]/4 + (Sqrt[
3]*ArcTan[(1 + 2*x)/Sqrt[3]])/4 - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2
```

```
]/8 - Log[1 + x + x^2]/8 - (Sqrt[3]*Log[1 - Sqrt[3]*x + x^2])/8 + (Sqrt[3]*
Log[1 + Sqrt[3]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1+2x^2}{-1-x^2-x^4}\,dx\right) - \frac{1}{2}\int \frac{1-2x^2}{-1+x^2-x^4}\,dx \\
&= \frac{1}{4}\int \frac{1+x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1-x}{1+x+x^2}\,dx + \frac{\int \frac{\sqrt{3}-3x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+3x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} \\
&= \frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx \\
&\quad - \frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx + \frac{3}{8}\int \frac{1}{1-x+x^2}\,dx + \frac{3}{8}\int \frac{1}{1+x+x^2}\,dx \\
&\quad - \frac{1}{8}\sqrt{3}\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{8}\sqrt{3}\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx \\
&= \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{1}{8}\sqrt{3}\log\left(1-\sqrt{3}x+x^2\right) + \frac{1}{8}\sqrt{3}\log\left(1 + \sqrt{3}x+x^2\right) \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x\right) \\
&\quad - \frac{3}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{3}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= -\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right) + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{4}\tan^{-1}\left(\sqrt{3} + 2x\right) \\
&\quad + \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{1}{8}\sqrt{3}\log\left(1-\sqrt{3}x + x^2\right) + \frac{1}{8}\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$
\begin{aligned}
\int \frac{1-x^4}{1+x^4+x^8}\,dx = \frac{1}{8}\Bigg(&-2\sqrt{-2-2i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) \\
&- 2\sqrt{-2+2i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right) + 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) \\
&+ 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + \log\left(1-x+x^2\right) - \log\left(1+x+x^2\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] (-2*Sqrt[-2 - (2*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - 2*Sqrt[-2 + (2
*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2] - Log[1 + x
 + x^2])/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\ln\left(4x^2+4x+4\right)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{4}+\frac{\ln\left(4x^2-4x+4\right)}{8}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2+1\right)}\_R\ln(\right.}{4}$
default	$\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{4}-\frac{\ln\left(x^2+x+1\right)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{4}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{8}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+$

```
[In] int((-x^4+1)/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(4*x^2+4*x+4)+1/4*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(4*x^2-4
*x+4)+1/4*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*sum(_R*ln(-_R^3+_R+x),_R=
RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{1-x^4}{1+x^4+x^8}\,dx = & -\frac{1}{8}\sqrt{2}\sqrt{\sqrt{-3}+1}\log\left(\sqrt{2}\sqrt{\sqrt{-3}+1}(\sqrt{-3}-1)+4x\right) \\ & +\frac{1}{8}\sqrt{2}\sqrt{\sqrt{-3}+1}\log\left(-\sqrt{2}\sqrt{\sqrt{-3}+1}(\sqrt{-3}-1)+4x\right) \\ & +\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-3}+1}\log\left(\sqrt{2}(\sqrt{-3}+1)\sqrt{-\sqrt{-3}+1}+4x\right) \\ & -\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-3}+1}\log\left(-\sqrt{2}(\sqrt{-3}+1)\sqrt{-\sqrt{-3}+1}+4x\right) \\ & +\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\ & -\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*sqrt(sqrt(-3) + 1)*log(sqrt(2)*sqrt(sqrt(-3) + 1)*(sqrt(-3) -
1) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(-3) + 1)*log(-sqrt(2)*sqrt(sqrt(-3) + 1)*
(sqrt(-3) - 1) + 4*x) + 1/8*sqrt(2)*sqrt(-sqrt(-3) + 1)*log(sqrt(2)*(sqrt(-
3) + 1)*sqrt(-sqrt(-3) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(-3) + 1)*log(-s
qrt(2)*(sqrt(-3) + 1)*sqrt(-sqrt(-3) + 1) + 4*x) + 1/4*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/8*log(x^2
 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{1-x^4}{1+x^4+x^8}\,dx = & -\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right) \\ & -\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right) \\ & -\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right) \\ & -\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right) \\ & -\operatorname{RootSum}\left(256t^4-16t^2+1,\left(t\mapsto t\log\left(1024t^5+x\right)\right)\right)\end{aligned}$$

```
[In] integrate((-x**4+1)/(x**8+x**4+1),x)
```

```
[Out] -(-1/8 - sqrt(3)*I/8)*log(x + 1024*(-1/8 - sqrt(3)*I/8)**5) - (-1/8 + sqrt(
3)*I/8)*log(x + 1024*(-1/8 + sqrt(3)*I/8)**5) - (1/8 - sqrt(3)*I/8)*log(x +
 1024*(1/8 - sqrt(3)*I/8)**5) - (1/8 + sqrt(3)*I/8)*log(x + 1024*(1/8 + sqr
t(3)*I/8)**5) - RootSum(256*_t**4 - 16*_t**2 + 1, Lambda(_t, _t*log(1024*_t
**5 + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8+x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((2*x^2 - 1)/(x^4 - x^2 + 1), x) - 1/8*log(x^2 +
x + 1) + 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{1-x^4}{1+x^4+x^8}\,dx &= \frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\&\quad+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)\\&\quad-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)\\&\quad-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/8*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/8*sqrt(3)*log(x^2 - s
qrt(3)*x + 1) - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(3)) - 1/8
*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{1-x^4}{1+x^4+x^8}\,dx &= -\mathrm{atan}\left(\frac{54\sqrt{3}\,x}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\mathrm{i}\right)\\&\quad+\mathrm{atan}\left(\frac{54\sqrt{3}\,x}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\mathrm{i}\right)\\&\quad+\mathrm{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\\&\quad-\mathrm{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\end{aligned}$$

```
[In] int(-(x^4 - 1)/(x^4 + x^8 + 1),x)
```

```
[Out] atan((54*3^(1/2)*x)/(3^(1/2)*27i + 81))*(3^(1/2)/4 - 1i/4) - atan((54*3^(1/
2)*x)/(3^(1/2)*27i - 81))*(3^(1/2)/4 + 1i/4) + atan((3^(1/2)*x*54i)/(3^(1/2
)*27i - 81))*((3^(1/2)*1i)/4 - 1/4) - atan((3^(1/2)*x*54i)/(3^(1/2)*27i + 8
1))*((3^(1/2)*1i)/4 + 1/4)
```

3.24 $\int \frac{1-x^4}{1+x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 347

$$\begin{aligned}\int \frac{1-x^4}{1+x^8}\,dx = & -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} \\ & -\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) \\ & -\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right) \\ & -\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right) \\ & +\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)\end{aligned}$$

```
[Out] 1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2+x*(2-2
^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2))/(2-2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(4+2*2^(1/
2))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/4*arctan
((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)-1/4*arctan((
2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1428, 1183, 648, 632, 210, 642}

$$\int \frac{1-x^4}{1+x^8} dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} + \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} - \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} + \frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right) - \frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right) - \frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right) + \frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)$$

```
[In] Int[(1 - x^4)/(1 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 - Sqrt[2]]
+ ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]])
 + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]]
) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]
]) + (Sqrt[(2 - Sqrt[2])/2]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(
2 - Sqrt[2])/2]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[2])
/2]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[2])/2]*Log[1 +
Sqrt[2 + Sqrt[2]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-2*d*e, 2]}, Dist[d/(2*a), Int[(d - q*x^(n/2))/(d - q*x^(n/2) - e*x^
n), x], x] + Dist[d/(2*a), Int[(d + q*x^(n/2))/(d + q*x^(n/2) - e*x^n), x],
 x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] &&
IGtQ[n/2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx + \frac{1}{2}\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}}$$

$$= -\left(\frac{1}{8}\sqrt{3-2\sqrt{2}}\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx\right)$$
$$-\frac{1}{8}\sqrt{3-2\sqrt{2}}\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx+\frac{\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{8\sqrt{2-\sqrt{2}}}$$
$$+\frac{\left(-1+\sqrt{2}\right)\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{8\sqrt{2-\sqrt{2}}}+\frac{\left(-1-\sqrt{2}\right)\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{8\sqrt{2+\sqrt{2}}}$$
$$+\frac{\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{8\sqrt{2+\sqrt{2}}}+\frac{1}{8}\sqrt{3+2\sqrt{2}}\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{3+2\sqrt{2}}\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx$$

$$= \frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)-\frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)$$
$$-\frac{1}{8}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)+\frac{1}{8}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)$$
$$+\frac{1}{4}\sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx,x,-\sqrt{2+\sqrt{2}}+2x\right)$$
$$+\frac{1}{4}\sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx,x,\sqrt{2+\sqrt{2}}+2x\right)$$
$$-\frac{1}{4}\sqrt{3+2\sqrt{2}}\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx,x,-\sqrt{2-\sqrt{2}}+2x\right)$$
$$-\frac{1}{4}\sqrt{3+2\sqrt{2}}\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx,x,\sqrt{2-\sqrt{2}}+2x\right)$$

$$= -\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)$$
$$+\frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)-\frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)$$
$$-\frac{1}{8}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)+\frac{1}{8}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.74

$$\int \frac{1-x^4}{1+x^8}\,dx = \frac{1}{8}\Big(2\arctan\left(\cot\left(\frac{\pi}{8}\right)-x\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right) + \log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) + \log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) - \log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right) + \log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)\Big)$$

```
[In] Integrate[(1 - x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 -
2*x*Sin[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]
]*(-Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(-Cos[Pi/8] + Sin
[Pi/8]) + 2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] + Sin[Pi/8]) + 2*A
rcTan[x*Sec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x*
Cos[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]]*(Cos[Pi/8
] + Sin[Pi/8]))/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	29
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	29
meijerg	Expression too large to display	566

```
[In] int((-x^4+1)/(x^8+1),x,method=_RETURNVERBOSE)
[Out] 1/8*sum((-_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.56

$$
\begin{aligned}
\int \frac{1-x^4}{1+x^8}\,dx = & -\frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(8\sqrt{2}\left((-1)^{\frac{5}{8}}-(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& +\frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left((-1)^{\frac{5}{8}}-(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& +\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(i\,(-1)^{\frac{5}{8}}-i\,(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& -\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(-i\,(-1)^{\frac{5}{8}}+i\,(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& +\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x+(16i+16)\,(-1)^{\frac{5}{8}}+(16i+16)\,(-1)^{\frac{1}{8}}\right) \\
& -\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x-(16i-16)\,(-1)^{\frac{5}{8}}-(16i-16)\,(-1)^{\frac{1}{8}}\right) \\
& +\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x+(16i-16)\,(-1)^{\frac{5}{8}}+(16i-16)\,(-1)^{\frac{1}{8}}\right) \\
& -\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x-(16i+16)\,(-1)^{\frac{5}{8}}-(16i+16)\,(-1)^{\frac{1}{8}}\right)
\end{aligned}
$$

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="fricas")
[Out] -1/8*sqrt(2)*(-1)^(1/8)*log(8*sqrt(2)*((-1)^(5/8) - (-1)^(1/8)) + 16*x) + 1
/8*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*((-1)^(5/8) - (-1)^(1/8)) + 16*x) + 1/
8*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(I*(-1)^(5/8) - I*(-1)^(1/8)) + 16*x)
 - 1/8*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(-I*(-1)^(5/8) + I*(-1)^(1/8)) +
 16*x) + (1/8*I + 1/8)*(-1)^(1/8)*log(32*x + (16*I + 16)*(-1)^(5/8) + (16*I
 + 16)*(-1)^(1/8)) - (1/8*I - 1/8)*(-1)^(1/8)*log(32*x - (16*I - 16)*(-1)^(
5/8) - (16*I - 16)*(-1)^(1/8)) + (1/8*I - 1/8)*(-1)^(1/8)*log(32*x + (16*I
- 16)*(-1)^(5/8) + (16*I - 16)*(-1)^(1/8)) - (1/8*I + 1/8)*(-1)^(1/8)*log(3
2*x - (16*I + 16)*(-1)^(5/8) - (16*I + 16)*(-1)^(1/8))
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.06

$$\int \frac{1-x^4}{1+x^8}\,dx = -\operatorname{RootSum}\left(1048576t^8+1,\left(t\mapsto t\log\left(4096t^5-4t+x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8+1),x)
[Out] -RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 - 4*_t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1+x^8}\,dx = \int -\frac{x^4-1}{x^8+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{1-x^4}{1+x^8}\,dx &= \frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)\\ &\quad+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)\\ &\quad-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)\\ &\quad-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)\\ &\quad+\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)\\ &\quad-\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)\\ &\quad-\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)\\ &\quad+\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2)
+ 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqr
t(2) + 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt
(-sqrt(2) + 2)) + 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) +
1) - 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqr
t(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-2*sqrt(2
) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.90

$$\int \frac{1-x^4}{1+x^8}\,dx = -\ln\left(\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)-256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)-\mathrm{atan}\left(-\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}-2}}+\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{2}\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{4}-\frac{3}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\,(1-\mathrm{i})\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{3}{4}+\frac{1}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\,(1+1\mathrm{i})-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-1+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{3}{4}-\frac{1}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

```
[In] int(-(x^4 - 1)/(x^8 + 1),x)
```

```
[Out] (atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(1/4
 + 3i/4))*(2^(1/2)*(1 - 1i) - 2)*(2^(1/2) + 2)^(1/2)*1i)/8 - atan((x*1i)/(2
^(1/2) + 2)^(1/2) - (x*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*x*1i)/(2*(2^(1/2)
 - 2)^(1/2)) + (2^(1/2)*x*1i)/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2)*(2^(1/2) -
 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - log(((- 2*2^(1/2) -
 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 16384*(- 2*2^(1/2) -
4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) - 256)*((- 2*2^(1/2) - 4)^(1/2)/16
- (4 - 2*2^(1/2))^(1/2)/16) + (atan(x*(2^(1/2) + 2)^(3/2)*(1 - 1i/2) - 2^(1
/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2^(1/2) +
2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1 - 1i/2) + 2^(1/2)*(
2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1/2) + 2)^(1
/2)/16)*1i
```

3.25 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 355

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1435, 1183, 648, 632, 210, 642}

$$\int \frac{1-x^4}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} -\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right) -\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right) -\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right) +\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[3*(2 - Sqrt[3
])] + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} \\
&= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} \\
&\quad + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}+\left(2+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}
\end{aligned}$$

$$= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int\frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx\right)$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int\frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\int\frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx+\frac{\left(-2+\sqrt{3}\right)\int\frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int\frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int\frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int\frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int\frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx$$

$$=\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx,x,-\sqrt{2+\sqrt{3}}+2x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx,x,\sqrt{2+\sqrt{3}}+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx,x,-\sqrt{2-\sqrt{3}}+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx,x,\sqrt{2-\sqrt{3}}+2x\right)$$

$$= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x + x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1 + \sqrt{2+\sqrt{3}}x+x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}}{4}$	44
risch	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}}{4}$	44

```
[In] int((-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.17

$$
\begin{aligned}
\int \frac{1-x^4}{1-x^4+x^8}\,dx = {} & \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12\,x\right) \\
& + \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)\right. \\
& \left. + 12\,x\right) \\
& - \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12\,x\right) \\
& - \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)\right. \\
& \left. + 12\,x\right) \\
& + \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12\,x\right) \\
& + \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)\right. \\
& \left. + 12\,x\right) \\
& - \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)\right. \\
& \left. + 12\,x\right) \\
& - \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)\right. \\
& \left. + 12\,x\right)
\end{aligned}
$$

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sq
rt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sq
```

```
rt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(I*sqrt
(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(
6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)
*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3
) + 1))*(I*sqrt(3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3)
+ 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x
) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(
2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2
)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*
sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log
(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3) + 12*x)
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(9216t^5-8t+x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.59

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

```
[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4
))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3
^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/
4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2
)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.26 $\int \frac{1-x^4}{1-2x^4+x^8} dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{1-x^4}{1-2x^4+x^8} dx = \frac{\arctan(x)}{2} + \frac{\text{arctanh}(x)}{2}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 21, 218, 212, 209}

$$\int \frac{1-x^4}{1-2x^4+x^8} dx = \frac{\arctan(x)}{2} + \frac{\text{arctanh}(x)}{2}$$

```
[In] Int[(1 - x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] ArcTan[x]/2 + ArcTanh[x]/2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1-x^4}{\left(-1+x^4\right)^2}\,dx \\
&= -\int \frac{1}{-1+x^4}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\
&= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = \frac{\arctan(x)}{2} - \frac{1}{4}\log(1-x) + \frac{1}{4}\log(1+x)$$

```
[In] Integrate[(1 - x^4)/(1 - 2*x^4 + x^8),x]
[Out] ArcTan[x]/2 - Log[1 - x]/4 + Log[1 + x]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$-\frac{\ln(x-1)}{4}+\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{4}$	18
parallelrisch	$-\frac{\ln(x-1)}{4}+\frac{i\ln(x+i)}{4}-\frac{i\ln(x-i)}{4}+\frac{\ln(x+1)}{4}$	30

```
[In] int((-x^4+1)/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*arctanh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx=\frac{1}{2}\arctan{(x)}+\frac{1}{4}\log{(x+1)}-\frac{1}{4}\log{(x-1)}$$

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx=-\frac{\log{(x-1)}}{4}+\frac{\log{(x+1)}}{4}+\frac{\operatorname{atan}{(x)}}{2}$$

```
[In] integrate((-x**4+1)/(x**8-2*x**4+1),x)
[Out] -log(x - 1)/4 + log(x + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = \frac{1}{2}\,\arctan\left(x\right) + \frac{1}{4}\,\log\left(x+1\right) - \frac{1}{4}\,\log\left(x-1\right)$$

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = \frac{1}{2}\,\arctan\left(x\right) + \frac{1}{4}\,\log\left(|x+1|\right) - \frac{1}{4}\,\log\left(|x-1|\right)$$

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
[Out] 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = \frac{\mathrm{atan}(x)}{2} + \frac{\mathrm{atanh}(x)}{2}$$

```
[In] int(-(x^4 - 1)/(x^8 - 2*x^4 + 1),x)
[Out] atan(x)/2 + atanh(x)/2
```

3.27 $\int \frac{1-x^4}{1-3x^4+x^8} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 129

$$\int \frac{1-x^4}{1-3x^4+x^8} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[Out] arctan(x*2^(1/2)/(5^(1/2)-1)^(1/2))/(-10+10*5^(1/2))^(1/2)+arctanh(x*2^(1/2
)/(5^(1/2)-1)^(1/2))/(-10+10*5^(1/2))^(1/2)+arctan(x*2^(1/2)/(5^(1/2)+1)^(1
/2))/(10+10*5^(1/2))^(1/2)+arctanh(x*2^(1/2)/(5^(1/2)+1)^(1/2))/(10+10*5^(1
/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1433, 1107, 213, 209}

$$\int \frac{1-x^4}{1-3x^4+x^8} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} + \frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[In] Int[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1
 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/
Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqr
t[5])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{-1-x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+x^2+x^4}\,dx\\
&= -\frac{\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}+\frac{\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}+\frac{\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}\\
&= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[In] Integrate[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1
 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/
Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqr
t[5])]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-5\_R^3+3\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(5\_R^3+3\_R+x\right)\right)}{4}$
default	$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}$

```
[In] int((-x^4+1)/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(-5*_R^3+3*_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))+1/4*sum(_R*ln(5*
_R^3+3*_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(93) = 186$.

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \frac{1-x^4}{1-3x^4+x^8}\,dx &= \frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\ &\quad+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\ &\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\\ &\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1
) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*(sqrt(5) + 5)*sqr
t(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sqrt(
sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(-s
qrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(
5) + 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1) + 20*x) - 1/40*sqrt(1
0)*sqrt(-sqrt(5) + 1)*log(-sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1) + 20*x
) - 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) - 5)*sqrt(-sqrt(
5) - 1) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(-sqrt(10)*(sqrt(5) -
 5)*sqrt(-sqrt(5) - 1) + 20*x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = -\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right) - \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8-3*x**4+1),x)
[Out] -RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5 - 16*_t +
 x))) - RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5 -
16*_t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{1-x^4}{1-3x^4+x^8}\,dx &= \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) \\ &+ \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) \\ &+ \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) + 1/40*sqrt(10*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.09

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = -\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20}$$

```
[In] int(-(x^4 - 1)/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5
^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2
))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*(3*
5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1/2)
+ 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1
)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7
i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) - 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1
/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(
- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/2)*1i)/20
```

3.28 $\int \frac{1-x^4}{1-4x^4+x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 165

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

```
[Out] 1/4*arctan(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(-3+3*3^(1/2))^(1/2)+1/4*ar
ctanh(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(-3+3*3^(1/2))^(1/2)+1/4*arctan(
2^(1/4)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(3+3*3^(1/2))^(1/2)+1/4*arctanh(2^(1/4
)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(3+3*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1433, 1107, 213, 209}

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

```
[In] Int[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sqrt[3])]) +
 ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])]) +
ArcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sqrt[3])])
+ ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{2}x^2+x^4}\,dx\right) - \frac{1}{2}\int \frac{1}{-1+\sqrt{2}x^2+x^4}\,dx$$

$$= -\frac{\int \frac{1}{-\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}} + \frac{\int \frac{1}{\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}} - \frac{\int \frac{1}{-\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}} + \frac{\int \frac{1}{\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.33

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = -\frac{1}{8}\text{RootSum}\left[1-4\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

```
[In] Integrate[(1 - x^4)/(1 - 4*x^4 + x^8),x]
[Out] -1/8*RootSum[1 - 4*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-2*#1
^3 + #1^7) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	42
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	42

```
[In] int((-x^4+1)/(x^8-4*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*sum((-_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(109) = 218.

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.92

$$
\begin{aligned}
\int \frac{1-x^4}{1-4x^4+x^8}\,dx = & -\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(\sqrt{6}\left(\sqrt{3}-3\right)\sqrt{-\sqrt{\sqrt{3}+2}}+6x\right) \\
& +\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(-\sqrt{6}\left(\sqrt{3}-3\right)\sqrt{-\sqrt{\sqrt{3}+2}}+6x\right) \\
& +\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(\sqrt{6}\left(\sqrt{3}+3\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+6x\right) \\
& -\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(-\sqrt{6}\left(\sqrt{3}+3\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+6x\right) \\
& -\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right) \\
& +\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right) \\
& +\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right) \\
& -\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right)
\end{aligned}
$$

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*sqrt(-sqrt(sqrt(3) + 2))*log(sqrt(6)*(sqrt(3) - 3)*sqrt(-sqrt
(sqrt(3) + 2)) + 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(sqrt(3) + 2))*log(-sqrt(6)*
(sqrt(3) - 3)*sqrt(-sqrt(sqrt(3) + 2)) + 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(-sq
rt(3) + 2))*log(sqrt(6)*(sqrt(3) + 3)*sqrt(-sqrt(-sqrt(3) + 2)) + 6*x) - 1/
24*sqrt(6)*sqrt(-sqrt(-sqrt(3) + 2))*log(-sqrt(6)*(sqrt(3) + 3)*sqrt(-sqrt(
-sqrt(3) + 2)) + 6*x) - 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4)*log(sqrt(6)*(sqrt(
3) + 2)^(1/4)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4)*log(-
sqrt(6)*(sqrt(3) + 2)^(1/4)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(-sqrt(3) +
 2)^(1/4)*log(sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2)^(1/4) + 6*x) - 1/24*sqrt
(6)*(-sqrt(3) + 2)^(1/4)*log(-sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2)^(1/4) +
6*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.16

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx$$
$$= -\operatorname{RootSum}\left(84934656t^8 - 36864t^4 + 1, \left(t \mapsto t\log\left(36864t^5 - 20t + x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8-4*x**4+1),x)
[Out] -RootSum(84934656*_t**8 - 36864*_t**4 + 1, Lambda(_t, _t*log(36864*_t**5 -
20*_t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-4\,x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-4\,x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
[Out] integrate(-(x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.42

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}}{80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{112\sqrt{3}\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}}{3\left(80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}\right)}\right)\left(\sqrt{3}+2\right)^{1/4}}{12} + \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}64\mathrm{i}}{48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}}-\frac{\sqrt{3}\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}112\mathrm{i}}{3\left(48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}\right)}\right)\left(2-\sqrt{3}\right)^{1/4}1\mathrm{i}}{12} - \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}}{48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}}-\frac{112\sqrt{3}\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}}{3\left(48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}\right)}\right)\left(2-\sqrt{3}\right)^{1/4}}{12} - \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}64\mathrm{i}}{80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}112\mathrm{i}}{3\left(80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}\right)}\right)\left(\sqrt{3}+2\right)^{1/4}1\mathrm{i}}{12}$$

```
[In] int(-(x^4 - 1)/(x^8 - 4*x^4 + 1),x)
```

```
[Out] (6^(1/2)*atan((6^(1/2)*x*(2 - 3^(1/2))^(1/4)*64i)/(48*3^(1/2)*(2 - 3^(1/2))
^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (3^(1/2)*6^(1/2)*x*(2 - 3^(1/2))^(1/4)*1
12i)/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2))))*(2 - 3^
(1/2))^(1/4)*1i)/12 - (6^(1/2)*atan((64*6^(1/2)*x*(2 - 3^(1/2))^(1/4))/(48*
3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (112*3^(1/2)*6^(1/2
)*x*(2 - 3^(1/2))^(1/4))/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/
2))^(1/2))))*(2 - 3^(1/2))^(1/4))/12 + (6^(1/2)*atan((64*6^(1/2)*x*(3^(1/2)
 + 2)^(1/4))/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2)) + (1
12*3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3*(80*(3^(1/2) + 2)^(1/2) + 48*3
^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)^(1/4))/12 - (6^(1/2)*atan((6^(1
/2)*x*(3^(1/2) + 2)^(1/4)*64i)/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2
) + 2)^(1/2)) + (3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4)*112i)/(3*(80*(3^(1/2
) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)^(1/4)*1i)/12
```

3.29 $\int \frac{1-x^4}{1-5x^4+x^8} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{1-x^4}{1-5x^4+x^8} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}} + \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

```
[Out] arctan(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-14*3^(1/2)+14*7^(1/2))^(1/2)+ar
ctanh(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-14*3^(1/2)+14*7^(1/2))^(1/2)+arc
tan(x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(14*3^(1/2)+14*7^(1/2))^(1/2)+arctan
h(x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(14*3^(1/2)+14*7^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1433, 1107, 213, 209}

$$\int \frac{1-x^4}{1-5x^4+x^8} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}} + \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

```
[In] Int[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7])] + ArcT
an[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sq
rt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[
2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sqrt[7])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int\frac{1}{-1-\sqrt{3}x^2+x^4}\,dx\right)-\frac{1}{2}\int\frac{1}{-1+\sqrt{3}x^2+x^4}\,dx\\
&= -\frac{\int\frac{1}{-\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}}-\frac{\int\frac{1}{\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}}+\frac{\int\frac{1}{-\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}}+\frac{\int\frac{1}{\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}}\\
&= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-5\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-5\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - 5*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-5*#1
^3 + 2*#1^7) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.26

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	44

```
[In] int((-x^4+1)/(x^8-5*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(121) = 242$.

Time = 0.30 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.82

$$\begin{aligned}
\int \frac{1-x^4}{1-5x^4+x^8}\,dx &= -\frac{1}{56}\,\sqrt{14}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\,\sqrt{14}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad-\frac{1}{56}\,\sqrt{14}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\,\sqrt{14}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\,\sqrt{14}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad-\frac{1}{56}\,\sqrt{14}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\,\sqrt{14}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad-\frac{1}{56}\,\sqrt{14}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)
\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7
)*sqrt(3) - 7)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(
14)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3)
- 7)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) - 1/56*sqrt(14)*sqrt(-
sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) - 7)*sqrt(
-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(-sqrt(2)*s
qrt(sqrt(7)*sqrt(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) - 7)*sqrt(-sqrt(2)
*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(-sqrt
(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*sqrt(-sqr
t(7)*sqrt(3) + 5)) + 28*x) - 1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(
3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqr
t(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)
```

```
)*log(sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) +
5)) + 28*x) - 1/56*sqrt(14)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-
sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) +
28*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.15

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx$$
$$= -\operatorname{RootSum}\left(157351936t^8-62720t^4+1,\left(t\mapsto t\log\left(50176t^5-24t+x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8-5*x**4+1),x)
[Out] -RootSum(157351936*_t**8 - 62720*_t**4 + 1, Lambda(_t, _t*log(50176*_t**5 -
 24*_t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-5\,x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-5\,x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
[Out] integrate(-(x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.86

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = \frac{2^{3/4}\sqrt{7}\operatorname{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\left(5-\sqrt{21}\right)^{1/4}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{28} - \frac{2^{3/4}\sqrt{7}\operatorname{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\left(5-\sqrt{21}\right)^{1/4}405\mathrm{i}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}621\mathrm{i}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}1\mathrm{i}}{28} + \frac{2^{3/4}\sqrt{7}\operatorname{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\left(\sqrt{21}+5\right)^{1/4}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{28} - \frac{2^{3/4}\sqrt{7}\operatorname{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\left(\sqrt{21}+5\right)^{1/4}405\mathrm{i}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}621\mathrm{i}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}1\mathrm{i}}{28}$$

```
[In] int(-(x^4 - 1)/(x^8 - 5*x^4 + 1),x)
```

```
[Out] (2^(3/4)*7^(1/2)*atan((405*2^(3/4)*7^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(243*
2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))) -
 (621*2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(14*(243*2^(1/2)*(5
- 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))*(5 - 21^(1/
2))^(1/4))/28 - (2^(3/4)*7^(1/2)*atan((2^(3/4)*7^(1/2)*x*(5 - 21^(1/2))^(1/
4)*405i)/(2*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21
^(1/2))^(1/2))) - (2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4)*621i)/(1
4*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1
/2))))*(5 - 21^(1/2))^(1/4)*1i)/28 + (2^(3/4)*7^(1/2)*atan((405*2^(3/4)*7^(
1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) + 54*2^(1
/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (621*2^(3/4)*7^(1/2)*21^(1/2)*x*(21^(
1/2) + 5)^(1/4))/(14*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) + 54*2^(1/2)*21^(1/2
)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4))/28 - (2^(3/4)*7^(1/2)*atan(
(2^(3/4)*7^(1/2)*x*(21^(1/2) + 5)^(1/4)*405i)/(2*(243*2^(1/2)*(21^(1/2) + 5
)^(1/2) + 54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (2^(3/4)*7^(1/2)*21^
(1/2)*x*(21^(1/2) + 5)^(1/4)*621i)/(14*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) +
54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4)*1i)/28
```

3.30 $\int \frac{1-x^4}{1-6x^4+x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 125

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[Out] 1/4*arctan(x/(2^(1/2)-1)^(1/2))/(-2+2*2^(1/2))^(1/2)+1/4*arctanh(x/(2^(1/2)
-1)^(1/2))/(-2+2*2^(1/2))^(1/2)+1/4*arctan(x/(1+2^(1/2))^(1/2))/(2+2*2^(1/2
))^(1/2)+1/4*arctanh(x/(1+2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1433, 1107, 213, 209}

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[In] Int[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[2*(-1 + Sqrt[2])]) + ArcTan[x/Sqrt[1 +
 Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqr
t[2*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])
])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{-1-2x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+2x^2+x^4}\,dx\\
&= -\frac{\int \frac{1}{-1-\sqrt{2}+x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{1}{1-\sqrt{2}+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{-1+\sqrt{2}+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{1+\sqrt{2}+x^2}\,dx}{4\sqrt{2}}\\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx$$
$$= \frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)+\sqrt{-1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)+\sqrt{-1+\sqrt{2}}}{4\sqrt{2}}$$

```
[In] Integrate[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] + Sqrt[-1 + Sqrt[2]]*ArcTan
[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] + S
qrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/(4*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-4\_Z^2-1\right)}\_R\ln\left(-2\_R^3+3\_R+x\right)\right)}{8}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4+4\_Z^2-1\right)}\_R\ln\left(2\_R^3+3\_R+x\right)\right)}{8}$
default	$\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\sqrt{2}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}$

```
[In] int((-x^4+1)/(x^8-6*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(_R*ln(-2*_R^3+3*_R+x),_R=RootOf(4*_Z^4-4*_Z^2-1))+1/8*sum(_R*ln(2*_
R^3+3*_R+x),_R=RootOf(4*_Z^4+4*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.96

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx = \frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right) - \frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right) + \frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right) - \frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right) + \frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}+1}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right) - \frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}+1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right) + \frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}-1}\log\left(\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right) - \frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}-1}\log\left(-\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right)$$

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*sqrt(sqrt(2) - 1)*log((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) - 1
/16*sqrt(2)*sqrt(sqrt(2) - 1)*log(-(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1
/16*sqrt(2)*sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) - 1/
16*sqrt(2)*sqrt(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) + 1/
16*sqrt(2)*sqrt(-sqrt(2) + 1)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 1) + x) - 1
/16*sqrt(2)*sqrt(-sqrt(2) + 1)*log(-(sqrt(2) + 1)*sqrt(-sqrt(2) + 1) + x) +
 1/16*sqrt(2)*sqrt(-sqrt(2) - 1)*log((sqrt(2) - 1)*sqrt(-sqrt(2) - 1) + x)
- 1/16*sqrt(2)*sqrt(-sqrt(2) - 1)*log(-(sqrt(2) - 1)*sqrt(-sqrt(2) - 1) + x
)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx = -\operatorname{RootSum}\left(16384t^4-256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right) - \operatorname{RootSum}\left(16384t^4+256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] -RootSum(16384*_t**4 - 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t**5 - 28*_t
 + x))) - RootSum(16384*_t**4 + 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t**
5 - 28*_t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-6\,x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 6*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1-x^4}{1-6x^4+x^8}\,dx = {} & \frac{1}{8}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right) \\ & + \frac{1}{8}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) \\ & + \frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right) \\ & - \frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right) \\ & + \frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right) \\ & - \frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) - 2)*arctan(x/sqrt(sqrt(2) + 1)) + 1/8*sqrt(2*sqrt(2) +
2)*arctan(x/sqrt(sqrt(2) - 1)) + 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x + sqrt(
sqrt(2) + 1))) - 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x - sqrt(sqrt(2) + 1))) +
 1/16*sqrt(2*sqrt(2) + 2)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/16*sqrt(2*sqr
t(2) + 2)*log(abs(x - sqrt(sqrt(2) - 1)))
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.96

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx = -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{1-\sqrt{2}}\,4352\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{2}}\,3072\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\,\sqrt{1-\sqrt{2}}\,1\mathrm{i}}{8} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{2}-1}\,4352\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{2}-1}\,3072\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\,\sqrt{-\sqrt{2}-1}\,1\mathrm{i}}{8} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{2}-1}\,4352\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}-1}\,3072\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\,\sqrt{\sqrt{2}-1}\,1\mathrm{i}}{8} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{2}+1}\,4352\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}+1}\,3072\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\,\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{8}$$

```
[In] int(-(x^4 - 1)/(x^8 - 6*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((x*(- 2^(1/2) - 1)^(1/2)*4352i)/(3072*2^(1/2) + 4352) + (2^(1
/2)*x*(- 2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) + 4352))*(- 2^(1/2) - 1)^(
1/2)*1i)/8 - (2^(1/2)*atan((x*(1 - 2^(1/2))^(1/2)*4352i)/(3072*2^(1/2) - 43
52) - (2^(1/2)*x*(1 - 2^(1/2))^(1/2)*3072i)/(3072*2^(1/2) - 4352))*(1 - 2^(
1/2))^(1/2)*1i)/8 + (2^(1/2)*atan((x*(2^(1/2) - 1)^(1/2)*4352i)/(3072*2^(1/
2) - 4352) - (2^(1/2)*x*(2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) - 4352))*(
2^(1/2) - 1)^(1/2)*1i)/8 - (2^(1/2)*atan((x*(2^(1/2) + 1)^(1/2)*4352i)/(307
2*2^(1/2) + 4352) + (2^(1/2)*x*(2^(1/2) + 1)^(1/2)*3072i)/(3072*2^(1/2) + 4
352))*(2^(1/2) + 1)^(1/2)*1i)/8
```

3.31 $\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}}$$

```
[Out] -1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/
2)+1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1
/2)-1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)+1/4*ln(1+x^2+x*(1/2*6
^(1/2)-1/2*2^(1/2)))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1437, 1175, 632, 210, 1178, 642}

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2}}$$

```
[In] Int[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2]) + ArcTan[(Sq
rt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - Log[1 - Sqrt[2 - Sqrt[3
]]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{3}\left(-1+\sqrt{3}\right)+\left(3-\sqrt{3}\right)x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}\left(-1+\sqrt{3}\right)+\left(-3+\sqrt{3}\right)x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2}\,dx}{2\sqrt{2}} + \frac{1}{4}\left(-1+\sqrt{3}\right)\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx + \frac{1}{4}\left(-1+\sqrt{3}\right)\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}} + \frac{1}{2}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right) + \frac{1}{2}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4 + \#1^8\&, \frac{-\log(x-\#1)+\sqrt{3}\log(x-\#1)+2\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Sqrt[3]*Log[x - #1] + 2*Log[x -
 #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-1+2\_R^{4}+\sqrt{3}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$
risch	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{3}-1}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^{3}}{\sqrt{3}-1}-\frac{\sqrt{2}\,x}{\sqrt{3}-1}+\frac{\sqrt{3}\,\sqrt{2}\,x}{2}-\frac{x\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\ln\left(2x^{2}+\left(\sqrt{3}\,\sqrt{2}-\sqrt{2}\right)x+2\right)}{4}-\frac{\sqrt{2}\,\ln\left(2x^{2}+\left(-\sqrt{3}\,\sqrt{2}+\right.\right.}{4}$

```
[In] int((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/(2*_R^7-_R^3)*(-1+2*_R^4+3^(1/2))*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$$
$$=\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x^3+\frac{1}{2}\sqrt{2}(x^3-2x)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x+\frac{1}{2}\sqrt{2}x\right)$$
$$+\frac{1}{4}\sqrt{2}\log\left(\frac{x^8+4x^6+5x^4+4x^2-\sqrt{2}(x^7+4x^5+4x^3+x)-\sqrt{3}(2x^6+4x^4+2x^2-\sqrt{2}(x^7+2x^5+}{x^8-x^4+1}\right.$$

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x^3 + 1/2*sqrt(2)*(x^3 - 2*x)) + 1/2
*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x + 1/2*sqrt(2)*x) + 1/4*sqrt(2)*log((x
^8 + 4*x^6 + 5*x^4 + 4*x^2 - sqrt(2)*(x^7 + 4*x^5 + 4*x^3 + x) - sqrt(3)*(2
*x^6 + 4*x^4 + 2*x^2 - sqrt(2)*(x^7 + 2*x^5 + 2*x^3 + x)) + 1)/(x^8 - x^4 +
 1))
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(x\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)\right)+2\operatorname{atan}\left(x^3\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)-\sqrt{2}x\right)\right)}{4} - \frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4} + \frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4}$$

```
[In] integrate((-1+2*x**4+3**(1/2))/(x**8-x**4+1),x)
[Out] sqrt(2)*(2*atan(x*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3)))) + 2*at
an(x**3*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3))) - sqrt(2)*x))/4 -
 sqrt(2)*log(x**2 - sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sqrt(3) + 2))/4
 + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sqrt(3
) + 2))/4 + 1)/4
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \int \frac{2\,x^4+\sqrt{3}-1}{x^8-x^4+1}\,dx$$

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{4}\sqrt{2}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{4}\sqrt{2}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/2*sqr
t(2)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/4*sqrt(2)*lo
g(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/4*sqrt(2)*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{72\,\sqrt{2}\,x}{144\,\sqrt{3}-144\,\sqrt{3}\,x^2-288\,x^2+288}+\frac{72\,\sqrt{2}\,\sqrt{3}\,x}{144\,\sqrt{3}-144\,\sqrt{3}\,x^2-288\,x^2+288}\right)}{2} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{72\,\sqrt{2}\,x}{144\,\sqrt{3}+144\,\sqrt{3}\,x^2+288\,x^2+288}+\frac{72\,\sqrt{2}\,\sqrt{3}\,x}{144\,\sqrt{3}+144\,\sqrt{3}\,x^2+288\,x^2+288}\right)}{2}$$

```
[In] int((3^(1/2) + 2*x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((72*2^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 288
) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 288))
)/2 + (2^(1/2)*atanh((72*2^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288*x^
2 + 288) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288*x^2
+ 288)))/2
```

3.32 $\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\begin{aligned}\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx &= -\frac{1}{2}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &\quad+\frac{1}{2}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\ &\quad-\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\ &\quad+\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\end{aligned}$$

```
[Out] -1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))+1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-
1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))-1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))+1/4*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*
(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

= {1437, 1175, 632, 210, 1178, 642}

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = -\frac{1}{2}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{2}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right) + \frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)$$

```
[In] Int[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/2*(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]
) + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])
/2 - (Sqrt[2 + Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 + (Sqrt[2 + S
qrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{3}+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}-\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&\quad - \frac{1}{4}\sqrt{2+\sqrt{3}}\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2}\,dx \\
&\quad - \frac{1}{4}\sqrt{2+\sqrt{3}}\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2}\,dx \\
&= -\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}} \\
&\quad - \frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4+\sqrt{3}\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4 + Sqrt[3]*Log[x
 - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.38

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(2\_R^4+2\sqrt{3}\_R^4+\left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{8}$	62

```
[In] int((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(2*_R^4+2*3^(1/2)*_R^4+(1+3^(1/2))*(3^(1/2)-1))*ln(
x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(-\left(x^3-\sqrt{3}x+x\right)\sqrt{\sqrt{3}+2}\right)+\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(x\sqrt{\sqrt{3}+2}\right)$$
$$+\frac{1}{4}\sqrt{\sqrt{3}+2}\log\left(\frac{x^8+4\,x^6+5\,x^4+4\,x^2-2\,\sqrt{3}(x^6+2\,x^4+x^2)+2\,(2\,x^7+5\,x^5+5\,x^3-\sqrt{3}(x^7+3\,x^5\right.}{x^8-x^4+1}$$

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(sqrt(3) + 2)*arctan(-(x^3 - sqrt(3)*x + x)*sqrt(sqrt(3) + 2)) + 1
/2*sqrt(sqrt(3) + 2)*arctan(x*sqrt(sqrt(3) + 2)) + 1/4*sqrt(sqrt(3) + 2)*lo
g((x^8 + 4*x^6 + 5*x^4 + 4*x^2 - 2*sqrt(3)*(x^6 + 2*x^4 + x^2) + 2*(2*x^7 +
 5*x^5 + 5*x^3 - sqrt(3)*(x^7 + 3*x^5 + 3*x^3 + x) + 2*x)*sqrt(sqrt(3) + 2)
 + 1)/(x^8 - x^4 + 1))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((1+x**4*(1+3**(1/2)))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(239467000838037598029035598269032581
07519197671516525068420004029031894115942 4*_t**88 + 13825633739587334576280
342370533073164132612616075147807283055647306312738 4064*sqrt(3)*_t**88 - 5
732624312622
```

Maxima [F]

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \int \frac{x^4\left(\sqrt{3}+1\right)+1}{x^8-x^4+1}\,dx$$

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx &= \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &\quad+\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &\quad+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &\quad-\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2)
)) + 1/4*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sq
rt(2))) + 1/8*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1)
- 1/8*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx=0$$

```
[In] int((x^4*(3^(1/2) + 1) + 1)/(x^8 - x^4 + 1),x)
```

```
[Out] 0
```

3.33 $\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 180

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)$$

```
[Out] 1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(3/2*2
^(1/2)-1/2*6^(1/2))-1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1
/2*2^(1/2)))*(3/2*2^(1/2)-1/2*6^(1/2))+1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1
/2)))*(3/2*2^(1/2)-1/2*6^(1/2))-1/4*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(
3/2*2^(1/2)-1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{1437, 1175, 632, 210, 1178, 642\}$

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right) - \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)$$

```
[In] Int[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])
/2 - (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[
3]]])/2 + (Sqrt[3*(2 - Sqrt[3])]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 - (S
qrt[3*(2 - Sqrt[3])]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{3}\left(3-2\sqrt{3}\right)+\left(-6+3\sqrt{3}\right)x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}\left(3-2\sqrt{3}\right)+\left(6-3\sqrt{3}\right)x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2}\,dx \\
&\quad +\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2}\,dx \\
&\quad -\frac{1}{4}\left(-3+2\sqrt{3}\right)\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&\quad -\frac{1}{4}\left(-3+2\sqrt{3}\right)\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&= \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) \\
&\quad -\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) \\
&\quad -\frac{1}{2}\left(3-2\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx,x,-\sqrt{2+\sqrt{3}}+2x\right) \\
&\quad -\frac{1}{2}\left(3-2\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx,x,\sqrt{2+\sqrt{3}}+2x\right)
\end{aligned}
$$

$$= \frac{1}{2}\sqrt{6-3\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{2}\sqrt{6-3\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x + x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4 + \#1^8\&, \frac{3\log(x-\#1)-2\sqrt{3}\log(x-\#1)-3\log(x-\#1)\#1^4+\sqrt{3}\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
[Out] RootSum[1 - #1^4 + #1^8 & , (3*Log[x - #1] - 2*Sqrt[3]*Log[x - #1] - 3*Log[
x - #1]*#1^4 + Sqrt[3]*Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-6\_R^4+2\sqrt{3}\_R^4+\left(-3+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}}{8}$	62

```
[In] int((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(-6*_R^4+2*3^(1/2)*_R^4+(-3+3^(1/2))*(3^(1/2)-1))*l
n(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}\left(3x^3+\sqrt{3}(2x^3-x)-3x\right)\sqrt{-3\sqrt{3}+6}\right)$$
$$-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}\left(2\sqrt{3}x+3x\right)\sqrt{-3\sqrt{3}+6}\right)$$
$$+\frac{1}{4}\sqrt{-3\sqrt{3}+6}\log\left(\frac{3x^8+12x^6+15x^4+12x^2-6\sqrt{3}(x^6+2x^4+x^2)+2\left(3x^5+3x^3-\sqrt{3}(x^7+x^5\right.}{x^8-x^4+1}\right.$$

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-3*sqrt(3) + 6)*arctan(1/3*(3*x^3 + sqrt(3)*(2*x^3 - x) - 3*x)*sq
rt(-3*sqrt(3) + 6)) - 1/2*sqrt(-3*sqrt(3) + 6)*arctan(1/3*(2*sqrt(3)*x + 3*
x)*sqrt(-3*sqrt(3) + 6)) + 1/4*sqrt(-3*sqrt(3) + 6)*log((3*x^8 + 12*x^6 + 1
5*x^4 + 12*x^2 - 6*sqrt(3)*(x^6 + 2*x^4 + x^2) + 2*(3*x^5 + 3*x^3 - sqrt(3)
*(x^7 + x^5 + x^3 + x))*sqrt(-3*sqrt(3) + 6) + 3)/(x^8 - x^4 + 1))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((3+x**4*(-3+3**(1/2))-2*3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(-36944369544063775196667969536*_t**
32 + 21329841701306232282053345280*sqrt(3)*_t**32 - 16711108317303678380308
7978496*sqrt(3)*_t**28 + 289444886563568182740740210688*_t**28 - 9921139603
646460044679
```

Maxima [F]

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx=\int \frac{x^4\left(\sqrt{3}-3\right)-2\sqrt{3}+3}{x^8-x^4+1}\,dx$$

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx=&\ \frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\&+\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\&+\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\&-\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6)
- sqrt(2))) + 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2))
 + 1) - 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx=0$$

```
[In] int((x^4*(3^(1/2) - 3) - 2*3^(1/2) + 3)/(x^8 - x^4 + 1),x)
[Out] 0
```

3.34 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} \, dx = \frac{dx}{c} - \frac{\sqrt{a} d \arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a+cx^2)}{2c}$$

```
[Out] d*x/c+1/2*e*ln(c*x^2+a)/c-d*arctan(x*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1408, 788, 649, 211, 266}

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} \, dx = -\frac{\sqrt{a} d \arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a+cx^2)}{2c} + \frac{dx}{c}$$

```
[In] Int[(d + e/x)/(c + a/x^2),x]
```

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x^
2])/(2*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(e+dx)}{a+cx^2}\,dx \\
&= \frac{dx}{c} + \frac{\int \frac{-ad+cex}{a+cx^2}\,dx}{c} \\
&= \frac{dx}{c} - \frac{(ad)\int \frac{1}{a+cx^2}\,dx}{c} + e\int \frac{x}{a+cx^2}\,dx \\
&= \frac{dx}{c} - \frac{\sqrt{a}d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e\log(a+cx^2)}{2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$
\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = \frac{dx}{c} - \frac{\sqrt{a}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e\log(a+cx^2)}{2c}
$$

```
[In] Integrate[(d + e/x)/(c + a/x^2),x]
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x^
2])/(2*c)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{dx}{c}+\frac{\frac{e\ln\left(c\,x^{2}+a\right)}{2}-\frac{da\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$	42
risch	$\frac{dx}{c}+\frac{\ln(-\sqrt{-ac}\,x-a)d\sqrt{-ac}}{2c^{2}}+\frac{\ln(-\sqrt{-ac}\,x-a)e}{2c}-\frac{\ln(\sqrt{-ac}\,x-a)d\sqrt{-ac}}{2c^{2}}+\frac{\ln(\sqrt{-ac}\,x-a)e}{2c}$	98

```
[In] int((d+e/x)/(c+a/x^2),x,method=_RETURNVERBOSE)
[Out] d*x/c+1/c*(1/2*e*ln(c*x^2+a)-d*a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = \left[\frac{d\sqrt{-\frac{a}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{a}{c}}-a}{cx^2+a}\right)+2\,dx+e\log\left(cx^2+a\right)}{2\,c},\ -\frac{2\,d\sqrt{\frac{a}{c}}\arctan\left(\frac{cx\sqrt{\frac{a}{c}}}{a}\right)-2\,dx-e\log\left(cx^2+a\right)}{2\,c}\right]$$

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="fricas")
[Out] [1/2*(d*sqrt(-a/c)*log((c*x^2 - 2*c*x*sqrt(-a/c) - a)/(c*x^2 + a)) + 2*d*x
+ e*log(c*x^2 + a))/c, -1/2*(2*d*sqrt(a/c)*arctan(c*x*sqrt(a/c)/a) - 2*d*x
- e*log(c*x^2 + a))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = \left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2c^3}\right)+e}{d}\right)+\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2c^3}\right)+e}{d}\right)+\frac{dx}{c}$$

```
[In] integrate((d+e/x)/(c+a/x**2),x)
```

```
[Out] (e/(2*c) - d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(e/(2*c) - d*sqrt(-a*c**
3)/(2*c**3)) + e)/d) + (e/(2*c) + d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(
e/(2*c) + d*sqrt(-a*c**3)/(2*c**3)) + e)/d) + d*x/c
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = -\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{dx}{c} + \frac{e\log\left(cx^2+a\right)}{2\,c}$$

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="maxima")
```

```
[Out] -a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = -\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{dx}{c} + \frac{e\log\left(cx^2+a\right)}{2\,c}$$

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="giac")
```

```
[Out] -a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = \frac{e\,\ln\left(c\,x^2+a\right)}{2\,c} + \frac{d\,x}{c} - \frac{\sqrt{a}\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

```
[In] int((d + e/x)/(c + a/x^2),x)
```

```
[Out] (e*log(a + c*x^2))/(2*c) + (d*x)/c - (a^(1/2)*d*atan((c^(1/2)*x)/a^(1/2)))/
c^(3/2)
```

3.35 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}} dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}} dx = \frac{dx}{c} - \frac{(b^2d-2acd-bce)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{(bd-ce)\log(a+bx+cx^2)}{2c^2}$$

```
[Out] d*x/c-1/2*(b*d-c*e)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*d+b^2*d-b*c*e)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1407, 787, 648, 632, 212, 642}

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}} dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-2acd+b^2d-bce)}{c^2\sqrt{b^2-4ac}} - \frac{(bd-ce)\log(a+bx+cx^2)}{2c^2} + \frac{dx}{c}$$

```
[In] Int[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] (d*x)/c - ((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(c^2*Sqrt[b^2 - 4*a*c]) - ((b*d - c*e)*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(e+dx)}{a+bx+cx^2}\,dx \\
&= \frac{dx}{c} + \frac{\int \frac{-ad+(-bd+ce)x}{a+bx+cx^2}\,dx}{c} \\
&= \frac{dx}{c} - \frac{(bd-ce)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} + \frac{(b^2d-2acd-bce)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}
\end{aligned}$$

$$= \frac{dx}{c} - \frac{(bd-ce)\log(a+bx+cx^2)}{2c^2} - \frac{(b^2d-2acd-bce)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^2}$$

$$= \frac{dx}{c} - \frac{(b^2d-2acd-bce)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{(bd-ce)\log(a+bx+cx^2)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{2cdx + \frac{2(b^2d-2acd-bce)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (-bd+ce)\log(a+x(b+cx))}{2c^2}$$

```
[In] Integrate[(d + e/x)/(c + a/x^2 + b/x),x]
[Out] (2*c*d*x + (2*(b^2*d - 2*a*c*d - b*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (-(b*d) + c*e)*Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{dx}{c}+\frac{\frac{(-bd+ec)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(-da-\frac{(-bd+ec)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$	90
risch	Expression too large to display	1357

```
[In] int((d+e/x)/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
[Out] d*x/c+1/c*(1/2*(-b*d+c*e)/c*ln(c*x^2+b*x+a)+2*(-d*a-1/2*(-b*d+c*e)*b/c)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.38

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= \left[\frac{2(b^2c-4ac^2)dx + (bce-(b^2-2ac)d)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right) - ((b^3-4abc}{2(b^2c^2-4ac^3)}\right.$$

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*d*x + (b*c*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 + b*
x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*d*x + 2*(b*c*e - (b^2
 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
 - 4*a*c)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 + b*x + a)
)/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(82) = 164$.

Time = 0.71 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.92

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}\right.}{2acd-b^2d+bce}\right. + \left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\right.}{2acd-b^2d+bce}\right. + \frac{dx}{c}$$

```
[In] integrate((d+e/x)/(c+a/x**2+b/x),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) -
(b*d - c*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(-sqrt(-4*a*c + b**2)*(2*a
*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) + 2*
a*c*e + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*
a*c - b**2)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + (sqrt(-
4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c
*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*d - b*
*2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) + 2*a*c*e + b
**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2
)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + d*x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{dx}{c} - \frac{(bd-ce)\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{(b^2d-2\,acd-bce)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] d*x/c - 1/2*(b*d - c*e)*log(c*x^2 + b*x + a)/c^2 + (b^2*d - 2*a*c*d - b*c*e
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.48

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(d\,b^3-e\,b^2\,c-4\,a\,d\,b\,c+4\,a\,e\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} + \frac{d\,x}{c} - \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(-d\,b^2+c\,e\,b+2\,a\,c\,d\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int((d + e/x)/(c + a/x^2 + b/x),x)
```

```
[Out] (log(a + b*x + c*x^2)*(b^3*d + 4*a*c^2*e - b^2*c*e - 4*a*b*c*d))/(2*(4*a*c^
3 - b^2*c^2)) + (d*x)/c - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^
2)^(1/2))*(2*a*c*d - b^2*d + b*c*e))/(c^2*(4*a*c - b^2)^(1/2))
```

3.36 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 253

$$\begin{aligned}\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = {} & \frac{dx}{c}+\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}} \\ & -\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}} \\ & +\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} \\ & -\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}\end{aligned}$$

```
[Out] d*x/c-1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)-e*c^(1/2))/a^(1/4
)/c^(5/4)*2^(1/2)-1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)-e*c^(1
/2))/a^(1/4)/c^(5/4)*2^(1/2)+1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(d*a^(1/2)+e*c^(1/2))/a^(1/4)/c^(5/4)*2^(1/2)-1/8*ln(a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(d*a^(1/2)+e*c^(1/2))/a^(1/4)/c^(5/4)*2^(
1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1408, 1294, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}d-\sqrt{c}e\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}d-\sqrt{c}e\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}} + \frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} - \frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} + \frac{dx}{c}$$

```
[In] Int[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5/4)) + ((Sqrt[a]*d + Sqrt[c]*e
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4
)*c^(5/4)) - ((Sqrt[a]*d + Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(e+dx^2)}{a+cx^4}\,dx\\ &= \frac{dx}{c} - \frac{\int \frac{ad-cex^2}{a+cx^4}\,dx}{c}\\ &= \frac{dx}{c} - \frac{\left(\frac{\sqrt{a}d}{\sqrt{c}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c} - \frac{\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}\end{aligned}$$

$$
\begin{aligned}
&= \frac{dx}{c} - \frac{\left(\frac{\sqrt{a}d}{\sqrt{c}} - e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} \, dx}{4c} - \frac{\left(\frac{\sqrt{a}d}{\sqrt{c}} - e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} \, dx}{4c} \\
&+ \frac{\left(\sqrt{a}d + \sqrt{c}e\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} \, dx}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} + \frac{\left(\sqrt{a}d + \sqrt{c}e\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} \, dx}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{dx}{c} + \frac{\left(\sqrt{a}d + \sqrt{c}e\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} \\
&- \frac{\left(\sqrt{a}d + \sqrt{c}e\right) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} \\
&- \frac{\left(\sqrt{a}d - \sqrt{c}e\right) \text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}} \\
&+ \frac{\left(\sqrt{a}d - \sqrt{c}e\right) \text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{dx}{c} + \frac{\left(\sqrt{a}d - \sqrt{c}e\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}} - \frac{\left(\sqrt{a}d - \sqrt{c}e\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}} \\
&+ \frac{\left(\sqrt{a}d + \sqrt{c}e\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} \\
&- \frac{\left(\sqrt{a}d + \sqrt{c}e\right) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.16

$$
\begin{aligned}
\int \frac{d + \frac{e}{x^2}}{c + \frac{a}{x^4}} \, dx &= \frac{dx}{c} + \frac{\left(-a^{5/4}\sqrt{c}d + a^{3/4}ce\right) \arctan\left(\frac{-\sqrt{2}\sqrt[4]{a} + 2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}ac^{7/4}} \\
&+ \frac{\left(-a^{5/4}\sqrt{c}d + a^{3/4}ce\right) \arctan\left(\frac{\sqrt{2}\sqrt[4]{a} + 2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}ac^{7/4}} \\
&+ \frac{\left(a^{5/4}\sqrt{c}d + a^{3/4}ce\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}ac^{7/4}} \\
&- \frac{\left(a^{5/4}\sqrt{c}d + a^{3/4}ce\right) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}ac^{7/4}}
\end{aligned}
$$

```
[In] Integrate[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((-(a^(5/4)*Sqrt[c]*d) + a^(3/4)*c*e)*ArcTan[(-(Sqrt[2]*a^(1/4))
+ 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*a*c^(7/4)) + ((-(a^(5/4)*Sqrt
[c]*d) + a^(3/4)*c*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/
4))])/(2*Sqrt[2]*a*c^(7/4)) + ((a^(5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a*c^(7/4)) - ((a^(
5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a*c^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.18

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^2 ce-da\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^2}$
default	$\frac{dx}{c}+\frac{-\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(-\cdots\right.\right.}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{c}$

```
[In] int((d+e/x^2)/(c+a/x^4),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/4/c^2*sum((_R^2*c*e-a*d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(172) = 344.

Time = 0.30 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.98

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \frac{c\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2\,acd^2e^2+c^2e^4}{ac^5}}+2\,de}{c^2}}\log\left(-(a^2d^4-c^2e^4)x+\left(ac^4e\sqrt{-\frac{a^2d^4-2\,acd^2e^2+c^2e^4}{ac^5}}+a^2cd^3-ac^2de^2\right)\sqrt{\frac{c^2\sqrt{-a\cdots}}{\cdots}}\right.}{\cdots}$$

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(c*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e
)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x + (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2
 + c^2*e^4)/(a*c^5)) + a^2*c*d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 -
```

```
2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)) - c*sqrt((c^2*sqrt(-(a^2*d
^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^
4)*x - (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + a^2*c*
d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c
^5)) + 2*d*e)/c^2)) - c*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4
)/(a*c^5)) - 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x + (a*c^4*e*sqrt(-(a^2*d
^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^
2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + c*sqr
t(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)*lo
g(-(a^2*d^4 - c^2*e^4)*x - (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^
4)/(a*c^5)) - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^
2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + 4*d*x)/c
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.43

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$$
$$= \operatorname{RootSum}\left(256t^4ac^5 - 64t^2ac^3de + a^2d^4 + 2acd^2e^2 + c^2e^4, \left(t \mapsto t\log\left(x + \frac{-64t^3ac^4e - 4ta^2cd^3 + 12tac}{a^2d^4 - c^2e^4}\right.\right.\right.$$
$$+ \frac{dx}{c}$$

```
[In] integrate((d+e/x**2)/(c+a/x**4),x)
```

```
[Out] RootSum(256*_t**4*a*c**5 - 64*_t**2*a*c**3*d*e + a**2*d**4 + 2*a*c*d**2*e**
2 + c**2*e**4, Lambda(_t, _t*log(x + (-64*_t**3*a*c**4*e - 4*_t*a**2*c*d**3
 + 12*_t*a*c**2*d*e**2)/(a**2*d**4 - c**2*e**4)))) + d*x/c
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.95

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \frac{dx}{c}$$
$$-\frac{\frac{2\sqrt{2}(a\sqrt{c}d-\sqrt{a}ce)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}(a\sqrt{c}d-\sqrt{a}ce)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(a\sqrt{c}d+\sqrt{a}ce)\log\left(\sqrt{c}x^2\right.}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8c}$$

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="maxima")
```

```
[Out] d*x/c - 1/8*(2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/2*s
qrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(a*sqrt(c)*d + sqrt(a)*c*e)*lo
g(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sq
rt(2)*(a*sqrt(c)*d + sqrt(a)*c*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)
*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \frac{dx}{c} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd-(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd-(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd+(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd+(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="giac")
```

```
[Out] d*x/c - 1/4*sqrt(2)*((a*c^3)^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) - 1/4*sqrt(2)*((a*c^3)
^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/8*sqrt(2)*((a*
c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.19

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx=\frac{d\,x}{c}-2\,\mathrm{atanh}\left(\frac{8\,a^2\,c\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{d\,e}{8\,c^2}-\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}-\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}-\frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{d\,e}{8\,c^2}-\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}-\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}\right)\sqrt{\frac{a\,d^2\,\sqrt{-a\,c^5}-c\,e^2\,\sqrt{-a\,c^5}+2\,a\,c^3\,d\,e}{16\,a\,c^5}}-2\,\mathrm{atanh}\left(\frac{8\,a^2\,c\,d^2\,x\,\sqrt{\frac{d\,e}{8\,c^2}-\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}+\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}-\frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d\,e}{8\,c^2}-\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}+\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}\right)\sqrt{\frac{c\,e^2\,\sqrt{-a\,c^5}-a\,d^2\,\sqrt{-a\,c^5}+2\,a\,c^3\,d\,e}{16\,a\,c^5}}$$

```
[In] int((d + e/x^2)/(c + a/x^4),x)
```

```
[Out] (d*x)/c - 2*atanh((8*a^2*c*d^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8*
c^2) - (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 + (
2*a^2*d^3*(-a*c^5)^(1/2))/c^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2) - (8*a*c^2*
e^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8*c^2) - (e^2*(-a*c^5)^(1/2))
/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 + (2*a^2*d^3*(-a*c^5)^(1/2))/c
^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2))*((a*d^2*(-a*c^5)^(1/2) - c*e^2*(-a*c^
5)^(1/2) + 2*a*c^3*d*e)/(16*a*c^5))^(1/2) - 2*atanh((8*a^2*c*d^2*x*((d*e)/(
8*c^2) - (d^2*(-a*c^5)^(1/2))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(
1/2))/(2*a^2*d^2*e - 2*a*c*e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 + (2*a*d*e^
2*(-a*c^5)^(1/2))/c^2) - (8*a*c^2*e^2*x*((d*e)/(8*c^2) - (d^2*(-a*c^5)^(1/2
))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*
e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 + (2*a*d*e^2*(-a*c^5)^(1/2))/c^2))*((c
*e^2*(-a*c^5)^(1/2) - a*d^2*(-a*c^5)^(1/2) + 2*a*c^3*d*e)/(16*a*c^5))^(1/2)
```

3.37 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 208

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] d*x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-c*e+(
2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-
c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1407, 1293, 1180, 211}

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}$$

```
[In] Int[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(e+dx^2)}{a+bx^2+cx^4}\,dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad+(bd-ce)x^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&\quad - \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}
\end{aligned}$$

$$= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.21

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \frac{dx}{c} - \frac{\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+bce-c\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b^2d-2acd+b\sqrt{b^2-4ac}d-bce-c\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[In] Integrate[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + b*c*e - c*Sqrt[b^2
 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*d - 2*a*
c*d + b*Sqrt[b^2 - 4*a*c]*d - b*c*e - c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.31

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left((-bd+ec)\_R^2-da\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{dx}{c}+\frac{\left(-bd\sqrt{-4ac+b^2}+ec\sqrt{-4ac+b^2}+2acd-b^2d+ebc\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-bd\sqrt{-4ac+b^2}+ec\sqrt{-4ac+b^2}-2acd-\right.}{2\sqrt{-4ac+b^2}\,c}$

```
[In] int((d+e/x^2)/(a/x^4+b/x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/2/c*sum(((-b*d+c*e)*_R^2-d*a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z
^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2540 vs. $2(172) = 344$.

Time = 0.46 (sec) , antiderivative size = 2540, normalized size of antiderivative = 12.21

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^
2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^
2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*
e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*
b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x + sqrt
(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b
^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*
e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^
3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))
*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^
3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 -
4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*
a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d
*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*
e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 -
 (b^3 + a*b*c)*d^3*e)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*
(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c
^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*
d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(
b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 -
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2
 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*
c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c
^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*
d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c
^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x + sqrt(1/2)*((b^4 - 5
*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^
```

```
3)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c
^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*
d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e
^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sq
rt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c -
 a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2
*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*
(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4
- (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^
2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2
*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*
d^3*e)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*
c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*
c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c
^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c
^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2
)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2
*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e
^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*d*x)/c
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\text{Timed out}$$

```
[In] integrate((d+e/x**2)/(c+a/x**4+b/x**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\int \frac{d+\frac{e}{x^2}}{c+\frac{b}{x^2}+\frac{a}{x^4}}\,dx$$

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate(-((b*d - c*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3179 vs. 2(172) = 344.

Time = 1.02 (sec) , antiderivative size = 3179, normalized size of antiderivative = 15.28

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] d*x/c + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*d -
(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 -
 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*
a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2
*c^4)*d*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^
5)*d + (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sq
```

rt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16 *a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*((2*b^5*c^2 - 16 *a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 *c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a *c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*d - (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4* c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*s qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8 *(b^2 - 4*a*c)*a*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a* b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2) *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c) *c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2* b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*d*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b ^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a *c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a *b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^ 4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2 *(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d + (2*b^4*c^5 - 8*a*b^2* c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4 *sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sq rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*s qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)* b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2)) /((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b ^2*c^5 - 4*a^2*c^6)*c^2)

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 6366, normalized size of antiderivative = 30.61

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

```
[In] int((d + e/x^2)/(c + a/x^4 + b/x^2),x)
```

```
[Out] (d*x)/c - atan((((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^
2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*
a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4
*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 -
 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4*d^2 - 2*a*c^3*e^2
 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*
e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2
 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^
2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*
c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4*d^2
- 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 +
 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2
*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c
^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 -
2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*
e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2
```

```
*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*
a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e
^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*d
^3 + a*b^2*d^2*e + a^2*c*d^2*e - 2*a*b*c*d*e^2))/c + (((16*a^2*c^3*d - 4*a*
b^2*c^2*d)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^
2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2
*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) + (2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3
*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*d^2 -
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^
2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*
a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4
*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 -
 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4*d^2 - 2*a*c^3*e^2
 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*
e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2
 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3
```

- 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^
2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*
c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4*d^2
- 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 +
6*a*b*c^2*d*e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2
*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c
^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 -
2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*
e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 + b^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*
a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e
^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*d
^3 + a*b^2*d^2*e + a^2*c*d^2*e - 2*a*b*c*d*e^2))/c + (((16*a^2*c^3*d - 4*a*
b^2*c^2*d)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^
2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2
*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) + (2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3
*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -

```
4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*d^2 +
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i
```

3.38 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 311

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx=\frac{dx}{c}-\frac{\sqrt[6]{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}}+\frac{\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{a}d+\sqrt{3}\sqrt{c}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}+\frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}$$

```
[Out] d*x/c-1/3*a^(1/6)*d*arctan(c^(1/6)*x/a^(1/6))/c^(7/6)-1/6*e*ln(a^(1/3)+c^(1
/3)*x^2)/a^(1/3)/c^(2/3)-1/12*ln(a^(1/3)+c^(1/3)*x^2+a^(1/6)*c^(1/6)*x*3^(1
/2))*(d*3^(1/2)*a^(1/2)-e*c^(1/2))/a^(1/3)/c^(7/6)+1/12*ln(a^(1/3)+c^(1/3)*
x^2-a^(1/6)*c^(1/6)*x*3^(1/2))*(d*3^(1/2)*a^(1/2)+e*c^(1/2))/a^(1/3)/c^(7/6
)-1/6*arctan(2*c^(1/6)*x/a^(1/6)-3^(1/2))*(d*a^(1/2)-e*3^(1/2)*c^(1/2))/a^(
1/3)/c^(7/6)-1/6*arctan(2*c^(1/6)*x/a^(1/6)+3^(1/2))*(d*a^(1/2)+e*3^(1/2)*c
^(1/2))/a^(1/3)/c^(7/6)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1408, 1517, 1430, 649, 209, 266, 648, 631, 210, 642}

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)}{6\sqrt[3]{a}c^{7/6}} - \frac{\arctan\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+\sqrt{3}\right)\left(\sqrt{a}d+\sqrt{3}\sqrt{c}e\right)}{6\sqrt[3]{a}c^{7/6}} - \frac{\sqrt[6]{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}} - \frac{\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} + \frac{dx}{c}$$

```
[In] Int[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((Sqrt[a]*d
 - Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6*a^(1/3)*c
^(7/6)) - ((Sqrt[a]*d + Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a
^(1/6)])/(6*a^(1/3)*c^(7/6)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^
(2/3)) + ((Sqrt[3]*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(
1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^(7/6)) - ((Sqrt[3]*Sqrt[a]*d - Sqrt[c]
*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^(
7/6))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

Rule 1517

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int
[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
```

```
1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(e+dx^3)}{a+cx^6}\,dx\\
&= \frac{dx}{c} - \frac{\int \frac{ad-cex^3}{a+cx^6}\,dx}{c}\\
&= \frac{dx}{c} - \frac{\int \frac{2a^{2/3}\sqrt[3]{c}d-\left(\sqrt{3}\sqrt{a}\sqrt{c}d+ce\right)x}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{2a^{2/3}\sqrt[3]{c}d+\left(\sqrt{3}\sqrt{a}\sqrt{c}d-ce\right)x}{1+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{a^{2/3}\sqrt[3]{c}d+cex}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}c^{4/3}}\\
&= \frac{dx}{c} - \frac{d\int \frac{1}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3c} - \frac{e\int \frac{x}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}} - \frac{\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\int \frac{\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}x}{\sqrt[3]{a}}}{1+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12\sqrt[3]{a}c^{7/6}}\\
&\quad + \frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\int \frac{-\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}x}{\sqrt[3]{a}}}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12\sqrt[3]{a}c^{7/6}}\\
&\quad - \frac{\left(d-\frac{\sqrt{3}\sqrt{c}e}{\sqrt{a}}\right)\int \frac{1}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12c} - \frac{\left(d+\frac{\sqrt{3}\sqrt{c}e}{\sqrt{a}}\right)\int \frac{1}{1+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12c}\\
&= \frac{dx}{c} - \frac{\sqrt[6]{a}d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}\\
&\quad + \frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}\\
&\quad - \frac{\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}\\
&\quad - \frac{\left(\sqrt{3}\sqrt{a}d-3\sqrt{c}e\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt[3]{a}c^{7/6}}\\
&\quad + \frac{\left(\sqrt{3}\sqrt{a}d+3\sqrt{c}e\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt[3]{a}c^{7/6}}
\end{aligned}
$$

$$= \frac{dx}{c} - \frac{\sqrt[6]{a}d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{\left(\sqrt{a}d - \sqrt{3}\sqrt{c}e\right)\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}}$$
$$- \frac{\left(\sqrt{a}d + \sqrt{3}\sqrt{c}e\right)\tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}} - \frac{e\log\left(\sqrt[3]{a} + \sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$
$$+ \frac{\left(\sqrt{3}\sqrt{a}d + \sqrt{c}e\right)\log\left(\sqrt[3]{a} - \sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x + \sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}$$
$$- \frac{\left(\sqrt{3}\sqrt{a}d - \sqrt{c}e\right)\log\left(\sqrt[3]{a} + \sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x + \sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.11

$$\int \frac{d + \frac{e}{x^3}}{c + \frac{a}{x^6}}\,dx = \frac{dx}{c} - \frac{\sqrt[6]{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}}$$
$$+ \frac{\left(-a^{7/6}\sqrt{c}d + \sqrt{3}a^{2/3}ce\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/3}}$$
$$+ \frac{\left(-a^{7/6}\sqrt{c}d - \sqrt{3}a^{2/3}ce\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/3}} - \frac{e\log\left(\sqrt[3]{a} + \sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$
$$- \frac{\left(-\sqrt{3}a^{7/6}\sqrt{c}d - a^{2/3}ce\right)\log\left(\sqrt[3]{a} - \sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x + \sqrt[3]{c}x^2\right)}{12ac^{5/3}}$$
$$- \frac{\left(\sqrt{3}a^{7/6}\sqrt{c}d - a^{2/3}ce\right)\log\left(\sqrt[3]{a} + \sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x + \sqrt[3]{c}x^2\right)}{12ac^{5/3}}$$

```
[In] Integrate[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((-(a^(7/6)
*Sqrt[c]*d) + Sqrt[3]*a^(2/3)*c*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x
)/a^(1/6)])/(6*a*c^(5/3)) + ((-(a^(7/6)*Sqrt[c]*d) - Sqrt[3]*a^(2/3)*c*e)*A
rcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(5/3)) - (e*Log[a^(1
/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((-(Sqrt[3]*a^(7/6)*Sqrt[c]*d) -
a^(2/3)*c*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*
c^(5/3)) - ((Sqrt[3]*a^(7/6)*Sqrt[c]*d - a^(2/3)*c*e)*Log[a^(1/3) + Sqrt[3]
*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(\_R^{3}ce-da\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c^{2}}$
default	$\frac{dx}{c}+\frac{c\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}e}{12a}+\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}d}{12}+\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)\sqrt{3}\,e}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right.}\right.}{6}$

```
[In] int((d+e/x^3)/(c+a/x^6),x,method=_RETURNVERBOSE)

[Out] d*x/c+1/6/c^2*sum((_R^3*c*e-a*d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(213) = 426.

Time = 0.39 (sec) , antiderivative size = 1608, normalized size of antiderivative = 5.17

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="fricas")

[Out] 1/12*(2*c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7))
+ 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d
*e^4)*x + (a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7))
 + a^2*c*d^4 - 3*a*c^2*d^2*e^2)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*
c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) - (sqrt(-3)*c +
c)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d
^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x
 - 1/2*(a^2*c*d^4 - 3*a*c^2*d^2*e^2 + sqrt(-3)*(a^2*c*d^4 - 3*a*c^2*d^2*e^2
) + (sqrt(-3)*a*c^5*e + a*c^5*e)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2
*e^4)/(a*c^7)))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*
c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) + (sqrt(-3)*c - c)*((a*c^3*sqrt(
-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a
*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x - 1/2*(a^2*c*d^
4 - 3*a*c^2*d^2*e^2 - sqrt(-3)*(a^2*c*d^4 - 3*a*c^2*d^2*e^2) - (sqrt(-3)*a*
c^5*e - a*c^5*e)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)))*
((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*
e - c*e^3)/(a*c^3))^(1/3)) + 2*c*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 +
```

```
9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 -
 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x - (a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 +
 9*c^2*d^2*e^4)/(a*c^7)) - a^2*c*d^4 + 3*a*c^2*d^2*e^2)*(-(a*c^3*sqrt(-(a^2
*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3)
)^(1/3)) - (sqrt(-3)*c + c)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2
*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*
c*d^3*e^2 - 3*c^2*d*e^4)*x - 1/2*(a^2*c*d^4 - 3*a*c^2*d^2*e^2 + sqrt(-3)*(a
^2*c*d^4 - 3*a*c^2*d^2*e^2) - (sqrt(-3)*a*c^5*e + a*c^5*e)*sqrt(-(a^2*d^6 -
 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)))*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d
^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)) + (sq
rt(-3)*c - c)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c
^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*
c^2*d*e^4)*x - 1/2*(a^2*c*d^4 - 3*a*c^2*d^2*e^2 - sqrt(-3)*(a^2*c*d^4 - 3*a
*c^2*d^2*e^2) + (sqrt(-3)*a*c^5*e - a*c^5*e)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
 + 9*c^2*d^2*e^4)/(a*c^7)))*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2
*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)) + 12*d*x)/c
```

Sympy [A] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.54

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$$
$$= \text{RootSum}\left(46656t^6a^2c^7 + t^3\left(-1296a^2c^4d^2e + 432ac^5e^3\right) + a^3d^6 + 3a^2cd^4e^2 + 3ac^2d^2e^4 + c^3e^6, \left(t \mapsto t\log\right.\right.$$
$$+ \frac{dx}{c}$$

```
[In] integrate((d+e/x**3)/(c+a/x**6),x)
```

```
[Out] RootSum(46656*_t**6*a**2*c**7 + _t**3*(-1296*a**2*c**4*d**2*e + 432*a*c**5*
e**3) + a**3*d**6 + 3*a**2*c*d**4*e**2 + 3*a*c**2*d**2*e**4 + c**3*e**6, La
mbda(_t, _t*log(x + (-1296*_t**4*a*c**5*e - 6*_t*a**2*c*d**4 + 36*_t*a*c**2
*d**2*e**2 - 6*_t*c**3*e**4)/(a**2*d**5 - 2*a*c*d**3*e**2 - 3*c**2*d*e**4))
)) + d*x/c
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.95

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = \frac{dx}{c} - \frac{\frac{2\,c^{\frac{1}{3}}\,e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}} + \frac{4\,a^{\frac{1}{3}}\,d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d-a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d+a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^2-\cdots\right)}{ac^{\frac{2}{3}}}\cdots}{12\,c}$$

[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="maxima")

[Out] d*x/c - 1/12*(2*c^(1/3)*e*log(c^(1/3)*x^2 + a^(1/3))/a^(1/3) + 4*a^(1/3)*d*arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sqrt(3)*a^(7/6)*sqrt(c)*d - a^(2/3)*c*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) - (sqrt(3)*a^(7/6)*sqrt(c)*d + a^(2/3)*c*e)*log(c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3)*a^(5/6)*c^(7/6)*e + a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2*(sqrt(3)*a^(5/6)*c^(7/6)*e - a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/c

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = &-\frac{e|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,(ac^5)^{\frac{1}{3}}} + \frac{dx}{c} - \frac{(ac^5)^{\frac{1}{6}}\,d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,c^2} \\ &- \frac{\left((ac^5)^{\frac{1}{6}}\,ac^2d+\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} \\ &- \frac{\left((ac^5)^{\frac{1}{6}}\,ac^2d-\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} \\ &- \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,ac^2d-(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} \\ &+ \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,ac^2d+(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}\end{aligned}$$

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="giac")
```

```
[Out] -1/6*e*abs(c)*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + d*x/c - 1/3*(a*c^5)^(1
/6)*d*arctan(x/(a/c)^(1/6))/c^2 - 1/6*((a*c^5)^(1/6)*a*c^2*d + sqrt(3)*(a*c
^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/6*
((a*c^5)^(1/6)*a*c^2*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/
c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d - (a*c
^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) + 1/12*
(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d + (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)
^(1/6) + (a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 1308, normalized size of antiderivative = 4.21

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$$
$$= \ln\left(e\,x\,\sqrt{-a^3\,c^7}-a^2\,c^4\left(-\frac{a\,c^5\,e^3+a\,d^3\,\sqrt{-a^3\,c^7}-3\,a^2\,c^4\,d^2\,e-3\,c\,d\,e^2\,\sqrt{-a^3\,c^7}}{a^2\,c^7}\right)^{1/3}\right.$$
$$\left.+a^2\,c^3\,d\,x\right)\left(-\frac{a\,c^5\,e^3+a\,d^3\,\sqrt{-a^3\,c^7}-3\,a^2\,c^4\,d^2\,e-3\,c\,d\,e^2\,\sqrt{-a^3\,c^7}}{216\,a^2\,c^7}\right)^{1/3}$$
$$+\ln\left(e\,x\,\sqrt{-a^3\,c^7}+a^2\,c^4\left(-\frac{a\,c^5\,e^3-a\,d^3\,\sqrt{-a^3\,c^7}-3\,a^2\,c^4\,d^2\,e+3\,c\,d\,e^2\,\sqrt{-a^3\,c^7}}{a^2\,c^7}\right)^{1/3}-a^2\,c^3\,d\,x\right)\left(-\right.$$

```
[In] int((d + e/x^3)/(c + a/x^6),x)
```

```
[Out] log(e*x*(-a^3*c^7)^(1/2) - a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) -
3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + a^2*c^3*d*
x)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^
3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(
a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)
^(1/2))/(a^2*c^7))^(1/3) - a^2*c^3*d*x)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/
2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + l
og(2*e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) -
 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 3^(1/2)*a
^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*
(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/
2)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^
3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) - log(2*e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(
-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^
7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7
)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i
```

```
 + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1
/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) -
log(a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d
*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 2*e*x*(-a^3*c^7)^(1/2) + 3^(1/2)*
a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2
*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1
/2)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a
^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(2*e*x*(-a^3*c^7)^(1/2) - a^2*c^4*
(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c
^7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^
7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1
i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(
1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) +
 (d*x)/c
```

3.39 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 716

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\frac{dx}{c}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] d*x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b*d-c*e+(2*a*
c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))
^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))
^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c
+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1
-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b*d-c*e+(2*a*c
*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b*d-c
*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^
2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^
2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b*d-c*e+(-2*a*c*d+b^2*d-b*c*
e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arc
```

```
tan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b*d-
c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1407, 1516, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{dx}{c}$$

```
[In] Int[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d -
2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
```

```
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e - (b^2*d -
 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d - 2*a*c
*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(e+dx^3)}{a+bx^3+cx^6}\,dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad+(bd-ce)x^3}{a+bx^3+cx^6}\,dx}{c} \\
&= \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} \\
&\quad - \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}
\end{aligned}
$$

$$= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$
= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}
$$

$$
- \frac{\left(bd - ce + \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}
$$

$$
+ \frac{\left(bd - ce - \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + 2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{6\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}
$$

$$
- \frac{\left(bd - ce - \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{2\ 2^{2/3}c\sqrt[3]{b - \sqrt{b^2-4ac}}}
$$

$$
+ \frac{\left(bd - ce + \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + 2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{6\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}
$$

$$
- \frac{\left(bd - ce + \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{2\ 2^{2/3}c\sqrt[3]{b + \sqrt{b^2-4ac}}}
$$

$$= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(bd - ce + \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(bd - ce - \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \log\left(\left(b - \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b - \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(bd - ce + \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \log\left(\left(b + \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b + \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(bd - ce - \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(bd - ce + \frac{b^2 d - 2acd - bce}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{dx}{c} + \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{dx}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ad\log(x-\#1)+bd\log(x-\#1)\#1^3-ce\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

```
[In] Integrate[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*d*Log[x - #1] + b*d*Log[x - #1
]*#1^3 - c*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-bd+ec\right)\_R^{3}-da\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-bd+ec\right)\_R^{3}-da\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67

```
[In] int((d+e/x^3)/(c+a/x^6+b/x^3),x,method=_RETURNVERBOSE)
[Out] d*x/c+1/3/c*sum(((-b*d+c*e)*_R^3-d*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(
_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8707 vs. 2(580) = 1160.

Time = 3.35 (sec) , antiderivative size = 8707, normalized size of antiderivative = 12.16

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((d+e/x**3)/(c+a/x**6+b/x**3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{d+\frac{e}{x^3}}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="maxima")
[Out] d*x/c + integrate(-((b*d - c*e)*x^3 + a*d)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{d+\frac{e}{x^3}}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="giac")
[Out] integrate((d + e/x^3)/(c + b/x^3 + a/x^6), x)
```

Mupad [B] (verification not implemented)

Time = 25.62 (sec) , antiderivative size = 11453, normalized size of antiderivative = 16.00

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\text{Too large to display}$$

```
[In] int((d + e/x^3)/(c + a/x^6 + b/x^3),x)
[Out] log((3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^3*e
^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^3
+ 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/3)*
((2^(1/3)*(81*a*c^3*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)
^2*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*
e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^
2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^
3))^(1/3))/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3
 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2
*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*
```

a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^
2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*
a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2*c^
2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e + 6
*a*b*c^2*d^2*e))/c)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c
^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d
^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*
e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*
c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c
^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^
3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^
2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3
*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2
 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3
*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 + log((3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^
3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e
^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b
^2)^2*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c
^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2
d^3(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48
*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/2)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*
e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3
 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e +
 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2
*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^
3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2
*c^2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e

```
+ 6*a*b*c^2*d^2*e))/c)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^
2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^
2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d
^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b
^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b
*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32
*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3
- 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e
^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e
- 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*e*
x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((
b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 -
 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) +
48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4
*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c
^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(
1/3))/4)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^
4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2
*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e +
 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
 b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2*c^2*d^3
 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e + 6*a*b*
c^2*d^2*e))/c)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^
3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 +
2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4
*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d
^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
```

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*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2
*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^
2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2
*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3
 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 +
 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d
^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*
d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2
*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))
^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3
*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2
*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^
3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*
b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3)
)^(1/3))/4)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 -
 b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a
^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*
b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*
e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*
c - b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2*c^2*
d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e + 6*a
*b*c^2*d^2*e))/c)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5
*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3
 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e
+ 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^
2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b
^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4
*(4*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e
+ 2*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c
*d^2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*
c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e
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^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^
2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*
c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c
^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*
d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6
)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)*(81*
a*c^3*e*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b
^2)^2*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c
^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48
*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/4)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*
e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3
+ 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e -
 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2
*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2
*c^2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e
+ 6*a*b*c^2*d^2*e))/c)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^
2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^
2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d
^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b
^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d
^3*e + 2*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*
b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a
*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*
c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2
*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*
b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2
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*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^
2*c^6)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)
*(81*a*c^3*e*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*
c - b^2)^2*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 -
b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^
2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b
^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e
 + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
 - b^2)^3))^(1/3))/4)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2
*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2
*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^
2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^
4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(c^4*(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3
+ a^2*c^2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d
^2*e + 6*a*b*c^2*d^2*e))/c)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b
^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^
6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 2
7*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^
2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 -
b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3
 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e
- 9*a*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^3 - b^4*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a
*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^
5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a
^2*b^2*c^6)))^(1/3) + (d*x)/c
```

3.40 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 753

$$\begin{aligned}\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx &= \frac{dx}{c}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}\\
&-\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}\\
&-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}\\
&+\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}\\
&-\frac{\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}\\
&+\frac{\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}\\
&+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}\\
&-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}\end{aligned}$$

```
[Out] d*x/c+1/8*arctan((-2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2
))^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))*(2-2^(1/2))^(1/2)/a^(3/8)/c^(9/
8)-1/8*arctan((2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(
1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))*(2-2^(1/2))^(1/2)/a^(3/8)/c^(9/8)-1
/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*((d-d*2^(1/2
))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(4-2*2^(1/2))^(1/2)+1/8*ln(a^(1/4)+c^
(1/4)*x^2+a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2)+e*c^(
1/2))/a^(3/8)/c^(9/8)/(4-2*2^(1/2))^(1/2)-1/8*arctan((-2*c^(1/8)*x+a^(1/8)*
(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2)+e*c^(1
/2))*(2+2^(1/2))^(1/2)/a^(3/8)/c^(9/8)+1/8*arctan((2*c^(1/8)*x+a^(1/8)*(2+2
^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2)+e*c^(1/2))
*(2+2^(1/2))^(1/2)/a^(3/8)/c^(9/8)+1/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/
8)*x*(2+2^(1/2))^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(
4+2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/4)*x^2+a^(1/8)*c^(1/8)*x*(2+2^(1/2))
^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(4+2*2^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used

$= \{1408, 1517, 1429, 1183, 648, 632, 210, 642\}$

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)}{8a^{3/8}c^{9/8}}$$
$$-\frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)}{8a^{3/8}c^{9/8}}$$
$$-\frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)}{8a^{3/8}c^{9/8}}$$
$$+\frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)}{8a^{3/8}c^{9/8}}$$
$$-\frac{\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}$$
$$+\frac{\left(\sqrt{a}(d-\sqrt{2}d)+\sqrt{c}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}$$
$$+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}$$
$$-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{dx}{c}$$

```
[In] Int[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (d*x)/c + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(
Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a
^(3/8)*c^(9/8)) - (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*
ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8)
)])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqr
t[c]*e)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]
*a^(1/8))])/(8*a^(3/8)*c^(9/8)) + (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*
d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 -
Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[
c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*
Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c
]*e)*Log[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*S
qrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]
*e)*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sq
```

```
rt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8)) - (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*
e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqr
t[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q - (d - e*q^2)*x
```

```
^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Dist[1/(2*Sqrt[2]*c*q^3),
 Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n), x
], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &
& NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

Rule 1517

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int
[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(e+dx^4)}{a+cx^8}\,dx$$

$$= \frac{dx}{c} - \frac{\int \frac{ad-cex^4}{a+cx^8}\,dx}{c}$$

$$= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}+\left(-ad-\sqrt{a}\sqrt{c}e\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}c^{5/4}} - \frac{\int \frac{\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}+\left(ad+\sqrt{a}\sqrt{c}e\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}c^{5/4}}$$

$$= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{11/8}d}{c^{3/8}} - \left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(ad+\sqrt{a}\sqrt{c}e)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$- \frac{\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{11/8}d}{c^{3/8}} + \left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(ad+\sqrt{a}\sqrt{c}e)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$- \frac{\int \frac{\frac{\sqrt{2\left(2+\sqrt{2}\right)}a^{11/8}d}{c^{3/8}} - \left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(-ad-\sqrt{a}\sqrt{c}e)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$- \frac{\int \frac{\frac{\sqrt{2\left(2+\sqrt{2}\right)}a^{11/8}d}{c^{3/8}} + \left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(-ad-\sqrt{a}\sqrt{c}e)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$= \frac{dx}{c} - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}\sqrt[4]{a}c^{5/4}}$$

$$- \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}\sqrt[4]{a}c^{5/4}}$$

$$+ \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}$$

$$- \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}$$

$$- \frac{\left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(ad+\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)\int \frac{\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$- \frac{\left(-\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}+\frac{\sqrt[4]{a}\left(ad+\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)\int \frac{-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$- \frac{\left(\frac{2\sqrt{2\left(2+\sqrt{2}\right)}a^{11/8}d}{c^{3/8}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(-ad-\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)}{\sqrt[8]{c}}\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$- \frac{\left(\frac{2\sqrt{2\left(2+\sqrt{2}\right)}a^{11/8}d}{c^{3/8}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(-\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}+\frac{\sqrt[4]{a}\left(-ad-\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)}{\sqrt[8]{c}}\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}c^{7/8}}$$

$$
\begin{aligned}
&= \frac{dx}{c} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&+ \frac{\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&+ \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&- \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&+ \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\text{Subst}\left(\int \frac{1}{-\frac{\left(2+\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx, x, -\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} \\
&+ \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\text{Subst}\left(\int \frac{1}{-\frac{\left(2+\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx, x, \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}+2x\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}} \\
&+ \frac{\left(\frac{2\sqrt{2\left(2+\sqrt{2}\right)}a^{11/8}d}{c^{3/8}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(-ad-\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)}{\sqrt[8]{c}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\left(2-\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx, x, \frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{\cdots}}{\sqrt[8]{c}}\cdots\right.}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}c^{7/8}} \\
&+ \frac{\left(\frac{2\sqrt{2\left(2+\sqrt{2}\right)}a^{11/8}d}{c^{3/8}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(-\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}+\frac{\sqrt[4]{a}\left(-ad-\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)}{\sqrt[8]{c}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\left(2-\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}-x^2}\,dx, x, -\frac{\sqrt{2+\sqrt{\cdots}}}{\sqrt[8]{\cdots}}\cdots\right.}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{9/8}c^{7/8}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{dx}{c} + \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&\quad - \frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}} \\
&\quad - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&\quad + \frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}} \\
&\quad - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&\quad + \frac{\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&\quad + \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}} \\
&\quad - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 551, normalized size of antiderivative = 0.73

$$
\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx
$$

$$
= \frac{8ac^{5/8}dx + 2\arctan\left(\cot\left(\frac{\pi}{8}\right)+\frac{\sqrt[8]{c}x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(a^{5/8}ce\cos\left(\frac{\pi}{8}\right)-a^{9/8}\sqrt{c}d\sin\left(\frac{\pi}{8}\right)\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{c}x^2+2\sqrt[8]{a}\right.}{}
$$

```
[In] Integrate[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (8*a*c^(5/8)*d*x + 2*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(a^(
5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x
^2 + 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(a^(5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c
]*d*Sin[Pi/8]) + 2*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(-(a^(
```

```
5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*
x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(-(a^(5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sq
rt[c]*d*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a
^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*S
ec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*S
in[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(
9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - Log[a^(1/4) + c^(1/4)*x
^2 + 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*
c*e*Sin[Pi/8]))/(8*a*c^(13/8))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.06

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\left(\_R^{4}ce-da\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c^{2}}$	45
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\left(\_R^{4}ce-da\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c^{2}}$	45

```
[In] int((d+e/x^4)/(c+a/x^8),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/8/c^2*sum((_R^4*c*e-a*d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2730 vs. $2(529) = 1058$.

Time = 0.66 (sec) , antiderivative size = 2730, normalized size of antiderivative = 3.63

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="fricas")
```

```
[Out] -1/8*(c*sqrt(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^
4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*
c^4)))*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x + (a^2
*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^
2*e^6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*e^2 + a*c^3*d*e^4)*
sqrt(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))))
```

- c*sqrt(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4)
))*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x - (a^2*c^6
*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^
6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*e^2 + a*c^3*d*e^4)*sqrt
(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4)))) - c*
sqrt(-sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))*
log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x + (a^2*c^6*e*
sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 +
 c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a*c^3*d*e^4)*sqrt(-s
qrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c
^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))) + c*sq
rt(-sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 1
2*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))*lo
g((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x - (a^2*c^6*e*sq
rt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c
^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a*c^3*d*e^4)*sqrt(-sqr
t(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3
*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))) + c*((a*
c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e
^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d
^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x + (a^2*c^6*e*sqrt(-(a^4
*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/
(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*e^2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^
4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)
/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)) - c*((a*c^4*sqrt(-(a^4
*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/
(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d^6 - 5*a^2*c*d
^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x - (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*
c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a
^3*c*d^5 - 6*a^2*c^2*d^3*e^2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3
*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) +
4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)) - c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3
*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) -
4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*
c^2*d^2*e^4 + c^3*e^6)*x + (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 3
8*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*
a^2*c^2*d^3*e^2 - a*c^3*d*e^4)*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e +
4*c*d*e^3)/(a*c^4))^(1/4)) + c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e +
4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4
+ c^3*e^6)*x - (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^

```
4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*
e^2 - a*c^3*d*e^4)*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*
d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(
a*c^4))^(1/4)) - 8*d*x)/c
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \text{Timed out}$$

```
[In] integrate((d+e/x**4)/(c+a/x**8),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$$

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="maxima")
[Out] d*x/c + integrate((c*e*x^4 - a*d)/(c*x^8 + a), x)/c
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 639, normalized size of antiderivative = 0.85

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \frac{dx}{c} - \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} - \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} + \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} + \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} - \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac} + \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac} + \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac} - \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac}$$

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="giac")
```

```
[Out] d*x/c - 1/8*(c*e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) + a*d*sqrt(sqrt(2) + 2)*(a/
c)^(1/8))*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*
(a/c)^(1/8)))/(a*c) - 1/8*(c*e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) + a*d*sqrt(sq
rt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*e*sqrt(sqrt(2) + 2)*(a/c)^(5/8)
- a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)
^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*e*sqrt(sqrt(2) + 2
)*(a/c)^(5/8) - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt
(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) - 1/16*(c*e*s
qrt(-sqrt(2) + 2)*(a/c)^(5/8) + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log(x^2
+ x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*e*sqrt(-sq
rt(2) + 2)*(a/c)^(5/8) + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqr
```

```
t(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*e*sqrt(sqrt(2) +
2)*(a/c)^(5/8) - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 + x*sqrt(-sqrt
(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) - 1/16*(c*e*sqrt(sqrt(2) + 2)*(a/
c)^(5/8) - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqrt(-sqrt(2) +
2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 2520, normalized size of antiderivative = 3.35

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \text{Too large to display}$$

```
[In] int((d + e/x^4)/(c + a/x^8),x)
```

```
[Out] (atan((a^3*d^6*x - c^3*e^6*x - a*c^2*d^2*e^4*x + a^2*c*d^4*e^2*x + (2*d*e*x
*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4
*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))/(a*c^4))/(a^2*c^6*e*(-(a^
2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3
*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^3*c*d^5*(
-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4
*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 2*a^2*c
^2*d^3*e^2*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c
^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/
4) + 3*a*c^3*d*e^4*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) +
 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c
^9))^(1/4)))*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2
*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(
1/4))/4 - (atan((c^3*e^6*x - a^3*d^6*x + a*c^2*d^2*e^4*x - a^2*c*d^4*e^2*x
+ (2*d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6
*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))/(a*c^4))/(a^2*c
^6*e*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^
3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^
3*c*d^5*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d
*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) +
 2*a^2*c^2*d^3*e^2*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) -
4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^
9))^(1/4) + 3*a*c^3*d*e^4*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(
1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/
(a^3*c^9))^(1/4)))*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) -
4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^
9))^(1/4))/4 + atan((c^3*e^6*x*1i - a^3*d^6*x*1i + a*c^2*d^2*e^4*x*1i - a^2
*c*d^4*e^2*x*1i + (d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/
2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))*2i
)/(a*c^4))/(a^2*c^6*e*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2)
```

```
 - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3
*c^9))^(5/4) - a^3*c*d^5*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1
/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(
a^3*c^9))^(1/4) + 2*a^2*c^2*d^3*e^2*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-
a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^
9)^(1/2))/(a^3*c^9))^(1/4) + 3*a*c^3*d*e^4*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2
*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(
-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4)))*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-
a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^
9)^(1/2))/(4096*a^3*c^9))^(1/4)*2i - atan((a^3*d^6*x*1i - c^3*e^6*x*1i - a*
c^2*d^2*e^4*x*1i + a^2*c*d^4*e^2*x*1i + (d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) +
c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^
2*(-a^3*c^9)^(1/2))*2i)/(a*c^4))/(a^2*c^6*e*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c
^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2
*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^3*c*d^5*(-(a^2*d^4*(-a^3*c^9)^(1/2)
 + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2
*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 2*a^2*c^2*d^3*e^2*(-(a^2*d^4*(-a^
3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e
 - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 3*a*c^3*d*e^4*(-(a^2*
d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c
^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4)))*(-(a^2*d^4*(-
a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3
*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(4096*a^3*c^9))^(1/4)*2i + (d*x)/c
```

3.41 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 433

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{dx}{c} + \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] d*x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b*d-c*e+
(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)
^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(b*d-c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4
*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))
^(1/4))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/
```

```
(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/
c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1407, 1516, 1436, 218, 214, 211}

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{dx}{c}$$

```
[In] Int[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

```
[Out] (d*x)/c + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e + (b^2*d
- 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
+ ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^4(e+dx^4)}{a+bx^4+cx^8}\,dx \\ &= \frac{dx}{c} - \frac{\int \frac{ad+(bd-ce)x^4}{a+bx^4+cx^8}\,dx}{c}\end{aligned}$$

$$=\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$

$$=\frac{dx}{c}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$=\frac{dx}{c}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.20

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{dx}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{ad\log(x-\#1)+bd\log(x-\#1)\#1^4-ce\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

```
[In] Integrate[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
[Out] (d*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*d*Log[x - #1] + b*d*Log[x - #1
]*#1^4 - c*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\left((-bd+ec)\_R^4-da\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4c}$	67
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\left((-bd+ec)\_R^4-da\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4c}$	67

```
[In] int((d+e/x^4)/(c+a/x^8+b/x^4),x,method=_RETURNVERBOSE)
[Out] d*x/c+1/4/c*sum(((-b*d+c*e)*_R^4-d*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12946 vs. $2(353) = 706$.

Time = 5.10 (sec) , antiderivative size = 12946, normalized size of antiderivative = 29.90

$$\int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} \, dx = \text{Too large to display}$$

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} \, dx = \text{Timed out}$$

```
[In] integrate((d+e/x**4)/(c+a/x**8+b/x**4),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} \, dx = \int \frac{d + \frac{e}{x^4}}{c + \frac{b}{x^4} + \frac{a}{x^8}} \, dx$$

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="maxima")
[Out] d*x/c + integrate(-((b*d - c*e)*x^4 + a*d)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F(-1)]

Timed out.

$$\int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} \, dx = \text{Timed out}$$

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 50213, normalized size of antiderivative = 115.97

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

```
[In] int((d + e/x^4)/(c + a/x^8 + b/x^4),x)
```

```
[Out] atan((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^
2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b
^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 1
28*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)
^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*
c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
- 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^
3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3
*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c
*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*
d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*
b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b
^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2
 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(
-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4
+ 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*
```

d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*
c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e
^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^
2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a
^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 +
 c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16
*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4
*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*
(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b
^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 + b^4*d
^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e
^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3
*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*
b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a
^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2
)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*
d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^

```
2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8
*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b
^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^
2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(3/4) + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b
*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*
c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^
3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 2
0*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*
c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*
d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*
b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13
*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^
4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) -
 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^
2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c
^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^
4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^
3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d
^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16
*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*
a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e
^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*
(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4
 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2
*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c -
b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^5*b*c^
6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 2
56*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^
```

4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d

^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^
5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5
*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 + b
^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6
*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3
*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*
b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^
2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*
a^4*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b
^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*
c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240
*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e -
 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*
e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(
16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^
7*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*
b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^
2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*
e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*
c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5
*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*d^5
 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a
^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6
*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22

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*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c
^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^
3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b
^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b
^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e +
240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*
b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*
e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d
^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2
*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*
a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^
3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d
^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^
3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*
c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48
*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 20
0*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4))*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*
e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4
 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e
^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^
3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^
3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) -
 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288
*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + ata
n((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 -
 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*
```

a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*
d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*
a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3
e(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5
*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(
4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 1
28*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d
^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e
^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e
^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e
+ 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^
5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5
- a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3
*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*
c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 -
19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b
^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 1
28*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
- 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2
*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6
+ a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d

^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^
4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*
b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^
3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^
4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^
2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*
a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^
2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*
(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^
5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c
^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*(40
96*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*
c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e -
1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 - b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
- 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^
3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3
*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*
b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5
)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4
 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3
*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*
d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*
c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b
^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4) + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^
5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4
*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 -

22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a
^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4
*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d
^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e
^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3
*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*
b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c
^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*
e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^
3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3
*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5
- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 +
128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^
4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*
d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2
)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2
- 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 3
20*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2
*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^5*b*c^6*d
^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*
a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c
^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2
)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*
d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^
2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8
*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b
^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^
2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))

/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12
288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*
c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5
*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3
*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*
b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (1
6*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^
7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c
^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^
2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e
 + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^
4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c -
b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c
^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*
b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*
a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200
*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5
*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^
4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*
e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 +
 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*
d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*
e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^
3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a
*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*
c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4
*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*

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c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4
*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c
^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 -
 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4
*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*
c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5
*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3
*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*
b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(163
84*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e
))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c
^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b
^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6
*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2
 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^
2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3
*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^
2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*
a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*d^5 -
4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*
c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^
2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^
4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*
d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 +
 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b
^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*
c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240
*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e +
 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*
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e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) +
 (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^
3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5
*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*
e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5
*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^
8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*
b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a
^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4)))*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4
 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 +
16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3
+ 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c
^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d
*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66
*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^
3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8
*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + 2*atan
((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 -
2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a
^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*d^4
+ b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*
c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*
a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(
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-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e +
 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(
4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 1
28*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d
^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e
^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e
^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e
- 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c
*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*
d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*
b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b
^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2
 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(
-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4
+ 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*
d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a
^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^
4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4
*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*
c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e -
4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*
c^3*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^
4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 +
 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3
 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*
c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 6
```

6*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a
^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + ((((4*x*
(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b
^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*d^4 + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
- 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^
3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3
*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*
b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5
*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^
8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*
b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a
^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*1i - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a
^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*
b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^
2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e
 - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*
b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4
+ b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*
c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*
a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^

2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6
 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^
2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e
^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^
2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*
c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^
3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*
c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^
2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d
^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/(((((4*x*(4096*a^5*
b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2
 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^
3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*d^4 + b^4*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*
c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48
*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 20
0*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e
 - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a
*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4
*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e
 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*
a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^
3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3

*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*

d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*
c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b
^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288
*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5
*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^
5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^
3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a
^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c
d^4(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4
*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e
- 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i
- (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^
2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a
^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^
4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d
^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 3
2*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*
c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a
^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4
- 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 +
 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*
e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 -
 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*
a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a
^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6
- 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2
*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c
^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*
d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2
*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4
 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3

*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120 *a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2 *e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40 *a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1 /2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a ^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3* b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e* (-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a ^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c ^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c ^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2 *c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4* b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48* a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(- (4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200 *a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^ 3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/ 2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b ^2*c^8)))^(1/4) + 2*atan((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a ^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^ 6*d*e))/c - ((-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16* a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 6 1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1 /2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4* d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a *c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^ 3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a* b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b *c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^ 5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8 *e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)* (-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4* a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2 *d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c ^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b ^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^ (1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e

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^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*d^5 - 4*a
^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5
*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b
^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b
^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e
^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*
a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*
c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5
*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3
*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*
b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i -
 (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^
3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5
*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*
e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5
*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^
8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*
b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a
^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4) + ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*
b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6
*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c
+ ((-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*
e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*
c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
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7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 -
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^
2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e
^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a
*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a
^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4
 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3
*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120
*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1
/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a
^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*
b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a
^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5
 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 1
3*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2
*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3
*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2
*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^
2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 -
c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*
a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 6
1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4
*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*
e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e
^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3
 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*
b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e
^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a

*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4
*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e
+ 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*
a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^
3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3
*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1
/4))/(((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^
2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*
d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*
a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3
e(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5
*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 - b^4*d^4
*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
- 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c
^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^
7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4
*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5
*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^
3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b
^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*
c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*
a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a
^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3
*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/
c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*

e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*
c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 -
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^
2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e
^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a
*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 +
 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^
2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2
*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*
b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*
e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^
4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^
4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e
^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d
*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^
2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*
b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*
c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
 - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 2
88*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (
(((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 204
8*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*
c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*d^4 - b
^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6
*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3
*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
+ 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 30
72*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^
4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*

a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4
- a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4
- 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3
+ 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3
e(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3
- 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6
*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^
5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5
- a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3
*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*
c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 -
19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b
^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 1
28*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
- 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*
c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e
^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^
2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a
^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 -
c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16
*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4
*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*d
^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^

2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a
^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 1
20*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e
^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128
*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^
3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*
e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (d*x)/c

3.42 $\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \frac{3de^2x}{c}+\frac{e^3x^{1+n}}{c(1+n)} + \frac{d(cd^2-3ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} + \frac{e(3cd^2-ae^2)\,x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(1+n)}$$

```
[Out] 3*d*e^2*x/c+e^3*x^(1+n)/c/(1+n)+d*(-3*a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[
1+1/2/n],-c*x^(2*n)/a)/a/c+e*(-a*e^2+3*c*d^2)*x^(1+n)*hypergeom([1, 1/2*(1+
n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/c/(1+n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1439, 1432, 251, 371}

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \frac{ex^{n+1}(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(n+1)} + \frac{dx(cd^2-3ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} + \frac{3de^2x}{c}+\frac{e^3x^{n+1}}{c(n+1)}$$

```
[In] Int[(d + e*x^n)^3/(a + c*x^(2*n)),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^(1 + n))/(c*(1 + n)) + (d*(c*d^2 - 3*a*e^2)*x*Hyperg
eometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) + (e*(3*c*
d^2 - a*e^2)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
-((c*x^(2*n))/a)])/(a*c*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3de^2}{c} + \frac{e^3x^n}{c} + \frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)\,x^n}{c\,(a + cx^{2n})}\right) dx \\
&= \frac{3de^2x}{c} + \frac{e^3x^{1+n}}{c(1+n)} + \frac{\int \frac{cd^3-3ade^2+(3cd^2e-ae^3)x^n}{a+cx^{2n}}\,dx}{c} \\
&= \frac{3de^2x}{c} + \frac{e^3x^{1+n}}{c(1+n)} + \frac{(d(cd^2-3ae^2))\int \frac{1}{a+cx^{2n}}\,dx}{c} + \frac{(e(3cd^2-ae^2))\int \frac{x^n}{a+cx^{2n}}\,dx}{c}
\end{aligned}$$

$$= \frac{3de^2x}{c} + \frac{e^3x^{1+n}}{c(1+n)} + \frac{d(cd^2-3ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac} + \frac{e(3cd^2-ae^2)\,x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(1+n)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \frac{x\left(d(cd^2-3ae^2)(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+e\left(ae(3d(1+n)+ex^n)+(3cd^2-\right.\right.}{ac(1+n)}$$

```
[In] Integrate[(d + e*x^n)^3/(a + c*x^(2*n)),x]
```

```
[Out] (x*(d*(c*d^2 - 3*a*e^2)*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/
2, -((c*x^(2*n))/a)] + e*(a*e*(3*d*(1 + n) + e*x^n) + (3*c*d^2 - a*e^2)*x^n
*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])))/(
a*c*(1 + n))
```

Maple [F]

$$\int \frac{(d+e\,x^n)^3}{a+c\,x^{2n}}dx$$

```
[In] int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+a}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c*x^(2*n) + a
), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.30

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}d^3x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)} + \frac{3a^{-\frac{5}{2}-\frac{1}{2n}}a^{\frac{3}{2}+\frac{1}{2n}}e^3x^{3n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{5}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{5}{2}-\frac{1}{2n}}a^{\frac{3}{2}+\frac{1}{2n}}e^3x^{3n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}{4n^2\Gamma\left(\frac{5}{2}+\frac{1}{2n}\right)} + \frac{3a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}d^2ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{3a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}d^2ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} - \frac{3a^{-\frac{1}{2n}}a^{1+\frac{1}{2n}}c^{\frac{1}{2n}}c^{-1-\frac{1}{2n}}de^2x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c},1,\frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1+\frac{1}{2n}\right)}$$

```
[In] integrate((d+e*x**n)**3/(a+c*x**(2*n)),x)
```

```
[Out] a**(1/(2*n))*a**(-1 - 1/(2*n))*d**3*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a
, 1, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + 3*a**(-5/2 - 1/(
2*n))*a**(3/2 + 1/(2*n))*e**3*x**(3*n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*
pi)/a, 1, 3/2 + 1/(2*n))*gamma(3/2 + 1/(2*n))/(4*n*gamma(5/2 + 1/(2*n))) +
a**(-5/2 - 1/(2*n))*a**(3/2 + 1/(2*n))*e**3*x**(3*n + 1)*lerchphi(c*x**(2*n
)*exp_polar(I*pi)/a, 1, 3/2 + 1/(2*n))*gamma(3/2 + 1/(2*n))/(4*n**2*gamma(5
/2 + 1/(2*n))) + 3*a**(-3/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*d**2*e*x**(n + 1)
*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*
n))/(4*n*gamma(3/2 + 1/(2*n))) + 3*a**(-3/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*d
**2*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*g
amma(1/2 + 1/(2*n))/(4*n**2*gamma(3/2 + 1/(2*n))) - 3*a**(1 + 1/(2*n))*c**(
1/(2*n))*c**(-1 - 1/(2*n))*d*e**2*x*lerchphi(a*exp_polar(I*pi)/(c*x**(2*n))
, 1, exp_polar(I*pi)/(2*n))*gamma(1/(2*n))/(4*a*a**(1/(2*n))*n**2*gamma(1 +
 1/(2*n)))
```

Maxima [F]

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+a}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (3*d*e^2*(n + 1)*x + e^3*x*x^n)/(c*(n + 1)) - integrate(-(c*d^3 - 3*a*d*e^2
 + (3*c*d^2*e - a*e^3)*x^n)/(c^2*x^(2*n) + a*c), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+a}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^3}{a+c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)^3/(a + c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)^3/(a + c*x^(2*n)), x)
```

3.43 $\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx=\frac{e^2x}{c}+\frac{(cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[Out] e^2*x/c+(-a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/c+2
*d*e*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1439, 1432, 251, 371}

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx=\frac{x(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)}+\frac{e^2x}{c}$$

```
[In] Int[(d + e*x^n)^2/(a + c*x^(2*n)),x]
```

```
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/
(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+2cdex^n}{c\,(a+cx^{2n})}\right) dx\\
&= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+2cdex^n}{a+cx^{2n}}\,dx}{c}\\
&= \frac{e^2x}{c}+(2de)\int \frac{x^n}{a+cx^{2n}}\,dx+\frac{(cd^2-ae^2)\int \frac{1}{a+cx^{2n}}\,dx}{c}\\
&= \frac{e^2x}{c}+\frac{(cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \frac{e^2x}{c} + \frac{(cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} + \frac{2dex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[In] Integrate[(d + e*x^n)^2/(a + c*x^(2*n)),x]
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/
(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

Maple [F]

$$\int \frac{(d+e\,x^n)^2}{a+c\,x^{2n}}\,dx$$

```
[In] int((d+e*x^n)^2/(a+c*x^(2*n)),x)
[Out] int((d+e*x^n)^2/(a+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2n}+a}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c*x^(2*n) + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}d^2x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}dex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{2n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}dex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{2n^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} - \frac{a^{-\frac{1}{2n}}a^{1+\frac{1}{2n}}c^{\frac{1}{2n}}c^{-1-\frac{1}{2n}}e^2x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c},1,\frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1+\frac{1}{2n}\right)}$$

```
[In] integrate((d+e*x**n)**2/(a+c*x**(2*n)),x)
```

```
[Out] a**(1/(2*n))*a**(-1 - 1/(2*n))*d**2*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a
, 1, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + a**(-3/2 - 1/(2*
n))*a**(1/2 + 1/(2*n))*d*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a
, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(2*n*gamma(3/2 + 1/(2*n))) + a**(-
3/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*d*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_po
lar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(2*n**2*gamma(3/2 + 1/(
2*n))) - a**(1 + 1/(2*n))*c**(1/(2*n))*c**(-1 - 1/(2*n))*e**2*x*lerchphi(a*
exp_polar(I*pi)/(c*x**(2*n)), 1, exp_polar(I*pi)/(2*n))*gamma(1/(2*n))/(4*a
*a**(1/(2*n))*n**2*gamma(1 + 1/(2*n)))
```

Maxima [F]

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2n}+a}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] e^2*x/c + integrate((2*c*d*e*x^n + c*d^2 - a*e^2)/(c^2*x^(2*n) + a*c), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2n}+a}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^2}{a+c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)^2/(a + c*x^(2*n)),x)
[Out] int((d + e*x^n)^2/(a + c*x^(2*n)), x)
```

3.44 $\int \frac{d+ex^n}{a+cx^{2n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[Out] d*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a+e*x^(1+n)*hypergeom([1,
1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1432, 251, 371}

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

```
[In] Int[(d + e*x^n)/(a + c*x^(2*n)),x]
```

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a + (
e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n
))/a)])/(a*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int \frac{1}{a+cx^{2n}}\,dx + e\int \frac{x^n}{a+cx^{2n}}\,dx \\ &= \frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n)),x]
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a + (
e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n
))/a)])/(a*(1 + n))
```

Maple [F]

$$\int \frac{d+e\,x^{n}}{a+c\,x^{2n}}dx$$

[In] int((d+e*x^n)/(a+c*x^(2*n)),x)

[Out] int((d+e*x^n)/(a+c*x^(2*n)),x)

Fricas [F]

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \int \frac{ex^n+d}{cx^{2\,n}+a}\,dx$$

[In] integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)/(c*x^(2*n) + a), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.52

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}dx\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

[In] integrate((d+e*x**n)/(a+c*x**(2*n)),x)

[Out] a**(1/(2*n))*a**(-1 - 1/(2*n))*d*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1
, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + a**(-3/2 - 1/(2*n))
*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1,
1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n*gamma(3/2 + 1/(2*n))) + a**(-3/2 -
 1/(2*n))*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*p
i)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n**2*gamma(3/2 + 1/(2*n)))

Maxima [F]

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx=\int \frac{ex^n+d}{cx^{2n}+a}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)/(c*x^(2*n) + a), x)
```

Giac [F]

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx=\int \frac{ex^n+d}{cx^{2n}+a}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)/(c*x^(2*n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx=\int \frac{d+e\,x^n}{a+c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)/(a + c*x^(2*n)),x)
[Out] int((d + e*x^n)/(a + c*x^(2*n)), x)
```

3.45 $\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx = \frac{cdx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)} + \frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)} - \frac{cex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)(1+n)}$$

```
[Out] c*d*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)+e^2*x*hy
pergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)-c*e*x^(1+n)*hypergeom([1
, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)/(1+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1439, 251, 1432, 371}

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx = -\frac{cex^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2+cd^2)} + \frac{cdx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)} + \frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(ae^2+cd^2)}$$

```
[In] Int[1/((d + e*x^n)*(a + c*x^(2*n))),x]
```

```
[Out] (c*d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*
(c*d^2 + a*e^2)) + (e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n
)/d)])/(d*(c*d^2 + a*e^2)) - (c*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{(cd^2+ae^2)\,(d+ex^n)} - \frac{c(-d+ex^n)}{(cd^2+ae^2)\,(a+cx^{2n})}\right) dx \\
&= -\frac{c\int \frac{-d+ex^n}{a+cx^{2n}}\,dx}{cd^2+ae^2} + \frac{e^2\int \frac{1}{d+ex^n}\,dx}{cd^2+ae^2} \\
&= \frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(cd^2+ae^2)} + \frac{(cd)\int \frac{1}{a+cx^{2n}}\,dx}{cd^2+ae^2} - \frac{(ce)\int \frac{x^n}{a+cx^{2n}}\,dx}{cd^2+ae^2}
\end{aligned}$$

$$= \frac{cdx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(cd^2+ae^2\right)} - \frac{cex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

$$\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)}\,dx = \frac{x\left(cd^2(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+e\left(ae(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n}\right.\right.\right.}{ad\left(cd^2+ae^2\right)(1+n)}$$

```
[In] Integrate[1/((d + e*x^n)*(a + c*x^(2*n))),x]
[Out] (x*(c*d^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n
))/a)] + e*(a*e*(1 + n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/
d)] - c*d*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])))/(a*d*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)\left(a+c\,x^{2n}\right)}dx$$

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n)),x)
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)}\,dx = \int \frac{1}{(cx^{2\,n}+a)(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(1/(a*e*x^n + a*d + (c*e*x^n + c*d)*x^(2*n)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + a)*(e*x^n + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + a)*(e*x^n + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})}\,dx = \int \frac{1}{(a+c\,x^{2\,n})\,(d+e\,x^n)}\,dx$$

```
[In] int(1/((a + c*x^(2*n))*(d + e*x^n)),x)
[Out] int(1/((a + c*x^(2*n))*(d + e*x^n)), x)
```

3.46 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx = \frac{c(cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^2} + \frac{2ce^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^2} - \frac{2c^2dex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^2\,(1+n)} + \frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\,(cd^2+ae^2)}$$

```
[Out] c*(-a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*
d^2)^2+2*c*e^2*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/(a*e^2+c*d^2)^2-2*c^2
*d*e*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+
c*d^2)^2/(1+n)+e^2*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*d^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{1439, 251, 1432, 371\}$

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})} dx = -\frac{2c^2dex^{n+1}\,\text{Hypergeometric2F1}\left(1, \frac{n+1}{2n}, \frac{1}{2}\left(3+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2} + \frac{cx(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1, \frac{1}{2n}, \frac{1}{2}\left(2+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2} + \frac{2ce^2x\,\text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1+\frac{1}{n}, -\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^2} + \frac{e^2x\,\text{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, -\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)}$$

```
[In] Int[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
```

```
[Out] (c*(c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(
2*n))/a)])/(a*(c*d^2 + a*e^2)^2) + (2*c*e^2*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^2 - (2*c^2*d*e*x^(1 + n)*Hyperge
ometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 +
 a*e^2)^2*(1 + n)) + (e^2*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x
^n)/d)])/(d^2*(c*d^2 + a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
```

```
e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^n)^2} + \frac{2cde^2}{(cd^2+ae^2)^2(d+ex^n)} - \frac{c(-cd^2+ae^2+2cdex^n)}{(cd^2+ae^2)^2(a+cx^{2n})}\right) dx\\
&= -\frac{c\int \frac{-cd^2+ae^2+2cdex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^2} + \frac{(2cde^2)\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^2} + \frac{e^2\int \frac{1}{(d+ex^n)^2}\,dx}{cd^2+ae^2}\\
&= \frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^2} + \frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)}\\
&\quad - \frac{(2c^2de)\int \frac{x^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^2} + \frac{(c(cd^2-ae^2))\int \frac{1}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^2}\\
&= \frac{c(cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2} + \frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^2}\\
&\quad - \frac{2c^2dex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2(1+n)} + \frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx = \frac{x\left(cd^2(cd^2-ae^2)(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+e\left(2acd^2e(1+n)\,\text{Hypergeometri}\right.\right.}{}$$

```
[In] Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
```

```
[Out] (x*(c*d^2*(c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1)
)/2, -((c*x^(2*n))/a)] + e*(2*a*c*d^2*e*(1 + n)*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((e*x^n)/d)] - 2*c^2*d^3*x^n*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)] + a*e*(c*d^2 + a*e^2)*(1 + n)*Hyperg
eometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])))/(a*(c*d^3 + a*d*e^2)^2*
(1 + n))
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)^2\,(a+c\,x^{2n})}dx$$

```
[In] int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^(2*n) + 2*a*d*e*x^n + a*d^2 + (c*e^2*x^(2*n) + 2*c*d*e*
x^n + c*d^2)*x^(2*n)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="maxima")
[Out] e^2*x/(c*d^4*n + a*d^2*e^2*n + (c*d^3*e*n + a*d*e^3*n)*x^n) + (c*d^2*e^2*(3
*n - 1) + a*e^4*(n - 1))*integrate(1/(c^2*d^6*n + 2*a*c*d^4*e^2*n + a^2*d^2
*e^4*n + (c^2*d^5*e*n + 2*a*c*d^3*e^3*n + a^2*d*e^5*n)*x^n), x) - integrate
((2*c^2*d*e*x^n - c^2*d^2 + a*c*e^2)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4
 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^(2*n)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+a)(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + a)*(e*x^n + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})}\,dx = \int \frac{1}{(a+c\,x^{2\,n})\,(d+e\,x^n)^2}\,dx$$

```
[In] int(1/((a + c*x^(2*n))*(d + e*x^n)^2),x)
[Out] int(1/((a + c*x^(2*n))*(d + e*x^n)^2), x)
```

3.47 $\int \frac{d+ex^n}{a-cx^{2n}} dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx=\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[Out] d*x*hypergeom([1, 1/2/n],[1+1/2/n],c*x^(2*n)/a)/a+e*x^(1+n)*hypergeom([1, 1
/2*(1+n)/n],[3/2+1/2/n],c*x^(2*n)/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1432, 251, 371}

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx=\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

```
[In] Int[(d + e*x^n)/(a - c*x^(2*n)),x]
```

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (c*x^(2*n))/a])/a + (e*x
^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, (c*x^(2*n))/a]
)/(a*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int \frac{1}{a-cx^{2n}}\,dx + e\int \frac{x^n}{a-cx^{2n}}\,dx\\ &= \frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[In] Integrate[(d + e*x^n)/(a - c*x^(2*n)),x]
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (c*x^(2*n))/a])/a + (e*x
^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, (c*x^(2*n))/a]
)/(a*(1 + n))
```

Maple [F]

$$\int \frac{d+e\,x^{n}}{a-c\,x^{2n}}dx$$

[In] int((d+e*x^n)/(a-c*x^(2*n)),x)

[Out] int((d+e*x^n)/(a-c*x^(2*n)),x)

Fricas [F]

$$\int \frac{d+ex^{n}}{a-cx^{2n}}\,dx = \int -\frac{ex^{n}+d}{cx^{2\,n}-a}\,dx$$

[In] integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="fricas")

[Out] integral(-(e*x^n + d)/(c*x^(2*n) - a), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.64

$$\int \frac{d+ex^{n}}{a-cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}dx\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^{2}\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n^{2}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

[In] integrate((d+e*x**n)/(a-c*x**(2*n)),x)

```
[Out] a**(1/(2*n))*a**(-1 - 1/(2*n))*d*x*lerchphi(c*x**(2*n)*exp_polar(2*I*pi)/a,
 1, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + a**(-3/2 - 1/(2*n
))*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(2*I*pi)/a,
 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n*gamma(3/2 + 1/(2*n))) + a**(-3
/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar
(2*I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n**2*gamma(3/2 + 1/(2
*n)))
```

Maxima [F]

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \int -\frac{ex^n+d}{cx^{2n}-a}\,dx$$

```
[In] integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="maxima")
[Out] -integrate((e*x^n + d)/(c*x^(2*n) - a), x)
```

Giac [F]

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \int -\frac{ex^n+d}{cx^{2n}-a}\,dx$$

```
[In] integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="giac")
[Out] integrate(-(e*x^n + d)/(c*x^(2*n) - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \int \frac{d+e\,x^n}{a-c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)/(a - c*x^(2*n)),x)
[Out] int((d + e*x^n)/(a - c*x^(2*n)), x)
```

3.48 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 288

$$\begin{aligned}
&\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx \\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{2acn\,(a+cx^{2n})} \\
&\quad + \frac{3de^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} \\
&\quad - \frac{d(cd^2-3ae^2)\,(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} \\
&\quad + \frac{e^3x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(1+n)} \\
&\quad - \frac{e(3cd^2-ae^2)\,(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(1+n)}
\end{aligned}$$

```
[Out] 1/2*x*(d*(-3*a*e^2+c*d^2)+e*(-a*e^2+3*c*d^2)*x^n)/a/c/n/(a+c*x^(2*n))+3*d*e
^2*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/c-1/2*d*(-3*a*e^2+c*d^2
)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n+e^3*x^(1+n
)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/c/(1+n)-1/2*e*(-a*
e^2+3*c*d^2)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n
)/a)/a^2/c/n/(1+n)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 1445, 1432, 251, 371}

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$$
$$= -\frac{e(1-n)x^{n+1}(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}$$
$$-\frac{d(1-2n)x(cd^2-3ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn}$$
$$+\frac{x(ex^n(3cd^2-ae^2)+d(cd^2-3ae^2))}{2acn\,(a+cx^{2n})}$$
$$+\frac{3de^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}$$
$$+\frac{e^3x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(n+1)}$$

```
[In] Int[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(2*a*c*n*(a + c*x^(2*n)
)) + (3*d*e^2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))
/a)])/(a*c) - (d*(c*d^2 - 3*a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n)
, (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n) + (e^3*x^(1 + n)*Hypergeom
etric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c*(1 + n))
 - (e*(3*c*d^2 - a*e^2)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n
), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{cd^3-3ade^2+\left(3cd^2e-ae^3\right)x^n}{c\left(a+cx^{2n}\right)^2}+\frac{e^2(3d+ex^n)}{c\left(a+cx^{2n}\right)}\right)\,dx\\
&= \frac{\int \frac{cd^3-3ade^2+\left(3cd^2e-ae^3\right)x^n}{\left(a+cx^{2n}\right)^2}\,dx}{c}+\frac{e^2\int \frac{3d+ex^n}{a+cx^{2n}}\,dx}{c}\\
&= \frac{x\left(d\left(cd^2-3ae^2\right)+e\left(3cd^2-ae^2\right)x^n\right)}{2acn\left(a+cx^{2n}\right)}+\frac{\left(3de^2\right)\int \frac{1}{a+cx^{2n}}\,dx}{c}\\
&\quad+\frac{e^3\int \frac{x^n}{a+cx^{2n}}\,dx}{c}-\frac{\int \frac{\left(cd^3-3ade^2\right)(1-2n)+\left(3cd^2e-ae^3\right)(1-n)x^n}{a+cx^{2n}}\,dx}{2acn}\\
&= \frac{x\left(d\left(cd^2-3ae^2\right)+e\left(3cd^2-ae^2\right)x^n\right)}{2acn\left(a+cx^{2n}\right)}+\frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}\\
&\quad+\frac{e^3x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(1+n)}\\
&\quad-\frac{\left(d\left(cd^2-3ae^2\right)(1-2n)\right)\int \frac{1}{a+cx^{2n}}\,dx}{2acn}-\frac{\left(e\left(3cd^2-ae^2\right)(1-n)\right)\int \frac{x^n}{a+cx^{2n}}\,dx}{2acn}
\end{aligned}
$$

$$= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{2acn\,(a+cx^{2n})} + \frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac} - \frac{d(cd^2-3ae^2)\,(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn} + \frac{e^3x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(1+n)} - \frac{e(3cd^2-ae^2)\,(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(1+n)}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \frac{x\left(3ade^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+\frac{ae^3x^n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}+d(cd^2-\right.}{a^2c}$$

```
[In] Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(3*a*d*e^2*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a
)] + (a*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^
(2*n))/a)])/(1 + n) + d*(c*d^2 - 3*a*e^2)*Hypergeometric2F1[2, 1/(2*n), (2
+ n^(-1))/2, -((c*x^(2*n))/a)] + (e*(3*c*d^2 - a*e^2)*x^n*Hypergeometric2F1
[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(a^2*c)
```

Maple [F]

$$\int \frac{(d+e\,x^n)^3}{(a+c\,x^{2n})^2}dx$$

```
[In] int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^2*x^(4*n) +
 2*a*c*x^(2*n) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$$

```
[In] integrate((d+e*x**n)**3/(a+c*x**(2*n))**2,x)
```

```
[Out] Integral((d + e*x**n)**3/(a + c*x**(2*n))**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*c*d^2*e - a*e^3)*x*x^n + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*n*x^(2*n) +
a^2*c*n) + integrate(1/2*(c*d^3*(2*n - 1) + 3*a*d*e^2 + (a*e^3*(n + 1) + 3*
c*d^2*e*(n - 1))*x^n)/(a*c^2*n*x^(2*n) + a^2*c*n), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx=\int \frac{{\left(d+e\,x^n\right)}^3}{{\left(a+c\,x^{2\,n}\right)}^2} \,d x$$

```
[In] int((d + e*x^n)^3/(a + c*x^(2*n))^2,x)
[Out] int((d + e*x^n)^3/(a + c*x^(2*n))^2, x)
```

3.49 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \frac{x(cd^2-ae^2+2cdex^n)}{2acn\,(a+cx^{2n})} + \frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} - \frac{(cd^2-ae^2)\,(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} - \frac{de(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2n(1+n)}$$

```
[Out] 1/2*x*(c*d^2-a*e^2+2*c*d*e*x^n)/a/c/n/(a+c*x^(2*n))+e^2*x*hypergeom([1, 1/2
/n],[1+1/2/n],-c*x^(2*n)/a)/a/c-1/2*(-a*e^2+c*d^2)*(1-2*n)*x*hypergeom([1,
1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n-d*e*(1-n)*x^(1+n)*hypergeom([1, 1/2*
(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/n/(1+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

= {1451, 1445, 1432, 251, 371}

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = -\frac{(1-2n)x(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} - \frac{de(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)} + \frac{x(-ae^2+cd^2+2cdex^n)}{2acn\,(a+cx^{2n})} + \frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}$$

```
[In] Int[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(2*a*c*n*(a + c*x^(2*n))) + (e^2*x*Hyperg
eometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) - ((c*d^2
- a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(
2*n))/a)])/(2*a^2*c*n) - (d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n
)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
```

```
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\text{integral} = \int \left(\frac{cd^2-ae^2+2cdex^n}{c\left(a+cx^{2n}\right)^2}+\frac{e^2}{c\left(a+cx^{2n}\right)}\right)dx$$

$$= \frac{\int \frac{cd^2-ae^2+2cdex^n}{\left(a+cx^{2n}\right)^2}\,dx}{c}+\frac{e^2\int \frac{1}{a+cx^{2n}}\,dx}{c}$$

$$= \frac{x(cd^2-ae^2+2cdex^n)}{2acn\left(a+cx^{2n}\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}-\frac{\int \frac{(cd^2-ae^2)(1-2n)+2cde(1-n)x^n}{a+cx^{2n}}\,dx}{2acn}$$

$$= \frac{x(cd^2-ae^2+2cdex^n)}{2acn\left(a+cx^{2n}\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac} - \frac{\left((cd^2-ae^2)(1-2n)\right)\int \frac{1}{a+cx^{2n}}\,dx}{2acn}-\frac{(de(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{an}$$

$$= \frac{x(cd^2-ae^2+2cdex^n)}{2acn\left(a+cx^{2n}\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac} - \frac{(cd^2-ae^2)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn} - \frac{de(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(1+n)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx$$
$$= \frac{x\left(ae^2(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+(cd^2-ae^2)\,(1+n)\,\text{Hypergeometric2F1}\left(2,\right.\right.}{a^2c(1+n)}$$

```
[In] Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(a*e^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n
))/a)] + (c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))
/2, -((c*x^(2*n))/a)] + 2*c*d*e*x^n*Hypergeometric2F1[2, (1 + n)/(2*n), (3
+ n^(-1))/2, -((c*x^(2*n))/a)]))/(a^2*c*(1 + n))
```

Maple [F]

$$\int \frac{(d+e\,x^n)^2}{(a+c\,x^{2n})^2}dx$$

```
[In] int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2\,n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2
), x)
```

Sympy [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx$$

```
[In] integrate((d+e*x**n)**2/(a+c*x**(2*n))**2,x)
[Out] Integral((d + e*x**n)**2/(a + c*x**(2*n))**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="maxima")
[Out] 1/2*(2*c*d*e*x*x^n + (c*d^2 - a*e^2)*x)/(a*c^2*n*x^(2*n) + a^2*c*n) + integ
rate(1/2*(2*c*d*e*(n - 1)*x^n + c*d^2*(2*n - 1) + a*e^2)/(a*c^2*n*x^(2*n) +
 a^2*c*n), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \int \frac{(d+e\,x^n)^2}{(a+c\,x^{2\,n})^2}\,dx$$

```
[In] int((d + e*x^n)^2/(a + c*x^(2*n))^2,x)
[Out] int((d + e*x^n)^2/(a + c*x^(2*n))^2, x)
```

3.50 $\int \frac{d+ex^n}{(a+cx^{2n})^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{d+ex^n}{(a+cx^{2n})^2} \, dx = \frac{x(d+ex^n)}{2an(a+cx^{2n})} - \frac{d(1-2n)x \,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n} - \frac{e(1-n)x^{1+n} \,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(1+n)}$$

```
[Out] 1/2*x*(d+e*x^n)/a/n/(a+c*x^(2*n))-1/2*d*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1
/2/n],-c*x^(2*n)/a)/a^2/n-1/2*e*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3
/2+1/2/n],-c*x^(2*n)/a)/a^2/n/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1445, 1432, 251, 371}

$$\int \frac{d+ex^n}{(a+cx^{2n})^2} \, dx = -\frac{d(1-2n)x \,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n} - \frac{e(1-n)x^{n+1} \,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)} + \frac{x(d+ex^n)}{2an(a+cx^{2n})}$$

```
[In] Int[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(d + e*x^n))/(2*a*n*(a + c*x^(2*n))) - (d*(1 - 2*n)*x*Hypergeometric2F1[
1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*n) - (e*(1 - n)*x^(1
+ n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])
/(2*a^2*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(d+ex^n)}{2an\,(a+cx^{2n})} - \frac{\int \frac{d(1-2n)+e(1-n)x^n}{a+cx^{2n}}\,dx}{2an} \\
&= \frac{x(d+ex^n)}{2an\,(a+cx^{2n})} - \frac{(d(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{2an} - \frac{(e(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{2an} \\
&= \frac{x(d+ex^n)}{2an\,(a+cx^{2n})} - \frac{d(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n} \\
&\quad - \frac{e(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(2,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2(1+n)}$$

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

```
[Out] (d*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^2 +
 (e*x^(1 + n)*Hypergeometric2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a^2*(1 + n))
```

Maple [F]

$$\int \frac{d+e\,x^n}{(a+c\,x^{2n})^2}dx$$

```
[In] int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 168.68 (sec) , antiderivative size = 994, normalized size of antiderivative = 7.42

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e*x**n)/(a+c*x**(2*n))**2,x)
```

```
[Out] d*(2*a*a**(1/(2*n))*a**(-2 - 1/(2*n))*n*x*lerchphi(c*x**(2*n)*exp_polar(I*p
i)/a, 1, 1/(2*n))*gamma(1/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x*
*(2*n)*gamma(1 + 1/(2*n))) + 2*a*a**(1/(2*n))*a**(-2 - 1/(2*n))*n*x*gamma(1
/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(1 + 1/(2*n))
) - a*a**(1/(2*n))*a**(-2 - 1/(2*n))*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/
a, 1, 1/(2*n))*gamma(1/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2
*n)*gamma(1 + 1/(2*n))) + 2*a**(1/(2*n))*a**(-2 - 1/(2*n))*c*n*x*x**(2*n)*l
erchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(8*a*n**3*
gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(1 + 1/(2*n))) - a**(1/(2*n))*a
**(-2 - 1/(2*n))*c*x*x**(2*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/(
2*n))*gamma(1/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma
(1 + 1/(2*n)))) + e*(a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*n**2*x**(n +
1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(
2*n))/(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n
))) + 2*a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*n**2*x**(n + 1)*gamma(1/2
+ 1/(2*n))/(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1
/(2*n))) + 2*a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*n*x**(n + 1)*gamma(1/
2 + 1/(2*n))/(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 +
 1/(2*n))) - a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*x**(n + 1)*lerchphi(c
*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(8*a*n*
*3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n))) + a**(-5/
2 - 1/(2*n))*a**(1/2 + 1/(2*n))*c*n**2*x**(2*n)*x**(n + 1)*lerchphi(c*x**(2
*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(8*a*n**3*gam
ma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n))) - a**(-5/2 - 1/
(2*n))*a**(1/2 + 1/(2*n))*c*x**(2*n)*x**(n + 1)*lerchphi(c*x**(2*n)*exp_pol
ar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(8*a*n**3*gamma(3/2 + 1/
(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n))))
```

Maxima [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2n}+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x*x^n + d*x)/(a*c*n*x^(2*n) + a^2*n) + integrate(1/2*(e*(n - 1)*x^n
+ d*(2*n - 1))/(a*c*n*x^(2*n) + a^2*n), x)
```

Giac [F]

$$\int \frac{d + ex^n}{(a + cx^{2n})^2}\,dx = \int \frac{ex^n + d}{(cx^{2\,n} + a)^2}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate((e*x^n + d)/(c*x^(2*n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d + ex^n}{(a + cx^{2n})^2}\,dx = \int \frac{d + e\,x^n}{(a + c\,x^{2\,n})^2}\,dx$$

```
[In] int((d + e*x^n)/(a + c*x^(2*n))^2,x)
[Out] int((d + e*x^n)/(a + c*x^(2*n))^2, x)
```

3.51 $\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 333

$$\begin{aligned}
\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx &= \frac{cx(d-ex^n)}{2a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)}+\frac{cde^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^2} \\
&\quad -\frac{cd(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)n} \\
&\quad +\frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(cd^2+ae^2\right)^2} \\
&\quad -\frac{ce^3x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^2(1+n)} \\
&\quad +\frac{ce(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)n(1+n)}
\end{aligned}$$

```
[Out] 1/2*c*x*(d-e*x^n)/a/(a*e^2+c*d^2)/n/(a+c*x^(2*n))+c*d*e^2*x*hypergeom([1, 1
/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^2-1/2*c*d*(1-2*n)*x*hypergeom
([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)/n+e^4*x*hypergeom([1,
 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)^2-c*e^3*x^(1+n)*hypergeom([1, 1/2*(
1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^2/(1+n)+1/2*c*e*(1-n)*x^(
1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)
/n/(1+n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 251, 1445, 1432, 371}

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx = \frac{ce(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)(ae^2+cd^2)} - \frac{cd(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2+cd^2)} + \frac{cde^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)^2} + \frac{cx(d-ex^n)}{2an(ae^2+cd^2)(a+cx^{2n})} + \frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(ae^2+cd^2)^2} - \frac{ce^3x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2+cd^2)^2}$$

```
[In] Int[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```

```
[Out] (c*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))) + (c*d*e^2*x*Hype
rgeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*
e^2)^2) - (c*d*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -(
(c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)^2) - (c*e^3*x^(1 + n)*H
ypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c
*d^2 + a*e^2)^2*(1 + n)) + (c*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 +
 n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)*n*(1 +
 n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^n)}-\frac{c(-d+ex^n)}{(cd^2+ae^2)(a+cx^{2n})^2}-\frac{ce^2(-d+ex^n)}{(cd^2+ae^2)^2(a+cx^{2n})}\right)dx\\
&= -\frac{(ce^2)\int\frac{-d+ex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^2}+\frac{e^4\int\frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^2}-\frac{c\int\frac{-d+ex^n}{(a+cx^{2n})^2}\,dx}{cd^2+ae^2}\\
&= \frac{cx(d-ex^n)}{2a(cd^2+ae^2)n(a+cx^{2n})}+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^2}\\
&\quad+\frac{(cde^2)\int\frac{1}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^2}-\frac{(ce^3)\int\frac{x^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^2}+\frac{c\int\frac{-d(1-2n)+e(1-n)x^n}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)n}\\
&= \frac{cx(d-ex^n)}{2a(cd^2+ae^2)n(a+cx^{2n})}+\frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2}\\
&\quad+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^2}-\frac{ce^3x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2(1+n)}\\
&\quad-\frac{(cd(1-2n))\int\frac{1}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)n}+\frac{(ce(1-n))\int\frac{x^n}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)n}
\end{aligned}
$$

$$= \frac{cx(d-ex^n)}{2a(cd^2+ae^2)n(a+cx^{2n})} + \frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2}$$
$$- \frac{cd(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)n}$$
$$+ \frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^2} - \frac{ce^3x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2(1+n)}$$
$$+ \frac{ce(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)n(1+n)}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx$$
$$= \frac{x\left(acd^2e^2(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+a^2e^4(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1-\right.\right.}{}$$

```
[In] Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```

```
[Out] (x*(a*c*d^2*e^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*
x^(2*n))/a)] + a^2*e^4*(1 + n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((
e*x^n)/d)] + c*d*(-(a*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-
1))/2, -((c*x^(2*n))/a)]) + (c*d^2 + a*e^2)*(d*(1 + n)*Hypergeometric2F1[2,
 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)] - e*x^n*Hypergeometric2F1[2, (1
 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)]))))/(a^2*d*(c*d^2 + a*e^2)^2
*(1 + n))
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)(a+c\,x^{2n})^2}dx$$

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

```
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+a)^2(ex^n+d)}\,dx$$

[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")

[Out] integral(1/(a^2*e*x^n + a^2*d + (c^2*e*x^n + c^2*d)*x^(4*n) + 2*(a*c*e*x^n + a*c*d)*x^(2*n)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx = \text{Timed out}$$

[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+a)^2(ex^n+d)}\,dx$$

[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")

[Out] e^4*integrate(1/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x^n), x) - 1/2*(c*e*x*x^n - c*d*x)/(a^2*c*d^2*n + a^3*e^2*n + (a*c^2*d^2*n + a^2*c*e^2*n)*x^(2*n)) - integrate(-1/2*(a*c*d*e^2*(4*n - 1) + c^2*d^3*(2*n - 1) - (a*c*e^3*(3*n - 1) + c^2*d^2*e*(n - 1))*x^n)/(a^2*c^2*d^4*n + 2*a^3*c*d^2*e^2*n + a^4*e^4*n + (a*c^3*d^4*n + 2*a^2*c^2*d^2*e^2*n + a^3*c*e^4*n)*x^(2*n)), x)

Giac [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+a)^2(ex^n+d)}\,dx$$

[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})^2}\,dx = \int \frac{1}{\left(a+c\,x^{2\,n}\right)^2\,\left(d+e\,x^n\right)}\,dx$$

```
[In] int(1/((a + c*x^(2*n))^2*(d + e*x^n)),x)
[Out] int(1/((a + c*x^(2*n))^2*(d + e*x^n)), x)
```

3.52 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 410

$$\begin{aligned}
&\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})} \\
&\quad+ \frac{ce^2(3cd^2-ae^2)\,x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3} \\
&\quad- \frac{c(cd^2-ae^2)(1-2n)x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)^2n} \\
&\quad+ \frac{4ce^4x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^3} \\
&\quad- \frac{4c^2de^3x^{1+n} \operatorname{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3(1+n)} \\
&\quad+ \frac{c^2de(1-n)x^{1+n} \operatorname{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2(cd^2+ae^2)^2n(1+n)} \\
&\quad+ \frac{e^4x \operatorname{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)^2}
\end{aligned}$$

```
[Out] 1/2*c*x*(c*d^2-a*e^2-2*c*d*e*x^n)/a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))+c*e^2*(
-a*e^2+3*c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d
^2)^3-1/2*c*(-a*e^2+c*d^2)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2
*n)/a)/a^2/(a*e^2+c*d^2)^2/n+4*c*e^4*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)
/(a*e^2+c*d^2)^3-4*c^2*d*e^3*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n]
```

```
,-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^3/(1+n)+c^2*d*e*(1-n)*x^(1+n)*hypergeom([1,
1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n/(1+n)+e^4*x*hy
pergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1451, 251, 1445, 1432, 371}

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} \, dx$$
$$= \frac{c^2 de(1-n)x^{n+1} \text{Hypergeometric2F1}\left(1, \frac{n+1}{2n}, \frac{1}{2}\left(3+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{a^2 n(n+1) \left(ae^2+cd^2\right)^2}$$
$$- \frac{c(1-2n)x(cd^2-ae^2) \text{Hypergeometric2F1}\left(1, \frac{1}{2n}, \frac{1}{2}\left(2+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{2a^2 n \left(ae^2+cd^2\right)^2}$$
$$- \frac{4c^2de^3x^{n+1} \text{Hypergeometric2F1}\left(1, \frac{n+1}{2n}, \frac{1}{2}\left(3+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{a(n+1) \left(ae^2+cd^2\right)^3}$$
$$+ \frac{ce^2x(3cd^2-ae^2) \text{Hypergeometric2F1}\left(1, \frac{1}{2n}, \frac{1}{2}\left(2+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{a \left(ae^2+cd^2\right)^3}$$
$$+ \frac{cx(-ae^2+cd^2-2cdex^n)}{2an \left(ae^2+cd^2\right)^2 (a+cx^{2n})} + \frac{4ce^4x \text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1+\frac{1}{n}, -\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^3}$$
$$+ \frac{e^4x \text{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, -\frac{ex^n}{d}\right)}{d^2 \left(ae^2+cd^2\right)^2}$$

```
[In] Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n)
)) + (c*e^2*(3*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/
2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) - (c*(c*d^2 - a*e^2)*(1 - 2*n)*
x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(
c*d^2 + a*e^2)^2*n) + (4*c*e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((e*x^n)/d)])/(c*d^2 + a*e^2)^3 - (4*c^2*d*e^3*x^(1 + n)*Hypergeometric2F1[
1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3*(
1 + n)) + (c^2*d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^2*n*(1 + n)) + (e^4*x
*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^
2)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^n)^2} + \frac{4cde^4}{(cd^2+ae^2)^3(d+ex^n)} - \frac{c(-cd^2+ae^2+2cdex^n)}{(cd^2+ae^2)^2(a+cx^{2n})^2} - \frac{ce^2(-3cd^2+ae^2+4cdex^n)}{(cd^2+ae^2)^3(a+cx^{2n})}\right) dx$$

$$= -\frac{(ce^2)\int \frac{-3cd^2+ae^2+4cdex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^3} + \frac{(4cde^4)\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^3} - \frac{c\int \frac{-cd^2+ae^2+2cdex^n}{(a+cx^{2n})^2}\,dx}{(cd^2+ae^2)^2} + \frac{e^4\int \frac{1}{(d+ex^n)^2}\,dx}{(cd^2+ae^2)^2}$$

$$= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{2a\left(cd^2 + ae^2\right)^2 n\left(a + cx^{2n}\right)} + \frac{4ce^4 x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{\left(cd^2 + ae^2\right)^3} + \frac{e^4 x\,{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\left(cd^2 + ae^2\right)^2} - \frac{(4c^2de^3)\int \frac{x^n}{a + cx^{2n}}\,dx}{\left(cd^2 + ae^2\right)^3} + \frac{(ce^2(3cd^2 - ae^2))\int \frac{1}{a + cx^{2n}}\,dx}{\left(cd^2 + ae^2\right)^3} + \frac{c\int \frac{(-cd^2 + ae^2)(1 - 2n) + 2cde(1 - n)x^n}{a + cx^{2n}}\,dx}{2a\left(cd^2 + ae^2\right)^2 n}$$

$$= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{2a\left(cd^2 + ae^2\right)^2 n\left(a + cx^{2n}\right)} + \frac{ce^2(3cd^2 - ae^2)\,x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^3} + \frac{4ce^4 x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{\left(cd^2 + ae^2\right)^3} - \frac{4c^2de^3x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^3(1 + n)} + \frac{e^4 x\,{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\left(cd^2 + ae^2\right)^2} - \frac{(c(cd^2 - ae^2)(1 - 2n))\int \frac{1}{a + cx^{2n}}\,dx}{2a\left(cd^2 + ae^2\right)^2 n} + \frac{(c^2de(1 - n))\int \frac{x^n}{a + cx^{2n}}\,dx}{a\left(cd^2 + ae^2\right)^2 n}$$

$$= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{2a\left(cd^2 + ae^2\right)^2 n\left(a + cx^{2n}\right)} + \frac{ce^2(3cd^2 - ae^2)\,x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^3} - \frac{c(cd^2 - ae^2)(1 - 2n)x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2 + ae^2\right)^2 n} + \frac{4ce^4 x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{\left(cd^2 + ae^2\right)^3} - \frac{4c^2de^3x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^3(1 + n)} + \frac{c^2de(1 - n)x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2\left(cd^2 + ae^2\right)^2 n(1 + n)} + \frac{e^4 x\,{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\left(cd^2 + ae^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(d + ex^n\right)^2\left(a + cx^{2n}\right)^2}\,dx = \frac{x\left(\frac{ce^2(3cd^2 - ae^2)\,\text{Hypergeometric2F1}\left(1, \frac{1}{2n}, \frac{1}{2}\left(2 + \frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{a} + 4ce^4\,\text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{ex^n}{d}\right) - \frac{4c^2de^3x^n\,\text{Hy}}{}\right.}{}$$

```
[In] Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```

```
[Out] (x*((c*e^2*(3*c*d^2 - a*e^2)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2,
-((c*x^(2*n))/a)])/a + 4*c*e^4*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((
e*x^n)/d)] - (4*c^2*d*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-
1))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) + (c*(c*d^2 - a*e^2)*(c*d^2 + a*e^2)*
```

```
Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^2 + (e^4
*(c*d^2 + a*e^2)*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d^
2 - (2*c^2*d*e*(c*d^2 + a*e^2)*x^n*Hypergeometric2F1[2, (1 + n)/(2*n), (3 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1 + n))))/(c*d^2 + a*e^2)^3
```

Maple [F]

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{2}\left(a+c\,x^{2n}\right)^{2}}dx$$

```
[In] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{\left(d+ex^{n}\right)^{2}\left(a+cx^{2n}\right)^{2}}\,dx=\int \frac{1}{\left(cx^{2\,n}+a\right)^{2}\left(ex^{n}+d\right)^{2}}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e^2*x^(2*n) + 2*a^2*d*e*x^n + a^2*d^2 + (c^2*e^2*x^(2*n) +
2*c^2*d*e*x^n + c^2*d^2)*x^(4*n) + 2*(a*c*e^2*x^(2*n) + 2*a*c*d*e*x^n + a*c
*d^2)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^{n}\right)^{2}\left(a+cx^{2n}\right)^{2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} dx = \int \frac{1}{(cx^{2n}+a)^2 (ex^n+d)^2} dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] (c*d^2*e^4*(5*n - 1) + a*e^6*(n - 1))*integrate(1/(c^3*d^8*n + 3*a*c^2*d^6*
e^2*n + 3*a^2*c*d^4*e^4*n + a^3*d^2*e^6*n + (c^3*d^7*e*n + 3*a*c^2*d^5*e^3*
n + 3*a^2*c*d^3*e^5*n + a^3*d*e^7*n)*x^n), x) - 1/2*(2*(c^2*d^2*e^2 - a*c*e
^4)*x*x^(2*n) + (c^2*d^3*e + a*c*d*e^3)*x*x^n - (c^2*d^4 - a*c*d^2*e^2 + 2*
a^2*e^4)*x)/(a^2*c^2*d^6*n + 2*a^3*c*d^4*e^2*n + a^4*d^2*e^4*n + (a*c^3*d^5
*e*n + 2*a^2*c^2*d^3*e^3*n + a^3*c*d*e^5*n)*x^(3*n) + (a*c^3*d^6*n + 2*a^2*
c^2*d^4*e^2*n + a^3*c*d^2*e^4*n)*x^(2*n) + (a^2*c^2*d^5*e*n + 2*a^3*c*d^3*e
^3*n + a^4*d*e^5*n)*x^n) - integrate(1/2*(a^2*c*e^4*(4*n - 1) - c^3*d^4*(2*
n - 1) - 6*a*c^2*d^2*e^2*n + 2*(a*c^2*d*e^3*(5*n - 1) + c^3*d^3*e*(n - 1))*
x^n)/(a^2*c^3*d^6*n + 3*a^3*c^2*d^4*e^2*n + 3*a^4*c*d^2*e^4*n + a^5*e^6*n +
 (a*c^4*d^6*n + 3*a^2*c^3*d^4*e^2*n + 3*a^3*c^2*d^2*e^4*n + a^4*c*e^6*n)*x^
(2*n)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} dx = \int \frac{1}{(cx^{2n}+a)^2 (ex^n+d)^2} dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} dx = \int \frac{1}{(a+c\,x^{2\,n})^2 (d+e\,x^n)^2} dx$$

```
[In] int(1/((a + c*x^(2*n))^2*(d + e*x^n)^2),x)
```

```
[Out] int(1/((a + c*x^(2*n))^2*(d + e*x^n)^2), x)
```

3.53 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 424

$$\begin{aligned}
&\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3} dx \\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)x^n)}{4acn(a+cx^{2n})^2} + \frac{e^2x(3d+ex^n)}{2acn(a+cx^{2n})} \\
&\quad - \frac{x(d(cd^2-3ae^2)(1-4n)+e(3cd^2-ae^2)(1-3n)x^n)}{8a^2cn^2(a+cx^{2n})} \\
&\quad + \frac{d(cd^2-3ae^2)(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2} \\
&\quad - \frac{3de^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} \\
&\quad + \frac{e(3cd^2-ae^2)(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(1+n)} \\
&\quad - \frac{e^3(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(1+n)}
\end{aligned}$$

```
[Out] 1/4*x*(d*(-3*a*e^2+c*d^2)+e*(-a*e^2+3*c*d^2)*x^n)/a/c/n/(a+c*x^(2*n))^2+1/2
*e^2*x*(3*d+e*x^n)/a/c/n/(a+c*x^(2*n))-1/8*x*(d*(-3*a*e^2+c*d^2)*(1-4*n)+e*
(-a*e^2+3*c*d^2)*(1-3*n)*x^n)/a^2/c/n^2/(a+c*x^(2*n))+1/8*d*(-3*a*e^2+c*d^2
)*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/c/n^2-
3/2*d*e^2*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n+1/
8*e*(-a*e^2+3*c*d^2)*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+
1/2/n],-c*x^(2*n)/a)/a^3/c/n^2/(1+n)-1/2*e^3*(1-n)*x^(1+n)*hypergeom([1, 1/
2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/c/n/(1+n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1451, 1445, 1432, 251, 371}

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \frac{e(1-3n)(1-n)x^{n+1}(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(n+1)} + \frac{d(1-4n)(1-2n)x(cd^2-3ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2} - \frac{x(e(1-3n)x^n(3cd^2-ae^2)+d(1-4n)(cd^2-3ae^2))}{8a^2cn^2(a+cx^{2n})} - \frac{3de^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} - \frac{e^3(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)} + \frac{x(ex^n(3cd^2-ae^2)+d(cd^2-3ae^2))}{4acn(a+cx^{2n})^2} + \frac{e^2x(3d+ex^n)}{2acn(a+cx^{2n})}$$

```
[In] Int[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(4*a*c*n*(a + c*x^(2*n)
)^2) + (e^2*x*(3*d + e*x^n))/(2*a*c*n*(a + c*x^(2*n))) - (x*(d*(c*d^2 - 3*a
*e^2)*(1 - 4*n) + e*(3*c*d^2 - a*e^2)*(1 - 3*n)*x^n))/(8*a^2*c*n^2*(a + c*x
^(2*n))) + (d*(c*d^2 - 3*a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1,
1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2) - (3*d*e^2*(1 - 2
*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a
^2*c*n) + (e*(3*c*d^2 - a*e^2)*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F
1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2*(1 + n)
) - (e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))
/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{cd^3-3ade^2+(3cd^2e-ae^3)\,x^n}{c\,(a+cx^{2n})^3}+\frac{e^2(3d+ex^n)}{c\,(a+cx^{2n})^2}\right)dx \\
&= \frac{\int \frac{cd^3-3ade^2+(3cd^2e-ae^3)x^n}{(a+cx^{2n})^3}\,dx}{c}+\frac{e^2\int \frac{3d+ex^n}{(a+cx^{2n})^2}\,dx}{c} \\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})} \\
&\quad -\frac{\int \frac{(cd^3-3ade^2)(1-4n)+(3cd^2e-ae^3)(1-3n)x^n}{(a+cx^{2n})^2}\,dx}{4acn}-\frac{e^2\int \frac{3d(1-2n)+e(1-n)x^n}{a+cx^{2n}}\,dx}{2acn}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2} + \frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})} \\
&\quad - \frac{x(d(cd^2-3ae^2)\,(1-4n)+e(3cd^2-ae^2)\,(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})} \\
&\quad + \frac{\int \frac{(cd^3-3ade^2)(1-4n)(1-2n)+(3cd^2e-ae^3)(1-3n)(1-n)x^n}{a+cx^{2n}}\,dx}{8a^2cn^2} \\
&\quad - \frac{(3de^2(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{2acn} - \frac{(e^3(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{2acn} \\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2} + \frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})} \\
&\quad - \frac{x(d(cd^2-3ae^2)\,(1-4n)+e(3cd^2-ae^2)\,(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})} \\
&\quad - \frac{3de^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn} \\
&\quad - \frac{e^3(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(1+n)} \\
&\quad + \frac{(d(cd^2-3ae^2)\,(1-4n)(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{8a^2cn^2} \\
&\quad + \frac{(e(3cd^2-ae^2)\,(1-3n)(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{8a^2cn^2} \\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2} + \frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})} \\
&\quad - \frac{x(d(cd^2-3ae^2)\,(1-4n)+e(3cd^2-ae^2)\,(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})} \\
&\quad + \frac{d(cd^2-3ae^2)\,(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2} \\
&\quad - \frac{3de^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn} \\
&\quad + \frac{e(3cd^2-ae^2)\,(1-3n)(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(1+n)} \\
&\quad - \frac{e^3(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.44

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \frac{x\left(3ade^2\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+\frac{ae^3x^n\,\text{Hypergeometric2F1}\left(2,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}+d(cd^2$$

```
[In] Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(3*a*d*e^2*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a
)] + (a*e^3*x^n*Hypergeometric2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^
(2*n))/a)])/(1 + n) + d*(c*d^2 - 3*a*e^2)*Hypergeometric2F1[3, 1/(2*n), (2
+ n^(-1))/2, -((c*x^(2*n))/a)] + (e*(3*c*d^2 - a*e^2)*x^n*Hypergeometric2F1
[3, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(a^3*c)
```

Maple [F]

$$\int \frac{(d+e\,x^n)^3}{(a+c\,x^{2n})^3}dx$$

```
[In] int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^3*x^(6*n) +
 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**3/(a+c*x**(2*n))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="maxima")
[Out] 1/8*((3*c^2*d^2*e*(3*n - 1) + a*c*e^3*(n + 1))*x*x^(3*n) + (c^2*d^3*(4*n -
1) + 3*a*c*d*e^2)*x*x^(2*n) + (3*a*c*d^2*e*(5*n - 1) - a^2*e^3*(n - 1))*x*x
^n + (a*c*d^3*(6*n - 1) - 3*a^2*d*e^2*(2*n - 1))*x)/(a^2*c^3*n^2*x^(4*n) +
2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^2) + integrate(1/8*((8*n^2 - 6*n + 1)*c*d^3
 + 3*a*d*e^2*(2*n - 1) + (3*(3*n^2 - 4*n + 1)*c*d^2*e + (n^2 - 1)*a*e^3)*x^
n)/(a^2*c^2*n^2*x^(2*n) + a^3*c*n^2), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(d+e\,x^n)^3}{(a+c\,x^{2\,n})^3}\,dx$$

```
[In] int((d + e*x^n)^3/(a + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)^3/(a + c*x^(2*n))^3, x)
```

3.54 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 272

$$\begin{aligned}\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx &= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}-\frac{x((cd^2-ae^2)\,(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}\\
&\quad+\frac{(cd^2-ae^2)\,(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}\\
&\quad+\frac{de(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3n^2(1+n)}\\
&\quad+\frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2c}\end{aligned}$$

```
[Out] 1/4*x*(c*d^2-a*e^2+2*c*d*e*x^n)/a/c/n/(a+c*x^(2*n))^2-1/8*x*((-a*e^2+c*d^2)
*(1-4*n)+2*c*d*e*(1-3*n)*x^n)/a^2/c/n^2/(a+c*x^(2*n))+1/8*(-a*e^2+c*d^2)*(1
-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/c/n^2+1/4*
d*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)
/a)/a^3/n^2/(1+n)+e^2*x*hypergeom([2, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1451, 1445, 1432, 251, 371}

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \frac{(1-4n)(1-2n)x(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2} + \frac{de(1-3n)(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)} - \frac{x((1-4n)\left(cd^2-ae^2\right)+2cde(1-3n)x^n)}{8a^2cn^2\left(a+cx^{2n}\right)} + \frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2c} + \frac{x(-ae^2+cd^2+2cdex^n)}{4acn\left(a+cx^{2n}\right)^2}$$

```
[In] Int[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(4*a*c*n*(a + c*x^(2*n))^2) - (x*((c*d^2
- a*e^2)*(1 - 4*n) + 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*c*n^2*(a + c*x^(2*n)))
+ ((c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2) + (d*e*(1 - 3*n)*(1 - n)*x^(1
+ n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])
/(4*a^3*n^2*(1 + n)) + (e^2*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2,
 -((c*x^(2*n))/a)])/(a^2*c)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
```

```
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{cd^2-ae^2+2cdex^n}{c\,(a+cx^{2n})^3}+\frac{e^2}{c\,(a+cx^{2n})^2}\right)dx\\
&= \frac{\int \frac{cd^2-ae^2+2cdex^n}{(a+cx^{2n})^3}\,dx}{c}+\frac{e^2\int \frac{1}{(a+cx^{2n})^2}\,dx}{c}\\
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}-\frac{\int \frac{(cd^2-ae^2)(1-4n)+2cde(1-3n)x^n}{(a+cx^{2n})^2}\,dx}{4acn}\\
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}-\frac{x((cd^2-ae^2)\,(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}\\
&\quad+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}+\frac{\int \frac{(cd^2-ae^2)(1-4n)(1-2n)+2cde(1-3n)(1-n)x^n}{a+cx^{2n}}\,dx}{8a^2cn^2}\\
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}-\frac{x((cd^2-ae^2)\,(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}\\
&\quad+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}\\
&\quad+\frac{((cd^2-ae^2)\,(1-4n)(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{8a^2cn^2}+\frac{(de(1-3n)(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{4a^2n^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2} - \frac{x((cd^2-ae^2)\,(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})} \\
&+ \frac{(cd^2-ae^2)\,(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2} \\
&+ \frac{de(1-3n)(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{4a^3n^2(1+n)} \\
&+ \frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.50

$$
\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx
= \frac{x\left(ae^2(1+n)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+(cd^2-ae^2)\,(1+n)\,\text{Hypergeometric2F1}\left(3,\right.\right.}{a^3c(1+n)}
$$

```
[In] Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(a*e^2*(1 + n)*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n
))/a)] + (c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1))
/2, -((c*x^(2*n))/a)] + 2*c*d*e*x^n*Hypergeometric2F1[3, (1 + n)/(2*n), (3
+ n^(-1))/2, -((c*x^(2*n))/a)]))/(a^3*c*(1 + n))
```

Maple [F]

$$
\int \frac{(d+e\,x^n)^2}{(a+c\,x^{2n})^3}dx
$$

```
[In] int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3} dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^3} dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="fricas")
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3
*a^2*c*x^(2*n) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3} dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**2/(a+c*x**(2*n))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3} dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^3} dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="maxima")
[Out] 1/8*(2*c^2*d*e*(3*n - 1)*x*x^(3*n) + 2*a*c*d*e*(5*n - 1)*x*x^n + (c^2*d^2*(
4*n - 1) + a*c*e^2)*x*x^(2*n) + (a*c*d^2*(6*n - 1) - a^2*e^2*(2*n - 1))*x)/
(a^2*c^3*n^2*x^(4*n) + 2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^2) + integrate(1/8*(
2*(3*n^2 - 4*n + 1)*c*d*e*x^n + (8*n^2 - 6*n + 1)*c*d^2 + a*e^2*(2*n - 1))/
(a^2*c^2*n^2*x^(2*n) + a^3*c*n^2), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3} dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^3} dx$$

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \int \frac{(d+e\,x^n)^2}{(a+c\,x^{2\,n})^3}\,dx$$

```
[In] int((d + e*x^n)^2/(a + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)^2/(a + c*x^(2*n))^3, x)
```

3.55 $\int \frac{d+ex^n}{(a+cx^{2n})^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{d+ex^n}{(a+cx^{2n})^3} dx = \frac{x(d+ex^n)}{4an(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2(a+cx^{2n})} + \frac{d(1-4n)(1-2n)x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2} + \frac{e(1-3n)(1-n)x^{1+n} \operatorname{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2(1+n)}$$

```
[Out] 1/4*x*(d+e*x^n)/a/n/(a+c*x^(2*n))^2-1/8*x*(d*(1-4*n)+e*(1-3*n)*x^n)/a^2/n^2
/(a+c*x^(2*n))+1/8*d*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^
(2*n)/a)/a^3/n^2+1/8*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/
2+1/2/n],-c*x^(2*n)/a)/a^3/n^2/(1+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1445, 1432, 251, 371}

$$\int \frac{d+ex^n}{(a+cx^{2n})^3} dx = \frac{d(1-4n)(1-2n)x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2} + \frac{e(1-3n)(1-n)x^{n+1} \operatorname{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2(a+cx^{2n})} + \frac{x(d+ex^n)}{4an(a+cx^{2n})^2}$$

```
[In] Int[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(d + e*x^n))/(4*a*n*(a + c*x^(2*n))^2) - (x*(d*(1 - 4*n) + e*(1 - 3*n)*x
^n))/(8*a^2*n^2*(a + c*x^(2*n))) + (d*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2
F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*n^2) + (e*(1 - 3*n
)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((
c*x^(2*n))/a)])/(8*a^3*n^2*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{\int \frac{d(1-4n)+e(1-3n)x^n}{(a+cx^{2n})^2}\,dx}{4an} \\ &= \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})} + \frac{\int \frac{d(1-4n)(1-2n)+e(1-3n)(1-n)x^n}{a+cx^{2n}}\,dx}{8a^2n^2}\end{aligned}$$

$$= \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})} + \frac{(d(1-4n)(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{8a^2n^2} + \frac{(e(1-3n)(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{8a^2n^2}$$

$$= \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})} + \frac{d(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2} + \frac{e(1-3n)(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(3,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^3} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(3,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^3(1+n)}$$

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

```
[Out] (d*x*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^3 +
 (e*x^(1 + n)*Hypergeometric2F1[3, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a^3*(1 + n))
```

Maple [F]

$$\int \frac{d+e\,x^n}{(a+c\,x^{2n})^3}dx$$

```
[In] int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(c*e*(3*n - 1)*x*x^(3*n) + c*d*(4*n - 1)*x*x^(2*n) + a*e*(5*n - 1)*x*x^
n + a*d*(6*n - 1)*x)/(a^2*c^2*n^2*x^(4*n) + 2*a^3*c*n^2*x^(2*n) + a^4*n^2)
+ integrate(1/8*((3*n^2 - 4*n + 1)*e*x^n + (8*n^2 - 6*n + 1)*d)/(a^2*c*n^2*
x^(2*n) + a^3*n^2), x)
```

Giac [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx=\int \frac{d+e\,x^n}{\left(a+c\,x^{2\,n}\right)^3}\,dx$$

```
[In] int((d + e*x^n)/(a + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)/(a + c*x^(2*n))^3, x)
```

3.56 $\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 582

$$\begin{aligned}\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx &= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2}+\frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})}\\
&-\frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2(cd^2+ae^2)n^2(a+cx^{2n})}+\frac{cde^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3}\\
&+\frac{cd(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3(cd^2+ae^2)n^2}\\
&-\frac{cde^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)^2n}\\
&+\frac{e^6x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^3}\\
&-\frac{ce^5x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3(1+n)}\\
&-\frac{ce(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3(cd^2+ae^2)n^2(1+n)}\\
&+\frac{ce^3(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)^2n(1+n)}\end{aligned}$$

```
[Out] 1/4*c*x*(d-e*x^n)/a/(a*e^2+c*d^2)/n/(a+c*x^(2*n))^2+1/2*c*e^2*x*(d-e*x^n)/a
/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))-1/8*c*x*(d*(1-4*n)-e*(1-3*n)*x^n)/a^2/(a*e
```

```
^2+c*d^2)/n^2/(a+c*x^(2*n))+c*d*e^4*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(
2*n)/a)/a/(a*e^2+c*d^2)^3+1/8*c*d*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1
+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)/n^2-1/2*c*d*e^2*(1-2*n)*x*hypergeom
([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n+e^6*x*hypergeom([
1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)^3-c*e^5*x^(1+n)*hypergeom([1, 1/2
*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^3/(1+n)-1/8*c*e*(1-3*n)
*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^3/(a*
e^2+c*d^2)/n^2/(1+n)+1/2*c*e^3*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/
2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n/(1+n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 251, 1445, 1432, 371}

$$
\begin{aligned}
\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^3}\,dx
&= -\frac{ce(1-3n)(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)\left(ae^2+cd^2\right)} \\
&\quad+\frac{cd(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)} \\
&\quad-\frac{cde^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^2} \\
&\quad-\frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2n^2\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)} \\
&\quad+\frac{ce^3(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\left(ae^2+cd^2\right)^2} \\
&\quad+\frac{ce^2x(d-ex^n)}{2an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}+\frac{cx(d-ex^n)}{4an\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)^2} \\
&\quad+\frac{e^6x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)^3} \\
&\quad-\frac{ce^5x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^3} \\
&\quad+\frac{cde^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^3}
\end{aligned}
$$

```
[In] Int[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]
```

```
[Out] (c*x*(d - e*x^n))/(4*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))^2) + (c*e^2*x*(d -
 e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n))) - (c*x*(d*(1 - 4*n) - e*
(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)*n^2*(a + c*x^(2*n))) + (c*d*e^4*x*Hy
pergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 +
a*e^2)^3) + (c*d*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n
^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2) - (c*d*e^2*(1 - 2*
n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^
2*(c*d^2 + a*e^2)^2*n) + (e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(
(e*x^n)/d)])/(d*(c*d^2 + a*e^2)^3) - (c*e^5*x^(1 + n)*Hypergeometric2F1[1,
(1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3*(1 +
 n)) - (c*e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n),
 (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2*(1 + n)) + (
c*e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
 -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
```

```
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^6}{(cd^2+ae^2)^3(d+ex^n)} - \frac{c(-d+ex^n)}{(cd^2+ae^2)(a+cx^{2n})^3} - \frac{ce^2(-d+ex^n)}{(cd^2+ae^2)^2(a+cx^{2n})^2} - \frac{ce^4(-d+ex^n)}{(cd^2+ae^2)^3(a+cx^{2n})}\right) dx$$

$$= -\frac{(ce^4)\int \frac{-d+ex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^3} + \frac{e^6\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^3} - \frac{(ce^2)\int \frac{-d+ex^n}{(a+cx^{2n})^2}\,dx}{(cd^2+ae^2)^2} - \frac{c\int \frac{-d+ex^n}{(a+cx^{2n})^3}\,dx}{cd^2+ae^2}$$

$$= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2} + \frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})} + \frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^3} + \frac{(cde^4)\int \frac{1}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^3} - \frac{(ce^5)\int \frac{x^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^3} + \frac{(ce^2)\int \frac{-d(1-2n)+e(1-n)x^n}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)^2n} + \frac{c\int \frac{-d(1-4n)+e(1-3n)x^n}{(a+cx^{2n})^2}\,dx}{4a(cd^2+ae^2)n}$$

$$= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2} + \frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})} - \frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2(cd^2+ae^2)n^2(a+cx^{2n})} + \frac{cde^4x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3} + \frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^3} - \frac{ce^5x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3(1+n)} - \frac{c\int \frac{-d(1-4n)(1-2n)+e(1-3n)(1-n)x^n}{a+cx^{2n}}\,dx}{8a^2(cd^2+ae^2)n^2} - \frac{(cde^2(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)^2n} + \frac{(ce^3(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)^2n}$$

$$
\begin{aligned}
&= \frac{cx(d-ex^n)}{4a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)^2} + \frac{ce^2x(d-ex^n)}{2a\left(cd^2+ae^2\right)^2 n\left(a+cx^{2n}\right)} \\
&\quad - \frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2\left(cd^2+ae^2\right)n^2\left(a+cx^{2n}\right)} + \frac{cde^4x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^3} \\
&\quad - \frac{cde^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)^2 n} \\
&\quad + \frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(cd^2+ae^2\right)^3} - \frac{ce^5x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^3(1+n)} \\
&\quad + \frac{ce^3(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)^2 n(1+n)} \\
&\quad + \frac{(cd(1-4n)(1-2n))\int\frac{1}{a+cx^{2n}}\,dx}{8a^2\left(cd^2+ae^2\right)n^2} - \frac{(ce(1-3n)(1-n))\int\frac{x^n}{a+cx^{2n}}\,dx}{8a^2\left(cd^2+ae^2\right)n^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{cx(d-ex^n)}{4a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)^2} + \frac{ce^2x(d-ex^n)}{2a\left(cd^2+ae^2\right)^2 n\left(a+cx^{2n}\right)} \\
&\quad - \frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2\left(cd^2+ae^2\right)n^2\left(a+cx^{2n}\right)} + \frac{cde^4x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^3} \\
&\quad + \frac{cd(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3\left(cd^2+ae^2\right)n^2} \\
&\quad - \frac{cde^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)^2 n} \\
&\quad + \frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(cd^2+ae^2\right)^3} - \frac{ce^5x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^3(1+n)} \\
&\quad - \frac{ce(1-3n)(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3\left(cd^2+ae^2\right)n^2(1+n)} \\
&\quad + \frac{ce^3(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)^2 n(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.59

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \frac{x\left(\frac{cde^4\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}+\frac{e^6\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d}-\frac{ce^5x^n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}(3\cdots\right.}{a(1+n)}\right.}{}$$

```
[In] Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]
```

```
[Out] (x*((c*d*e^4*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)
])/a + (e^6*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d - (c*
e^5*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a
)])/(a*(1 + n)) + (c*d*e^2*(c*d^2 + a*e^2)*Hypergeometric2F1[2, 1/(2*n), (2
 + n^(-1))/2, -((c*x^(2*n))/a)])/a^2 - (c*e^3*(c*d^2 + a*e^2)*x^n*Hypergeom
etric2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1 + n))
 + (c*d*(c*d^2 + a*e^2)^2*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1))/2, -((
c*x^(2*n))/a)])/a^3 - (c*e*(c*d^2 + a*e^2)^2*x^n*Hypergeometric2F1[3, (1 +
n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*(1 + n))))/(c*d^2 + a*e^2
)^3
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)\,(a+c\,x^{2n})^3}dx$$

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*e*x^n + a^3*d + (c^3*e*x^n + c^3*d)*x^(6*n) + 3*(a*c^2*e*x^
n + a*c^2*d)*x^(4*n) + 3*(a^2*c*e*x^n + a^2*c*d)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="maxima")
[Out] e^6*integrate(1/(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6 +
(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*x^n), x) - 1/8*((
a*c^2*e^3*(7*n - 1) + c^3*d^2*e*(3*n - 1))*x*x^(3*n) - (a*c^2*d*e^2*(8*n -
1) + c^3*d^3*(4*n - 1))*x*x^(2*n) + (a^2*c*e^3*(9*n - 1) + a*c^2*d^2*e*(5*n
 - 1))*x*x^n - (a^2*c*d*e^2*(10*n - 1) + a*c^2*d^3*(6*n - 1))*x)/(a^4*c^2*d
^4*n^2 + 2*a^5*c*d^2*e^2*n^2 + a^6*e^4*n^2 + (a^2*c^4*d^4*n^2 + 2*a^3*c^3*d
^2*e^2*n^2 + a^4*c^2*e^4*n^2)*x^(4*n) + 2*(a^3*c^3*d^4*n^2 + 2*a^4*c^2*d^2*
e^2*n^2 + a^5*c*e^4*n^2)*x^(2*n)) - integrate(-1/8*((8*n^2 - 6*n + 1)*c^3*d
^5 + 2*(12*n^2 - 8*n + 1)*a*c^2*d^3*e^2 + (24*n^2 - 10*n + 1)*a^2*c*d*e^4 -
 ((3*n^2 - 4*n + 1)*c^3*d^4*e + 2*(5*n^2 - 6*n + 1)*a*c^2*d^2*e^3 + (15*n^2
 - 8*n + 1)*a^2*c*e^5)*x^n)/(a^3*c^3*d^6*n^2 + 3*a^4*c^2*d^4*e^2*n^2 + 3*a^
5*c*d^2*e^4*n^2 + a^6*e^6*n^2 + (a^2*c^4*d^6*n^2 + 3*a^3*c^3*d^4*e^2*n^2 +
3*a^4*c^2*d^2*e^4*n^2 + a^5*c*e^6*n^2)*x^(2*n)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})^3}\,dx = \int \frac{1}{(a+c\,x^{2\,n})^3\,(d+e\,x^n)}\,dx$$

```
[In] int(1/((a + c*x^(2*n))^3*(d + e*x^n)),x)
[Out] int(1/((a + c*x^(2*n))^3*(d + e*x^n)), x)
```

3.57 $\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 701

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^3}\,dx = \frac{cx(cd^2-ae^2-2cdex^n)}{4a(cd^2+ae^2)^2n(a+cx^{2n})^2}+\frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a(cd^2+ae^2)^3n(a+cx^{2n})} - \frac{cx((cd^2-ae^2)(1-4n)-2cde(1-3n)x^n)}{8a^2(cd^2+ae^2)^2n^2(a+cx^{2n})} + \frac{ce^4(5cd^2-ae^2)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^4} + \frac{c(cd^2-ae^2)(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3(cd^2+ae^2)^2n^2} - \frac{ce^2(3cd^2-ae^2)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)^3n} + \frac{6ce^6x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^4} - \frac{6c^2de^5x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^4(1+n)} - \frac{c^2de(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3(cd^2+ae^2)^2n^2(1+n)} + \frac{2c^2de^3(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2(cd^2+ae^2)^3n(1+n)} + \frac{e^6x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)^3}$$

```
[Out] 1/4*c*x*(c*d^2-a*e^2-2*c*d*e*x^n)/a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))^2+1/2*c
*e^2*x*(3*c*d^2-a*e^2-4*c*d*e*x^n)/a/(a*e^2+c*d^2)^3/n/(a+c*x^(2*n))-1/8*c*
x*((-a*e^2+c*d^2)*(1-4*n)-2*c*d*e*(1-3*n)*x^n)/a^2/(a*e^2+c*d^2)^2/n^2/(a+c
*x^(2*n))+c*e^4*(-a*e^2+5*c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n
)/a)/a/(a*e^2+c*d^2)^4+1/8*c*(-a*e^2+c*d^2)*(1-4*n)*(1-2*n)*x*hypergeom([1,
 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)^2/n^2-1/2*c*e^2*(-a*e^2+3
*c*d^2)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c
*d^2)^3/n+6*c*e^6*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/(a*e^2+c*d^2)^4-6*
c^2*d*e^5*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a
*e^2+c*d^2)^4/(1+n)-1/4*c^2*d*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+
n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)^2/n^2/(1+n)+2*c^2*d*e^3*(
```

```
1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^
2+c*d^2)^3/n/(1+n)+e^6*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*
d^2)^3
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1451, 251, 1445, 1432, 371}

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^3}\,dx$$
$$= -\frac{c^2de(1-3n)(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)\left(ae^2+cd^2\right)^2}$$
$$+\frac{c(1-4n)(1-2n)x(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)^2}$$
$$+\frac{2c^2de^3(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)\left(ae^2+cd^2\right)^3}$$
$$-\frac{ce^2(1-2n)x(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^3}$$
$$-\frac{cx((1-4n)\left(cd^2-ae^2\right)-2cde(1-3n)x^n)}{8a^2n^2\left(ae^2+cd^2\right)^2(a+cx^{2n})}$$
$$-\frac{6c^2de^5x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^4}$$
$$+\frac{ce^2x(-ae^2+3cd^2-4cdex^n)}{2an\left(ae^2+cd^2\right)^3(a+cx^{2n})}+\frac{cx(-ae^2+cd^2-2cdex^n)}{4an\left(ae^2+cd^2\right)^2(a+cx^{2n})^2}$$
$$+\frac{6ce^6x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^4}$$
$$+\frac{e^6x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)^3}$$
$$+\frac{ce^4x(5cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^4}$$

```
[In] Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(4*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n)
)^2) + (c*e^2*x*(3*c*d^2 - a*e^2 - 4*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^3*n*(
a + c*x^(2*n))) - (c*x*((c*d^2 - a*e^2)*(1 - 4*n) - 2*c*d*e*(1 - 3*n)*x^n))
```

```
/(8*a^2*(c*d^2 + a*e^2)^2*n^2*(a + c*x^(2*n))) + (c*e^4*(5*c*d^2 - a*e^2)*x
*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2
 + a*e^2)^4) + (c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1
, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)^2*n^2)
 - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 +
n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^3*n) + (6*c*e^6*x*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^4 - (6*
c^2*d*e^5*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((
c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3*n)*(1 - n)*
x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))
/a)])/(4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1 - n)*x^(1 + n
)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a
^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hypergeometric2F1[2, n^(-1), 1 + n
^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2)^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
```

```
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^6}{(cd^2+ae^2)^3 (d+ex^n)^2} + \frac{6cde^6}{(cd^2+ae^2)^4 (d+ex^n)} - \frac{c(-cd^2+ae^2+2cdex^n)}{(cd^2+ae^2)^2 (a+cx^{2n})^3} \right. \\
&\qquad \left. - \frac{ce^2(-3cd^2+ae^2+4cdex^n)}{(cd^2+ae^2)^3 (a+cx^{2n})^2} - \frac{ce^4(-5cd^2+ae^2+6cdex^n)}{(cd^2+ae^2)^4 (a+cx^{2n})} \right) dx \\
&= -\frac{(ce^4)\int \frac{-5cd^2+ae^2+6cdex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^4} + \frac{(6cde^6)\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^4} \\
&\quad - \frac{(ce^2)\int \frac{-3cd^2+ae^2+4cdex^n}{(a+cx^{2n})^2}\,dx}{(cd^2+ae^2)^3} + \frac{e^6\int \frac{1}{(d+ex^n)^2}\,dx}{(cd^2+ae^2)^3} - \frac{c\int \frac{-cd^2+ae^2+2cdex^n}{(a+cx^{2n})^3}\,dx}{(cd^2+ae^2)^2} \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{4a(cd^2+ae^2)^2 n (a+cx^{2n})^2} + \frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a(cd^2+ae^2)^3 n (a+cx^{2n})} + \frac{6ce^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^4} \\
&\quad + \frac{e^6x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)^3} - \frac{(6c^2de^5)\int \frac{x^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^4} + \frac{(ce^4(5cd^2-ae^2))\int \frac{1}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^4} \\
&\quad + \frac{(ce^2)\int \frac{(-3cd^2+ae^2)(1-2n)+4cde(1-n)x^n}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)^3 n} + \frac{c\int \frac{(-cd^2+ae^2)(1-4n)+2cde(1-3n)x^n}{(a+cx^{2n})^2}\,dx}{4a(cd^2+ae^2)^2 n} \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{4a(cd^2+ae^2)^2 n (a+cx^{2n})^2} + \frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a(cd^2+ae^2)^3 n (a+cx^{2n})} \\
&\quad - \frac{cx((cd^2-ae^2)(1-4n)-2cde(1-3n)x^n)}{8a^2(cd^2+ae^2)^2 n^2 (a+cx^{2n})} \\
&\quad + \frac{ce^4(5cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^4} \\
&\quad + \frac{6ce^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^4} - \frac{6c^2de^5x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^4(1+n)} \\
&\quad + \frac{e^6x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)^3} - \frac{c\int \frac{(-cd^2+ae^2)(1-4n)(1-2n)+2cde(1-3n)(1-n)x^n}{a+cx^{2n}}\,dx}{8a^2(cd^2+ae^2)^2 n^2} \\
&\quad - \frac{(ce^2(3cd^2-ae^2)(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{2a(cd^2+ae^2)^3 n} + \frac{(2c^2de^3(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{a(cd^2+ae^2)^3 n}
\end{aligned}$$

$$= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{4a\left(cd^2 + ae^2\right)^2 n\left(a + cx^{2n}\right)^2} + \frac{ce^2x(3cd^2 - ae^2 - 4cdex^n)}{2a\left(cd^2 + ae^2\right)^3 n\left(a + cx^{2n}\right)}$$
$$- \frac{cx((cd^2 - ae^2)(1-4n) - 2cde(1-3n)x^n)}{8a^2\left(cd^2 + ae^2\right)^2 n^2\left(a + cx^{2n}\right)}$$
$$+ \frac{ce^4(5cd^2 - ae^2)\,x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^4}$$
$$- \frac{ce^2(3cd^2 - ae^2)(1-2n)x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2 + ae^2\right)^3 n}$$
$$+ \frac{6ce^6x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{\left(cd^2 + ae^2\right)^4} - \frac{6c^2de^5x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^4(1+n)}$$
$$+ \frac{2c^2de^3(1-n)x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2\left(cd^2 + ae^2\right)^3 n(1+n)} + \frac{e^6x\,{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\left(cd^2 + ae^2\right)^3}$$
$$+ \frac{(c(cd^2 - ae^2)(1-4n)(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{8a^2\left(cd^2 + ae^2\right)^2 n^2} - \frac{(c^2de(1-3n)(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{4a^2\left(cd^2 + ae^2\right)^2 n^2}$$

$$= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{4a\left(cd^2 + ae^2\right)^2 n\left(a + cx^{2n}\right)^2} + \frac{ce^2x(3cd^2 - ae^2 - 4cdex^n)}{2a\left(cd^2 + ae^2\right)^3 n\left(a + cx^{2n}\right)}$$
$$- \frac{cx((cd^2 - ae^2)(1-4n) - 2cde(1-3n)x^n)}{8a^2\left(cd^2 + ae^2\right)^2 n^2\left(a + cx^{2n}\right)}$$
$$+ \frac{ce^4(5cd^2 - ae^2)\,x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^4}$$
$$+ \frac{c(cd^2 - ae^2)(1-4n)(1-2n)x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3\left(cd^2 + ae^2\right)^2 n^2}$$
$$- \frac{ce^2(3cd^2 - ae^2)(1-2n)x\,{}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2 + ae^2\right)^3 n}$$
$$+ \frac{6ce^6x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{\left(cd^2 + ae^2\right)^4} - \frac{6c^2de^5x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\left(cd^2 + ae^2\right)^4(1+n)}$$
$$- \frac{c^2de(1-3n)(1-n)x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{4a^3\left(cd^2 + ae^2\right)^2 n^2(1+n)}$$
$$+ \frac{2c^2de^3(1-n)x^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2\left(cd^2 + ae^2\right)^3 n(1+n)} + \frac{e^6x\,{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\left(cd^2 + ae^2\right)^3}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})^3}\,dx$$
$$= \frac{x\left(\frac{ce^4(5cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}+6ce^6\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)-\frac{6c^2de^5x^n\,\text{Hy}}{}\right.}{}$$

```
[In] Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

```
[Out] (x*((c*e^4*(5*c*d^2 - a*e^2)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2,
-((c*x^(2*n))/a)])/a + 6*c*e^6*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((
e*x^n)/d)] - (6*c^2*d*e^5*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-
1))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) + (c*e^2*(3*c*d^2 - a*e^2)*(c*d^2 + a
*e^2)*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^2
+ (e^6*(c*d^2 + a*e^2)*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d
)])/d^2 - (4*c^2*d*e^3*(c*d^2 + a*e^2)*x^n*Hypergeometric2F1[2, (1 + n)/(2*
n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1 + n)) + (c*(c*d^2 - a*e^2)*(
c*d^2 + a*e^2)^2*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n)
)/a)])/a^3 - (2*c^2*d*e*(c*d^2 + a*e^2)^2*x^n*Hypergeometric2F1[3, (1 + n)/
(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*(1 + n))))/(c*d^2 + a*e^2)^4
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)^2\,(a+c\,x^{2n})^3}dx$$

```
[In] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*e^2*x^(2*n) + 2*a^3*d*e*x^n + a^3*d^2 + (c^3*e^2*x^(2*n) +
2*c^3*d*e*x^n + c^3*d^2)*x^(6*n) + 3*(a*c^2*e^2*x^(2*n) + 2*a*c^2*d*e*x^n +
 a*c^2*d^2)*x^(4*n) + 3*(a^2*c*e^2*x^(2*n) + 2*a^2*c*d*e*x^n + a^2*c*d^2)*x
^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^n\right)^2\left(a+cx^{2n}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^n\right)^2\left(a+cx^{2n}\right)^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="maxima")
[Out] (c*d^2*e^6*(7*n - 1) + a*e^8*(n - 1))*integrate(1/(c^4*d^10*n + 4*a*c^3*d^8
*e^2*n + 6*a^2*c^2*d^6*e^4*n + 4*a^3*c*d^4*e^6*n + a^4*d^2*e^8*n + (c^4*d^9
*e*n + 4*a*c^3*d^7*e^3*n + 6*a^2*c^2*d^5*e^5*n + 4*a^3*c*d^3*e^7*n + a^4*d*
e^9*n)*x^n), x) - 1/8*(2*(a*c^3*d^2*e^4*(11*n - 1) + c^4*d^4*e^2*(3*n - 1)
- 4*a^2*c^2*e^6*n)*x*x^(4*n) + (a^2*c^2*d*e^5*(8*n - 1) + 2*a*c^3*d^3*e^3*(
5*n - 1) + c^4*d^5*e*(2*n - 1))*x*x^(3*n) + (a^2*c^2*d^2*e^4*(34*n - 3) - c
^4*d^6*(4*n - 1) - 2*a*c^3*d^4*e^2*(n + 1) - 16*a^3*c*e^6*n)*x*x^(2*n) + (a
^3*c*d*e^5*(10*n - 1) + 2*a^2*c^2*d^3*e^3*(7*n - 1) + a*c^3*d^5*e*(4*n - 1)
)*x*x^n + (a^3*c*d^2*e^4*(10*n - 1) - a*c^3*d^6*(6*n - 1) - 12*a^2*c^2*d^4*
e^2*n - 8*a^4*e^6*n)*x)/(a^4*c^3*d^8*n^2 + 3*a^5*c^2*d^6*e^2*n^2 + 3*a^6*c*
d^4*e^4*n^2 + a^7*d^2*e^6*n^2 + (a^2*c^5*d^7*e*n^2 + 3*a^3*c^4*d^5*e^3*n^2
+ 3*a^4*c^3*d^3*e^5*n^2 + a^5*c^2*d*e^7*n^2)*x^(5*n) + (a^2*c^5*d^8*n^2 + 3
*a^3*c^4*d^6*e^2*n^2 + 3*a^4*c^3*d^4*e^4*n^2 + a^5*c^2*d^2*e^6*n^2)*x^(4*n)
 + 2*(a^3*c^4*d^7*e*n^2 + 3*a^4*c^3*d^5*e^3*n^2 + 3*a^5*c^2*d^3*e^5*n^2 + a
^6*c*d*e^7*n^2)*x^(3*n) + 2*(a^3*c^4*d^8*n^2 + 3*a^4*c^3*d^6*e^2*n^2 + 3*a^
5*c^2*d^4*e^4*n^2 + a^6*c*d^2*e^6*n^2)*x^(2*n) + (a^4*c^3*d^7*e*n^2 + 3*a^5
*c^2*d^5*e^3*n^2 + 3*a^6*c*d^3*e^5*n^2 + a^7*d*e^7*n^2)*x^n) - integrate(-1
/8*((8*n^2 - 6*n + 1)*c^4*d^6 + (32*n^2 - 18*n + 1)*a*c^3*d^4*e^2 + (48*n^2
 - 2*n - 1)*a^2*c^2*d^2*e^4 - (24*n^2 - 10*n + 1)*a^3*c*e^6 - 2*((3*n^2 - 4
*n + 1)*c^4*d^5*e + 2*(7*n^2 - 8*n + 1)*a*c^3*d^3*e^3 + (35*n^2 - 12*n + 1)
*a^2*c^2*d*e^5)*x^n)/(a^3*c^4*d^8*n^2 + 4*a^4*c^3*d^6*e^2*n^2 + 6*a^5*c^2*d
^4*e^4*n^2 + 4*a^6*c*d^2*e^6*n^2 + a^7*e^8*n^2 + (a^2*c^5*d^8*n^2 + 4*a^3*c
^4*d^6*e^2*n^2 + 6*a^4*c^3*d^4*e^4*n^2 + 4*a^5*c^2*d^2*e^6*n^2 + a^6*c*e^8*
n^2)*x^(2*n)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3} \, dx = \int \frac{1}{(cx^{2n}+a)^3 (ex^n+d)^2} \, dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3} \, dx = \int \frac{1}{(a+c\,x^{2n})^3 \,(d+e\,x^n)^2} \, dx$$

```
[In] int(1/((a + c*x^(2*n))^3*(d + e*x^n)^2),x)
[Out] int(1/((a + c*x^(2*n))^3*(d + e*x^n)^2), x)
```

3.58 $\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \frac{x\sqrt{1+\frac{cx^{2n}}{a}}\,\text{AppellF1}\left(\frac{1}{2n},\frac{1}{2},1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}} - \frac{ex^{1+n}\sqrt{1+\frac{cx^{2n}}{a}}\,\text{AppellF1}\left(\frac{1+n}{2n},\frac{1}{2},1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)\sqrt{a+cx^{2n}}}$$

```
[Out] x*AppellF1(1/2/n,1,1/2,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)*(1+c*x^(2*n)/a
)^(1/2)/d/(a+c*x^(2*n))^(1/2)-e*x^(1+n)*AppellF1(1/2*(1+n)/n,1,1/2,3/2+1/2/
n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)*(1+c*x^(2*n)/a)^(1/2)/d^2/(1+n)/(a+c*x^(2*n
))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1452, 441, 440, 525, 524}

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \frac{x\sqrt{\frac{cx^{2n}}{a}+1}\,\text{AppellF1}\left(\frac{1}{2n},\frac{1}{2},1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}} - \frac{ex^{n+1}\sqrt{\frac{cx^{2n}}{a}+1}\,\text{AppellF1}\left(\frac{n+1}{2n},\frac{1}{2},1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)\sqrt{a+cx^{2n}}}$$

```
[In] Int[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
```

```
[Out] (x*Sqrt[1 + (c*x^(2*n))/a]*AppellF1[1/(2*n), 1/2, 1, (2 + n^(-1))/2, -((c*x
^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*Sqrt[a + c*x^(2*n)]) - (e*x^(1 + n)*Sqrt
[1 + (c*x^(2*n))/a]*AppellF1[(1 + n)/(2*n), 1/2, 1, (3 + n^(-1))/2, -((c*x^
(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + n)*Sqrt[a + c*x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d}{\sqrt{a+cx^{2n}}\,(d^2-e^2x^{2n})} + \frac{ex^n}{\sqrt{a+cx^{2n}}\,(-d^2+e^2x^{2n})} \right) dx$$

$$= d\int \frac{1}{\sqrt{a+cx^{2n}}\,(d^2-e^2x^{2n})}\,dx + e\int \frac{x^n}{\sqrt{a+cx^{2n}}\,(-d^2+e^2x^{2n})}\,dx$$

$$= \frac{\left(d\sqrt{1+\frac{cx^{2n}}{a}}\right)\int \frac{1}{\sqrt{1+\frac{cx^{2n}}{a}}(d^2-e^2x^{2n})}\,dx}{\sqrt{a+cx^{2n}}} + \frac{\left(e\sqrt{1+\frac{cx^{2n}}{a}}\right)\int \frac{x^n}{\sqrt{1+\frac{cx^{2n}}{a}}(-d^2+e^2x^{2n})}\,dx}{\sqrt{a+cx^{2n}}}$$

$$= \frac{x\sqrt{1+\frac{cx^{2n}}{a}}F_1\left(\frac{1}{2n};\frac{1}{2},1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}} - \frac{ex^{1+n}\sqrt{1+\frac{cx^{2n}}{a}}F_1\left(\frac{1+n}{2n};\frac{1}{2},1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)\sqrt{a+cx^{2n}}}$$

Mathematica [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$$

```
[In] Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
[Out] Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]), x]
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)\sqrt{a+c\,x^{2n}}}dx$$

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^(2*n) + a)/(a*e*x^n + a*d + (c*e*x^n + c*d)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a+cx^{2n}}\,(d+ex^n)}\,dx$$

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**(2*n))*(d + e*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a+c\,x^{2\,n}}\,\left(d+e\,x^n\right)}\,dx$$

```
[In] int(1/((a + c*x^(2*n))^(1/2)*(d + e*x^n)),x)
[Out] int(1/((a + c*x^(2*n))^(1/2)*(d + e*x^n)), x)
```

3.59 $\int (d+ex^n)^q (a+cx^{2n})^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \text{Int}\left((d+ex^n)^q \left(a+cx^{2n}\right)^p, x\right)$$

```
[Out] Unintegrable((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

```
[In] Int[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
[Out] Defer[Int][(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi steps

$$\text{integral} = \int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q (a+cx^{2n})^p \, dx = \int (d+ex^n)^q (a+cx^{2n})^p \, dx$$

```
[In] Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
[Out] Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (d+e\,x^n)^q \left(a+c\,x^{2n}\right)^p dx$$

```
[In] int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
[Out] int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q (a+cx^{2n})^p \, dx = \int (cx^{2n}+a)^p (ex^n+d)^q \, dx$$

```
[In] integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^q (a+cx^{2n})^p \, dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**q*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (cx^{2n}+a)^p (ex^n+d)^q \, dx$$

```
[In] integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (cx^{2n}+a)^p (ex^n+d)^q \, dx$$

```
[In] integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

Mupad [N/A]

Not integrable

Time = 8.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int \left(a+c\,x^{2\,n}\right)^p \left(d+e\,x^n\right)^q \, dx$$

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n)^q,x)
[Out] int((a + c*x^(2*n))^p*(d + e*x^n)^q, x)
```

3.60 $\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 299

$$\begin{aligned}
&\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx \\
&= \frac{3de^2x^{1+2n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right), -p, \frac{1}{2}\left(4+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{1+2n} \\
&\quad + \frac{e^3x^{1+3n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right), -p, \frac{1}{2}\left(5+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{1+3n} \\
&\quad + d^3x\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n}, -p, \frac{1}{2}\left(2+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right) \\
&\quad + \frac{3d^2ex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+n}{2n}, -p, \frac{1}{2}\left(3+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{1+n}
\end{aligned}$$

```
[Out] 3*d*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*hypergeom([-p, 1+1/2/n],[2+1/2/n],-c*x^(2
*n)/a)/(1+2*n)/((1+c*x^(2*n)/a)^p)+e^3*x^(1+3*n)*(a+c*x^(2*n))^p*hypergeom(
[-p, 3/2+1/2/n],[5/2+1/2/n],-c*x^(2*n)/a)/(1+3*n)/((1+c*x^(2*n)/a)^p)+d^3*x
*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^(2*n
)/a)^p)+3*d^2*e*x^(1+n)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+n)/n],[3/2+1/
2/n],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1451, 252, 251, 372, 371}

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p dx$$
$$= d^3x\left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)$$
$$+ \frac{3d^2ex^{n+1}\left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{n+1}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{n+1}$$
$$+ \frac{3de^2x^{2n+1}\left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2n+1}$$
$$+ \frac{e^3x^{3n+1}\left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{3n+1}$$

```
[In] Int[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (3*d*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))/2, -p
, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(2*n))/a)^p) + (e
^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(3 + n^(-1))/2, -p, (5 +
 n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 3*n)*(1 + (c*x^(2*n))/a)^p) + (d^3*x*(
a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n
))/a)])/(1 + (c*x^(2*n))/a)^p + (3*d^2*e*x^(1 + n)*(a + c*x^(2*n))^p*Hyperg
eometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rubi steps

$$\text{integral} = \int \left(d^3\left(a+cx^{2n}\right)^p+3d^2ex^n\left(a+cx^{2n}\right)^p+3de^2x^{2n}\left(a+cx^{2n}\right)^p+e^3x^{3n}\left(a+cx^{2n}\right)^p\right)\,dx$$

$$= d^3\int \left(a+cx^{2n}\right)^p\,dx+\left(3d^2e\right)\int x^n\left(a+cx^{2n}\right)^p\,dx + \left(3de^2\right)\int x^{2n}\left(a+cx^{2n}\right)^p\,dx+e^3\int x^{3n}\left(a+cx^{2n}\right)^p\,dx$$

$$= \left(d^3\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \left(1+\frac{cx^{2n}}{a}\right)^p\,dx + \left(3d^2e\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^n\left(1+\frac{cx^{2n}}{a}\right)^p\,dx + \left(3de^2\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^{2n}\left(1+\frac{cx^{2n}}{a}\right)^p\,dx + \left(e^3\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^{3n}\left(1+\frac{cx^{2n}}{a}\right)^p\,dx$$

$$= \frac{3de^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+2n}$$
$$+\frac{e^3x^{1+3n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+3n}$$
$$+d^3x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)$$
$$+\frac{3d^2ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+n}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.71

$$\int (d+ex^n)^3\left(a+cx^{2n}\right)^p\,dx$$
$$= x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(\frac{3de^2x^{2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+2n}\right.$$
$$+\frac{e^3x^{3n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+3n}$$
$$+d^2\left(d\,\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)\right.$$
$$\left.\left.+\frac{3ex^n\,\text{Hypergeometric2F1}\left(\frac{1+n}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}\right)\right)$$

```
[In] Integrate[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + c*x^(2*n))^p*((3*d*e^2*x^(2*n)*Hypergeometric2F1[(2 + n^(-1))/2, -p
, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + 2*n) + (e^3*x^(3*n)*Hypergeometri
c2F1[(3 + n^(-1))/2, -p, (5 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + 3*n) + d^2
*(d*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)] + (3*e
*x^n*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)]
)/(1 + n))))/(1 + (c*x^(2*n))/a)^p
```

Maple [F]

$$\int \left(d+e\,x^{n}\right)^{3} \left(a+c\,x^{2n}\right)^{p} dx$$

```
[In] int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)^3 (cx^{2\,n}+a)^p \, dx$$

```
[In] integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) + a
)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)^3 (cx^{2\,n}+a)^p \, dx$$

```
[In] integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^3 (a+cx^{2n})^p \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{96,[1,0,6,4,3,5,4,1,2]%%%}+%%%{480,[1,0,6,4,3,4,4,1,2]%%%}+%%
%{960,
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^3 (a+cx^{2n})^p \, dx = \int (a+c\,x^{2\,n})^p \,(d+e\,x^n)^3 \, dx$$

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n)^3,x)
```

```
[Out] int((a + c*x^(2*n))^p*(d + e*x^n)^3, x)
```

3.61 $\int (d+ex^n)^2 (a+cx^{2n})^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int (d+ex^n)^2 (a+cx^{2n})^p \, dx = \frac{e^2x^{1+2n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+2n} + d^2x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right) + \frac{2dex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+n}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}$$

```
[Out] e^2*x^(1+2*n)*(a+c*x^(2*n))^p*hypergeom([-p, 1+1/2/n],[2+1/2/n],-c*x^(2*n)/
a)/(1+2*n)/((1+c*x^(2*n)/a)^p)+d^2*x*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],
[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^(2*n)/a)^p)+2*d*e*x^(1+n)*(a+c*x^(2*n))^p*h
ypergeom([-p, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)
^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{1451, 252, 251, 372, 371\}$

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$$
$$= d^2x\left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n}, -p, \frac{1}{2}\left(2+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)$$
$$+ \frac{2dex^{n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{n+1}{2n}, -p, \frac{1}{2}\left(3+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{n+1}$$
$$+ \frac{e^2x^{2n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right), -p, \frac{1}{2}\left(4+\frac{1}{n}\right), -\frac{cx^{2n}}{a}\right)}{2n+1}$$

```
[In] Int[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))/2, -p, (4
 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(2*n))/a)^p) + (d^2*x
*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2
*n))/a)])/(1 + (c*x^(2*n))/a)^p + (2*d*e*x^(1 + n)*(a + c*x^(2*n))^p*Hyperg
eometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
```

```
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^2\left(a+cx^{2n}\right)^p+2dex^n\left(a+cx^{2n}\right)^p+e^2x^{2n}\left(a+cx^{2n}\right)^p\right)\,dx \\
&= d^2\int \left(a+cx^{2n}\right)^p\,dx+(2de)\int x^n\left(a+cx^{2n}\right)^p\,dx+e^2\int x^{2n}\left(a+cx^{2n}\right)^p\,dx \\
&= \left(d^2\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \left(1+\frac{cx^{2n}}{a}\right)^p dx \\
&\quad+\left(2de\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^n\left(1+\frac{cx^{2n}}{a}\right)^p dx \\
&\quad+\left(e^2\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^{2n}\left(1+\frac{cx^{2n}}{a}\right)^p dx \\
&= \frac{e^2x^{1+2n}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+2n} \\
&\quad+d^2x\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right) \\
&\quad+\frac{2dex^{1+n}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+n}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.79

$$\int \left(d+ex^n\right)^2\left(a+cx^{2n}\right)^p\,dx$$

$$=\frac{x\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(e^2(1+n)x^{2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+d(1+2}{(}$$

```
[In] Integrate[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + c*x^(2*n))^p*(e^2*(1 + n)*x^(2*n)*Hypergeometric2F1[(2 + n^(-1))/2,
 -p, (4 + n^(-1))/2, -((c*x^(2*n))/a)] + d*(1 + 2*n)*(d*(1 + n)*Hypergeomet
ric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)] + 2*e*x^n*Hypergeomet
ric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])))/((1 + n)*(1
+ 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F]

$$\int \left(d+e\,x^{n}\right)^{2} \left(a+c\,x^{2 n}\right)^{p}dx$$

```
[In] int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int \left(d+ex^{n}\right)^{2}\left(a+cx^{2n}\right)^{p}\,dx=\int \left(ex^{n}+d\right)^{2}\left(cx^{2\,n}+a\right)^{p}\,dx$$

```
[In] integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^{n}\right)^{2}\left(a+cx^{2n}\right)^{p}\,dx=\text{Timed out}$$

```
[In] integrate((d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)^2 (cx^{2n}+a)^p \, dx$$

```
[In] integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{64,[1,0,4,3,1,4,3,1,1]%%%}+%%%{256,[1,0,4,3,1,3,3,1,1]%%%}+%%
%{384,
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = \int \left(a+c\,x^{2\,n}\right)^p\,\left(d+e\,x^n\right)^2 \,dx$$

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n)^2,x)
```

```
[Out] int((a + c*x^(2*n))^p*(d + e*x^n)^2, x)
```

3.62 $\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 135

$$\begin{aligned}&\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx\\&=dx\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)\\&\quad+\frac{ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+n}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}\end{aligned}$$

```
[Out] d*x*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^(
2*n)/a)^p)+e*x^(1+n)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+n)/n],[3/2+1/2/n
],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1447, 252, 251, 372, 371}

$$\begin{aligned}&\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx\\&=dx\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)\\&\quad+\frac{ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{n+1}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{n+1}\end{aligned}$$

```
[In] Int[(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (d*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*
x^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (e*x^(1 + n)*(a + c*x^(2*n))^p*Hyperg
eometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1447

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(d\left(a+cx^{2n}\right)^p+ex^n\left(a+cx^{2n}\right)^p\right)\,dx\\&=d\int \left(a+cx^{2n}\right)^p\,dx+e\int x^n\left(a+cx^{2n}\right)^p\,dx\end{aligned}$$

$$= \left(d(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right) \int \left(1+\frac{cx^{2n}}{a}\right)^p dx + \left(e(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right) \int x^n \left(1+\frac{cx^{2n}}{a}\right)^p dx$$

$$= dx(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right) + \frac{ex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+n}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int (d+ex^n)\left(a+cx^{2n}\right)^p dx = \frac{x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(d(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+ex^n\,\text{Hypergeomet}\right.}{1+n}$$

```
[In] Integrate[(d + e*x^n)*(a + c*x^(2*n))^p,x]
[Out] (x*(a + c*x^(2*n))^p*(d*(1 + n)*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, -((c*x^(2*n))/a)] + e*x^n*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(
-1))/2, -((c*x^(2*n))/a)]))/((1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F]

$$\int (d+e\,x^n)\left(a+c\,x^{2n}\right)^p dx$$

```
[In] int((d+e*x^n)*(a+c*x^(2*n))^p,x)
[Out] int((d+e*x^n)*(a+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (d+ex^n)\left(a+cx^{2n}\right)^p dx = \int (ex^n+d)\left(cx^{2n}+a\right)^p dx$$

```
[In] integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 133.83 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx = \frac{a^{\frac{1}{2n}}a^{p-\frac{1}{2n}}dx\Gamma\left(\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2n},-p\\1+\frac{1}{2n}\end{matrix}\middle|\frac{cx^{2n}e^{i\pi}}{a}\right)}{2n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{\frac{1}{2}+\frac{1}{2n}}a^{p-\frac{1}{2}-\frac{1}{2n}}ex^{n+1}\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-p,\frac{1}{2}+\frac{1}{2n}\\\frac{3}{2}+\frac{1}{2n}\end{matrix}\middle|\frac{cx^{2n}e^{i\pi}}{a}\right)}{2n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

```
[In] integrate((d+e*x**n)*(a+c*x**(2*n))**p,x)
[Out] a**(1/(2*n))*a**(p - 1/(2*n))*d*x*gamma(1/(2*n))*hyper((1/(2*n), -p), (1 +
1/(2*n),), c*x**(2*n)*exp_polar(I*pi)/a)/(2*n*gamma(1 + 1/(2*n))) + a**(1/2
 + 1/(2*n))*a**(p - 1/2 - 1/(2*n))*e*x**(n + 1)*gamma(1/2 + 1/(2*n))*hyper(
(-p, 1/2 + 1/(2*n)), (3/2 + 1/(2*n),), c*x**(2*n)*exp_polar(I*pi)/a)/(2*n*g
amma(3/2 + 1/(2*n)))
```

Maxima [F]

$$\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx = \int (ex^n+d)\left(cx^{2n}+a\right)^p\,dx$$

```
[In] integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

Giac [F]

$$\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx = \int (ex^n+d)\left(cx^{2n}+a\right)^p\,dx$$

```
[In] integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx=\int \left(a+c\,x^{2\,n}\right)^p\,(d+e\,x^n)\,dx$$

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n),x)
[Out] int((a + c*x^(2*n))^p*(d + e*x^n), x)
```

3.63 $\int \frac{(a+cx^{2n})^p}{d+ex^n} dx$

Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{(a+cx^{2n})^p}{d+ex^n} dx = \frac{x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2n},-p,1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d} - \frac{ex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{AppellF1}\left(\frac{1+n}{2n},-p,1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)}$$

```
[Out] x*(a+c*x^(2*n))^p*AppellF1(1/2/n,1,-p,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)
/d/((1+c*x^(2*n)/a)^p)-e*x^(1+n)*(a+c*x^(2*n))^p*AppellF1(1/2*(1+n)/n,1,-p,
3/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^2/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1452, 441, 440, 525, 524}

$$\int \frac{(a+cx^{2n})^p}{d+ex^n} dx = \frac{x(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2n},-p,1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d} - \frac{ex^{n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{n+1}{2n},-p,1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)}$$

```
[In] Int[(a + c*x^(2*n))^p/(d + e*x^n),x]
```

```
[Out] (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, (2 + n^(-1))/2, -((c*x^(2*n))
/a), (e^2*x^(2*n))/d^2])/(d*(1 + (c*x^(2*n))/a)^p) - (e*x^(1 + n)*(a + c*x^
(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 1, (3 + n^(-1))/2, -((c*x^(2*n))/a), (
e^2*x^(2*n))/d^2])/(d^2*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d(a + cx^{2n})^p}{d^2 - e^2x^{2n}} + \frac{ex^n(a + cx^{2n})^p}{-d^2 + e^2x^{2n}} \right) dx$$

$$= d\int \frac{(a+cx^{2n})^p}{d^2-e^2x^{2n}}\,dx + e\int \frac{x^n(a+cx^{2n})^p}{-d^2+e^2x^{2n}}\,dx$$

$$= \left(d(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{cx^{2n}}{a}\right)^p}{d^2-e^2x^{2n}}\,dx$$

$$+ \left(e(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{x^n\left(1+\frac{cx^{2n}}{a}\right)^p}{-d^2+e^2x^{2n}}\,dx$$

$$= \frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2n};-p,1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d}$$
$$- \frac{ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+n}{2n};-p,1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)}$$

Mathematica [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx$$

```
[In] Integrate[(a + c*x^(2*n))^p/(d + e*x^n),x]
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n), x]
```

Maple [F]

$$\int \frac{(a+c\,x^{2n})^p}{d+e\,x^n}dx$$

```
[In] int((a+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((a+c*x^(2*n))^p/(d+e*x^n),x)
```

Fricas [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+a)^p}{ex^n+d}\,dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+a)^p}{ex^n+d}\,dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

Giac [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+a)^p}{ex^n+d}\,dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(a+c\,x^{2\,n})^p}{d+e\,x^n}\,dx$$

```
[In] int((a + c*x^(2*n))^p/(d + e*x^n),x)
[Out] int((a + c*x^(2*n))^p/(d + e*x^n), x)
```

3.64 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx &= \frac{e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,2,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+2n)}\\ &\quad+\frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2n},-p,2,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2}\\ &\quad-\frac{2ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+n}{2n},-p,2,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+n)}\end{aligned}$$

```
[Out] e^2*x^(1+2*n)*(a+c*x^(2*n))^p*AppellF1(1+1/2/n,2,-p,2+1/2/n,e^2*x^(2*n)/d^2
,-c*x^(2*n)/a)/d^4/(1+2*n)/((1+c*x^(2*n)/a)^p)+x*(a+c*x^(2*n))^p*AppellF1(1
/2/n,2,-p,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^2/((1+c*x^(2*n)/a)^p)-2*e
*x^(1+n)*(a+c*x^(2*n))^p*AppellF1(1/2*(1+n)/n,2,-p,3/2+1/2/n,e^2*x^(2*n)/d^
2,-c*x^(2*n)/a)/d^3/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1452, 441, 440, 525, 524}

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \frac{x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},-p,2,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2} + \frac{e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,2,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(2n+1)} - \frac{2ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{n+1}{2n},-p,2,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(n+1)}$$

```
[In] Int[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 2, (4 + n^(
-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + 2*n)*(1 + (c*x^(2*n
))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 2, (2 + n^(-1))/2, -(
(c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + (c*x^(2*n))/a)^p) - (2*e*x^(1
 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 2, (3 + n^(-1))/2, -((c
*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2}-\frac{2dex^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}+\frac{e^2x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\right)dx\\
&= d^2\int \frac{(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2}\,dx-(2de)\int \frac{x^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\,dx+e^2\int \frac{x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&= \left(d^2(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{cx^{2n}}{a}\right)^p}{(d^2-e^2x^{2n})^2}\,dx\\
&\quad -\left(2de(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{x^n\left(1+\frac{cx^{2n}}{a}\right)^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&\quad +\left(e^2(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{x^{2n}\left(1+\frac{cx^{2n}}{a}\right)^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&= \frac{e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,2;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+2n)}\\
&\quad +\frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2n};-p,2;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2}\\
&\quad -\frac{2ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+n}{2n};-p,2;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+n)}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx=\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

```
[In] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Maple [F]

$$\int \frac{(a+c\,x^{2n})^p}{(d+e\,x^n)^2}dx$$

```
[In] int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
[Out] int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx=\int \frac{(cx^{2\,n}+a)^p}{(ex^n+d)^2}\,dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2\,n}+a)^p}{(ex^n+d)^2}\,dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2, x)
```

Giac [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2\,n}+a)^p}{(ex^n+d)^2}\,dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(a+c\,x^{2\,n})^p}{(d+e\,x^n)^2}\,dx$$

```
[In] int((a + c*x^(2*n))^p/(d + e*x^n)^2,x)
[Out] int((a + c*x^(2*n))^p/(d + e*x^n)^2, x)
```

3.65 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 357

$$\begin{aligned}\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx &= \frac{3e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(1+2n)}\\ &\quad-\frac{e^3x^{1+3n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(1+3n)}\\ &\quad+\frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2n},-p,3,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}\\ &\quad-\frac{3ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+n}{2n},-p,3,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+n)}\end{aligned}$$

```
[Out] 3*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*AppellF1(1+1/2/n,3,-p,2+1/2/n,e^2*x^(2*n)/d
^2,-c*x^(2*n)/a)/d^5/(1+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(a+c*x^(2*n)
)^p*AppellF1(3/2+1/2/n,3,-p,5/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^6/(1+
3*n)/((1+c*x^(2*n)/a)^p)+x*(a+c*x^(2*n))^p*AppellF1(1/2/n,3,-p,1+1/2/n,e^2*
x^(2*n)/d^2,-c*x^(2*n)/a)/d^3/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(a+c*x^(2*n))
^p*AppellF1(1/2*(1+n)/n,3,-p,3/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^4/(1
+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1452, 441, 440, 525, 524}

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$
$$= -\frac{e^3x^{3n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(3n+1)}$$
$$+\frac{3e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(2n+1)}$$
$$-\frac{3ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{n+1}{2n},-p,3,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(n+1)}$$
$$+\frac{x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},-p,3,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}$$

```
[In] Int[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] (3*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 3, (4 + n
^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^5*(1 + 2*n)*(1 + (c*x^(2
*n))/a)^p) - (e^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*AppellF1[(3 + n^(-1))/2, -p
, 3, (5 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + 3*n)*(
1 + (c*x^(2*n))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 3, (2 +
n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + (c*x^(2*n))/a)^p
) - (3*e*x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 3, (3 + n^
(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + n)*(1 + (c*x^(2*n)
)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{d^3(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3}+\frac{3d^2ex^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}-\frac{3de^2x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\right.\\ &\qquad\left.+\frac{e^3x^{3n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\right)dx\\ &= d^3\int \frac{(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3}\,dx+\left(3d^2e\right)\int \frac{x^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx\\ &\quad-\left(3de^2\right)\int \frac{x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx+e^3\int \frac{x^{3n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx\end{aligned}$$

$$= \left(d^3\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(d^2-e^2x^{2n}\right)^3}\,dx$$
$$+\left(3d^2e\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^n\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}\,dx$$
$$-\left(3de^2\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{2n}\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}\,dx$$
$$+\left(e^3\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{3n}\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}\,dx$$
$$=\frac{3e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(1+2n)}$$
$$-\frac{e^3x^{1+3n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(1+3n)}$$
$$+\frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2n};-p,3;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}$$
$$-\frac{3ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+n}{2n};-p,3;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+n)}$$

Mathematica [F]

$$\int\frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx=\int\frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

```
[In] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Maple [F]

$$\int\frac{(a+c\,x^{2n})^p}{(d+e\,x^n)^3}dx$$

```
[In] int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
[Out] int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \int \frac{(cx^{2\,n} + a)^p}{(ex^n + d)^3}\, dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d
^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \text{Timed out}$$

```
[In] integrate((a+c*x**(2*n))**p/(d+e*x**n)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \int \frac{(cx^{2\,n} + a)^p}{(ex^n + d)^3}\, dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3, x)
```

Giac [F]

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \int \frac{(cx^{2\,n} + a)^p}{(ex^n + d)^3}\, dx$$

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \int \frac{(a + c\,x^{2\,n})^p}{(d + e\,x^n)^3}\, dx$$

```
[In] int((a + c*x^(2*n))^p/(d + e*x^n)^3,x)
[Out] int((a + c*x^(2*n))^p/(d + e*x^n)^3, x)
```

3.66 $\int (d+ex^n)(a+bx^n+cx^{2n})\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = adx+\frac{(bd+ae)x^{1+n}}{1+n}+\frac{(cd+be)x^{1+2n}}{1+2n}+\frac{cex^{1+3n}}{1+3n}$$

```
[Out] a*d*x+(a*e+b*d)*x^(1+n)/(1+n)+(b*e+c*d)*x^(1+2*n)/(1+2*n)+c*e*x^(1+3*n)/(1+
3*n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1421}

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = \frac{x^{n+1}(ae+bd)}{n+1}+adx+\frac{x^{2n+1}(be+cd)}{2n+1}+\frac{cex^{3n+1}}{3n+1}$$

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^(1 + n))/(1 + n) + ((c*d + b*e)*x^(1 + 2*n))/(1 + 2*
n) + (c*e*x^(1 + 3*n))/(1 + 3*n)
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(ad + (bd + ae)x^n + (cd + be)x^{2n} + cex^{3n}\right) dx$$
$$= adx + \frac{(bd + ae)x^{1+n}}{1+n} + \frac{(cd + be)x^{1+2n}}{1+2n} + \frac{cex^{1+3n}}{1+3n}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int (d + ex^n)\left(a + bx^n + cx^{2n}\right) dx = x\left(ad + \frac{(bd + ae)x^n}{1+n} + \frac{(cd + be)x^{2n}}{1+2n} + \frac{cex^{3n}}{1+3n}\right)$$

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(a*d + ((b*d + a*e)*x^n)/(1 + n) + ((c*d + b*e)*x^(2*n))/(1 + 2*n) + (c*e
*x^(3*n))/(1 + 3*n))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

method	result
risch	$adx + \frac{(ae+bd)x\,x^n}{1+n} + \frac{(be+cd)x\,x^{2n}}{1+2n} + \frac{ecx\,x^{3n}}{1+3n}$
norman	$adx + \frac{(ae+bd)x\,e^{n\ln(x)}}{1+n} + \frac{(be+cd)x\,e^{2n\ln(x)}}{1+2n} + \frac{ecx\,e^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{3x\,x^{2n}be\,n^2+2x\,x^n x^{2n}ce\,n^2+4x\,x^{2n}ben+3x\,x^n x^{2n}cen+6x\,x^n ae\,n^2+6x\,x^n bd\,n^2+3x\,x^{2n}cd\,n^2+6xad\,n^3+x\,x^{2n}be+x\,x^n x^{2n}ce+5}{(1+n)(1+2n)(1+3n)}$

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+(a*e+b*d)/(1+n)*x*x^n+(b*e+c*d)/(1+2*n)*x*(x^n)^2+e*c/(1+3*n)*x*(x^n)
^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.21

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = \frac{(2\,cen^2+3\,cen+ce)xx^{3\,n}+(3\,(cd+be)n^2+cd+be+4\,(cd+be)n)xx^{2\,n}+(6\,(bd+ae)n^2+bd+ae+5}{6\,n^3+11\,n^2+6\,n+1}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] ((2*c*e*n^2 + 3*c*e*n + c*e)*x*x^(3*n) + (3*(c*d + b*e)*n^2 + c*d + b*e + 4
*(c*d + b*e)*n)*x*x^(2*n) + (6*(b*d + a*e)*n^2 + b*d + a*e + 5*(b*d + a*e)*
n)*x*x^n + (6*a*d*n^3 + 11*a*d*n^2 + 6*a*d*n + a*d)*x)/(6*n^3 + 11*n^2 + 6*
n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(53) = 106$.

Time = 0.39 (sec) , antiderivative size = 656, normalized size of antiderivative = 10.58

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = \begin{cases} adx+ae\log(x)+bd\log(x)-\frac{be}{x}-\frac{cd}{x}-\frac{ce}{2x^2} \\ adx+2ae\sqrt{x}+2bd\sqrt{x}+be\log(x)+cd\log(x)-\frac{2ce}{\sqrt{x}} \\ adx+\frac{3aex^{\frac{2}{3}}}{2}+\frac{3bdx^{\frac{2}{3}}}{2}+3be\sqrt[3]{x}+3cd\sqrt[3]{x}+ce\log(x) \\ \frac{6adn^3x}{6n^3+11n^2+6n+1}+\frac{11adn^2x}{6n^3+11n^2+6n+1}+\frac{6adnx}{6n^3+11n^2+6n+1}+\frac{adx}{6n^3+11n^2+6n+1}+\frac{6aen^2xx^n}{6n^3+11n^2+6n+1}+\frac{5aenxx^n}{6n^3+11n^2+6n+1}+\frac{}{6n^3+} \end{cases}$$

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((a*d*x + a*e*log(x) + b*d*log(x) - b*e/x - c*d/x - c*e/(2*x**2),
Eq(n, -1)), (a*d*x + 2*a*e*sqrt(x) + 2*b*d*sqrt(x) + b*e*log(x) + c*d*log(x
) - 2*c*e/sqrt(x), Eq(n, -1/2)), (a*d*x + 3*a*e*x**(2/3)/2 + 3*b*d*x**(2/3)
/2 + 3*b*e*x**(1/3) + 3*c*d*x**(1/3) + c*e*log(x), Eq(n, -1/3)), (6*a*d*n**
3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a*d*n**2*x/(6*n**3 + 11*n**2 + 6*n +
1) + 6*a*d*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*d*x/(6*n**3 + 11*n**2 + 6*n
 + 1) + 6*a*e*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a*e*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + a*e*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*d
*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*d*n*x*x**n/(6*n**3 + 11*n**
2 + 6*n + 1) + b*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*e*n**2*x*x**(2
*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b*e*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6
```

```
*n + 1) + b*e*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*d*n**2*x*x**(2*
n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*c*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*
n + 1) + c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*c*e*n**2*x*x**(3*n
)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*e*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n
 + 1) + c*e*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right) \, dx = adx + \frac{cex^{3\,n+1}}{3\,n+1} + \frac{cdx^{2\,n+1}}{2\,n+1} + \frac{bex^{2\,n+1}}{2\,n+1} + \frac{bdx^{n+1}}{n+1} + \frac{aex^{n+1}}{n+1}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] a*d*x + c*e*x^(3*n + 1)/(3*n + 1) + c*d*x^(2*n + 1)/(2*n + 1) + b*e*x^(2*n
+ 1)/(2*n + 1) + b*d*x^(n + 1)/(n + 1) + a*e*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.19

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right) \, dx$$
$$= \frac{6\,adn^3x + 2\,cen^2xx^{3\,n} + 3\,cdn^2xx^{2\,n} + 3\,ben^2xx^{2\,n} + 6\,bdn^2xx^n + 6\,aen^2xx^n + 11\,adn^2x + 3\,cenxx^{3\,n} + 4\,c}{6\,n^3 + 11\,}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (6*a*d*n^3*x + 2*c*e*n^2*x*x^(3*n) + 3*c*d*n^2*x*x^(2*n) + 3*b*e*n^2*x*x^(2
*n) + 6*b*d*n^2*x*x^n + 6*a*e*n^2*x*x^n + 11*a*d*n^2*x + 3*c*e*n*x*x^(3*n)
+ 4*c*d*n*x*x^(2*n) + 4*b*e*n*x*x^(2*n) + 5*b*d*n*x*x^n + 5*a*e*n*x*x^n + 6
*a*d*n*x + c*e*x*x^(3*n) + c*d*x*x^(2*n) + b*e*x*x^(2*n) + b*d*x*x^n + a*e*
x*x^n + a*d*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = a\,d\,x+\frac{x\,x^{2\,n}\,(b\,e+c\,d)}{2\,n+1}+\frac{x\,x^n\,(a\,e+b\,d)}{n+1}+\frac{c\,e\,x\,x^{3\,n}}{3\,n+1}$$

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] a*d*x + (x*x^(2*n)*(b*e + c*d))/(2*n + 1) + (x*x^n*(a*e + b*d))/(n + 1) + (
c*e*x*x^(3*n))/(3*n + 1)
```

3.67 $\int (d+ex^n)(a+bx^n+cx^{2n})^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int (d+ex^n)(a+bx^n+cx^{2n})^2 \, dx = a^2dx + \frac{a(2bd+ae)x^{1+n}}{1+n} + \frac{(b^2d+2acd+2abe)\,x^{1+2n}}{1+2n} + \frac{(2bcd+b^2e+2ace)\,x^{1+3n}}{1+3n} + \frac{c(cd+2be)x^{1+4n}}{1+4n} + \frac{c^2ex^{1+5n}}{1+5n}$$

```
[Out] a^2*d*x+a*(a*e+2*b*d)*x^(1+n)/(1+n)+(2*a*b*e+2*a*c*d+b^2*d)*x^(1+2*n)/(1+2*
n)+(2*a*c*e+b^2*e+2*b*c*d)*x^(1+3*n)/(1+3*n)+c*(2*b*e+c*d)*x^(1+4*n)/(1+4*n
)+c^2*e*x^(1+5*n)/(1+5*n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1446}

$$\int (d+ex^n)(a+bx^n+cx^{2n})^2 \, dx = a^2dx + \frac{x^{2n+1}(2abe+2acd+b^2d)}{2n+1} + \frac{x^{3n+1}(2ace+b^2e+2bcd)}{3n+1} + \frac{ax^{n+1}(ae+2bd)}{n+1} + \frac{cx^{4n+1}(2be+cd)}{4n+1} + \frac{c^2ex^{5n+1}}{5n+1}$$

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*e
)*x^(1 + 2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(1 + 3*n))/(1 + 3
```

```
*n) + (c*(c*d + 2*b*e)*x^(1 + 4*n))/(1 + 4*n) + (c^2*e*x^(1 + 5*n))/(1 + 5*
n)
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2d + a(2bd+ae)x^n + \left(b^2d+2acd+2abe\right)x^{2n} + \left(2bcd+b^2e+2ace\right)x^{3n} + c(cd+2be)x^{4n} + c^2ex^{5n}\right)\,dx$$
$$= a^2dx + \frac{a(2bd+ae)x^{1+n}}{1+n} + \frac{\left(b^2d+2acd+2abe\right)x^{1+2n}}{1+2n} + \frac{\left(2bcd+b^2e+2ace\right)x^{1+3n}}{1+3n} + \frac{c(cd+2be)x^{1+4n}}{1+4n} + \frac{c^2ex^{1+5n}}{1+5n}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = x\left(a^2d + \frac{a(2bd+ae)x^n}{1+n} + \frac{\left(b^2d+2acd+2abe\right)x^{2n}}{1+2n} + \frac{\left(2bcd+b^2e+2ace\right)x^{3n}}{1+3n} + \frac{c(cd+2be)x^{4n}}{1+4n} + \frac{c^2ex^{5n}}{1+5n}\right)$$

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(a^2*d + (a*(2*b*d + a*e)*x^n)/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^(
2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(3*n))/(1 + 3*n) + (c*(c*d
 + 2*b*e)*x^(4*n))/(1 + 4*n) + (c^2*e*x^(5*n))/(1 + 5*n))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$a^2dx+\frac{(2ace+b^2e+2bcd)x\,x^{3n}}{1+3n}+\frac{(2abe+2acd+b^2d)x\,x^{2n}}{1+2n}+\frac{a(ae+2bd)x\,x^n}{1+n}+\frac{c(2be+cd)x\,x^{4n}}{1+4n}+\frac{e\,c^2x\,x^{5n}}{1+5n}$
norman	$a^2dx+\frac{(2ace+b^2e+2bcd)x\,e^{3n\ln(x)}}{1+3n}+\frac{(2abe+2acd+b^2d)x\,e^{2n\ln(x)}}{1+2n}+\frac{a(ae+2bd)x\,e^{n\ln(x)}}{1+n}+\frac{c(2be+cd)x\,e^{4n\ln(x)}}{1+4n}+\frac{e\,c}{}$
parallelrisch	$\frac{142x\,x^nabd\,n^2+118x\,x^{2n}acd\,n^2+2x\,x^nx^{2n}ace+2x\,x^nx^{2n}bcd+28x\,x^nabdn+26x\,x^{2n}acdn+120x\,x^na^2e\,n^4+154x\,x^na^2e\,n^3+22x}{}$

[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)

[Out] a^2*d*x+(2*a*c*e+b^2*e+2*b*c*d)/(1+3*n)*x*(x^n)^3+(2*a*b*e+2*a*c*d+b^2*d)/(
1+2*n)*x*(x^n)^2+a*(a*e+2*b*d)/(1+n)*x*x^n+c*(2*b*e+c*d)/(1+4*n)*x*(x^n)^4+
e*c^2/(1+5*n)*x*(x^n)^5

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(132) = 264$.

Time = 0.37 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.75

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx$$
$$=\frac{(24\,c^2en^4+50\,c^2en^3+35\,c^2en^2+10\,c^2en+c^2e)xx^{5\,n}+(30\,(c^2d+2\,bce)n^4+61\,(c^2d+2\,bce)n^3+c^2d+2\,}{}$$

[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")

[Out] ((24*c^2*e*n^4 + 50*c^2*e*n^3 + 35*c^2*e*n^2 + 10*c^2*e*n + c^2*e)*x*x^(5*n
) + (30*(c^2*d + 2*b*c*e)*n^4 + 61*(c^2*d + 2*b*c*e)*n^3 + c^2*d + 2*b*c*e
+ 41*(c^2*d + 2*b*c*e)*n^2 + 11*(c^2*d + 2*b*c*e)*n)*x*x^(4*n) + (40*(2*b*c
*d + (b^2 + 2*a*c)*e)*n^4 + 78*(2*b*c*d + (b^2 + 2*a*c)*e)*n^3 + 2*b*c*d +
49*(2*b*c*d + (b^2 + 2*a*c)*e)*n^2 + (b^2 + 2*a*c)*e + 12*(2*b*c*d + (b^2 +
2*a*c)*e)*n)*x*x^(3*n) + (60*(2*a*b*e + (b^2 + 2*a*c)*d)*n^4 + 107*(2*a*b*
e + (b^2 + 2*a*c)*d)*n^3 + 2*a*b*e + 59*(2*a*b*e + (b^2 + 2*a*c)*d)*n^2 + (
b^2 + 2*a*c)*d + 13*(2*a*b*e + (b^2 + 2*a*c)*d)*n)*x*x^(2*n) + (120*(2*a*b*
d + a^2*e)*n^4 + 154*(2*a*b*d + a^2*e)*n^3 + 2*a*b*d + a^2*e + 71*(2*a*b*d
+ a^2*e)*n^2 + 14*(2*a*b*d + a^2*e)*n)*x*x^n + (120*a^2*d*n^5 + 274*a^2*d*n
^4 + 225*a^2*d*n^3 + 85*a^2*d*n^2 + 15*a^2*d*n + a^2*d)*x)/(120*n^5 + 274*n
^4 + 225*n^3 + 85*n^2 + 15*n + 1)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3128 vs. 2(124) = 248.

Time = 1.34 (sec) , antiderivative size = 3128, normalized size of antiderivative = 23.70

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*log(x) + 2*a*b*d*log(x) - 2*a*b*e/x - 2*a*c*d/
x - a*c*e/x**2 - b**2*d/x - b**2*e/(2*x**2) - b*c*d/x**2 - 2*b*c*e/(3*x**3)
 - c**2*d/(3*x**3) - c**2*e/(4*x**4), Eq(n, -1)), (a**2*d*x + 2*a**2*e*sqrt
(x) + 4*a*b*d*sqrt(x) + 2*a*b*e*log(x) + 2*a*c*d*log(x) - 4*a*c*e/sqrt(x) +
 b**2*d*log(x) - 2*b**2*e/sqrt(x) - 4*b*c*d/sqrt(x) - 2*b*c*e/x - c**2*d/x
- 2*c**2*e/(3*x**(3/2)), Eq(n, -1/2)), (a**2*d*x + 3*a**2*e*x**(2/3)/2 + 3*
a*b*d*x**(2/3) + 6*a*b*e*x**(1/3) + 6*a*c*d*x**(1/3) + 2*a*c*e*log(x) + 3*b
**2*d*x**(1/3) + b**2*e*log(x) + 2*b*c*d*log(x) - 6*b*c*e/x**(1/3) - 3*c**2
*d/x**(1/3) - 3*c**2*e/(2*x**(2/3)), Eq(n, -1/3)), (a**2*d*x + 4*a**2*e*x**
(3/4)/3 + 8*a*b*d*x**(3/4)/3 + 4*a*b*e*sqrt(x) + 4*a*c*d*sqrt(x) + 8*a*c*e*
x**(1/4) + 2*b**2*d*sqrt(x) + 4*b**2*e*x**(1/4) + 8*b*c*d*x**(1/4) + 2*b*c*
e*log(x) + c**2*d*log(x) - 4*c**2*e/x**(1/4), Eq(n, -1/4)), (a**2*d*x + 5*a
**2*e*x**(4/5)/4 + 5*a*b*d*x**(4/5)/2 + 10*a*b*e*x**(3/5)/3 + 10*a*c*d*x**(
3/5)/3 + 5*a*c*e*x**(2/5) + 5*b**2*d*x**(3/5)/3 + 5*b**2*e*x**(2/5)/2 + 5*b
*c*d*x**(2/5) + 10*b*c*e*x**(1/5) + 5*c**2*d*x**(1/5) + c**2*e*log(x), Eq(n
, -1/5)), (120*a**2*d*n**5*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 274*a**2*d*n**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*
n + 1) + 225*a**2*d*n**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 85*a**2*d*n**2*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 15*a**2*d*n*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 a**2*d*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 120*a**2*
e*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 154*a
**2*e*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 7
1*a**2*e*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
+ 14*a**2*e*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
+ a**2*e*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240
*a*b*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
308*a*b*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 142*a*b*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 28*a*b*d*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 2*a*b*d*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 120*a*b*e*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 214*a*b*e*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 118*a*b*e*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + 26*a*b*e*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
```

```
 + 85*n**2 + 15*n + 1) + 2*a*b*e*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 120*a*c*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 214*a*c*d*n**3*x*x**(2*n)/(120*n**5 + 274*n
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 118*a*c*d*n**2*x*x**(2*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 26*a*c*d*n*x*x**(2*n)/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*c*d*x*x**(2*n)/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 80*a*c*e*n**4*x*x**(3*n)/(
120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*a*c*e*n**3*x*x**
(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*a*c*e*n**2
*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*a*c*
e*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*
c*e*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b
**2*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 107*b**2*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 59*b**2*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 13*b**2*d*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + b**2*d*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 40*b**2*e*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 78*b**2*e*n**3*x*x**(3*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 49*b**2*e*n**2*x*x**(3*n)/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12*b**2*e*n*x*x**(3*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + b**2*e*x*x**(3*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 80*b*c*d*n**4*x*x**(3*n)/(120*
n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*b*c*d*n**3*x*x**(3*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*b*c*d*n**2*x*x
**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*c*d*n*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*b*c*d*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b*c*e
*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12
2*b*c*e*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 82*b*c*e*n**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 22*b*c*e*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 2*b*c*e*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 30*c**2*d*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 61*c**2*d*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 41*c**2*d*n**2*x*x**(4*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 11*c**2*d*n*x*x**(4*n)/(120*n**5 + 274*n
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*d*x*x**(4*n)/(120*n**5 + 274*n*
*4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*c**2*e*n**4*x*x**(5*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*c**2*e*n**3*x*x**(5*n)/(120
*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 35*c**2*e*n**2*x*x**(5*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 10*c**2*e*n*x*x*
*(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*e*x*x**
(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.58

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = a^2dx + \frac{c^2ex^{5n+1}}{5n+1} + \frac{c^2dx^{4n+1}}{4n+1} + \frac{2\,bcex^{4n+1}}{4n+1} + \frac{2\,bcdx^{3n+1}}{3n+1} + \frac{b^2ex^{3n+1}}{3n+1} + \frac{2\,acex^{3n+1}}{3n+1} + \frac{b^2dx^{2n+1}}{2n+1} + \frac{2\,acdx^{2n+1}}{2n+1} + \frac{2\,abex^{2n+1}}{2n+1} + \frac{2\,abdx^{n+1}}{n+1} + \frac{a^2ex^{n+1}}{n+1}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] a^2*d*x + c^2*e*x^(5*n + 1)/(5*n + 1) + c^2*d*x^(4*n + 1)/(4*n + 1) + 2*b*c
*e*x^(4*n + 1)/(4*n + 1) + 2*b*c*d*x^(3*n + 1)/(3*n + 1) + b^2*e*x^(3*n + 1
)/(3*n + 1) + 2*a*c*e*x^(3*n + 1)/(3*n + 1) + b^2*d*x^(2*n + 1)/(2*n + 1) +
 2*a*c*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*e*x^(2*n + 1)/(2*n + 1) + 2*a*b*d*x^
(n + 1)/(n + 1) + a^2*e*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. 2(132) = 264.

Time = 0.38 (sec) , antiderivative size = 798, normalized size of antiderivative = 6.05

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = \frac{120\,a^2dn^5x + 24\,c^2en^4xx^{5n} + 30\,c^2dn^4xx^{4n} + 60\,bcen^4xx^{4n} + 80\,bcdn^4xx^{3n} + 40\,b^2en^4xx^{3n} + 80\,acen^4xx}{}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (120*a^2*d*n^5*x + 24*c^2*e*n^4*x*x^(5*n) + 30*c^2*d*n^4*x*x^(4*n) + 60*b*c
*e*n^4*x*x^(4*n) + 80*b*c*d*n^4*x*x^(3*n) + 40*b^2*e*n^4*x*x^(3*n) + 80*a*c
*e*n^4*x*x^(3*n) + 60*b^2*d*n^4*x*x^(2*n) + 120*a*c*d*n^4*x*x^(2*n) + 120*a
*b*e*n^4*x*x^(2*n) + 240*a*b*d*n^4*x*x^n + 120*a^2*e*n^4*x*x^n + 274*a^2*d*
n^4*x + 50*c^2*e*n^3*x*x^(5*n) + 61*c^2*d*n^3*x*x^(4*n) + 122*b*c*e*n^3*x*x
^(4*n) + 156*b*c*d*n^3*x*x^(3*n) + 78*b^2*e*n^3*x*x^(3*n) + 156*a*c*e*n^3*x
*x^(3*n) + 107*b^2*d*n^3*x*x^(2*n) + 214*a*c*d*n^3*x*x^(2*n) + 214*a*b*e*n^
3*x*x^(2*n) + 308*a*b*d*n^3*x*x^n + 154*a^2*e*n^3*x*x^n + 225*a^2*d*n^3*x +
 35*c^2*e*n^2*x*x^(5*n) + 41*c^2*d*n^2*x*x^(4*n) + 82*b*c*e*n^2*x*x^(4*n) +
 98*b*c*d*n^2*x*x^(3*n) + 49*b^2*e*n^2*x*x^(3*n) + 98*a*c*e*n^2*x*x^(3*n) +
 59*b^2*d*n^2*x*x^(2*n) + 118*a*c*d*n^2*x*x^(2*n) + 118*a*b*e*n^2*x*x^(2*n)
 + 142*a*b*d*n^2*x*x^n + 71*a^2*e*n^2*x*x^n + 85*a^2*d*n^2*x + 10*c^2*e*n*x
```

```
*x^(5*n) + 11*c^2*d*n*x*x^(4*n) + 22*b*c*e*n*x*x^(4*n) + 24*b*c*d*n*x*x^(3*
n) + 12*b^2*e*n*x*x^(3*n) + 24*a*c*e*n*x*x^(3*n) + 13*b^2*d*n*x*x^(2*n) + 2
6*a*c*d*n*x*x^(2*n) + 26*a*b*e*n*x*x^(2*n) + 28*a*b*d*n*x*x^n + 14*a^2*e*n*
x*x^n + 15*a^2*d*n*x + c^2*e*x*x^(5*n) + c^2*d*x*x^(4*n) + 2*b*c*e*x*x^(4*n
) + 2*b*c*d*x*x^(3*n) + b^2*e*x*x^(3*n) + 2*a*c*e*x*x^(3*n) + b^2*d*x*x^(2*
n) + 2*a*c*d*x*x^(2*n) + 2*a*b*e*x*x^(2*n) + 2*a*b*d*x*x^n + a^2*e*x*x^n +
a^2*d*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = a^2\,d\,x+\frac{x\,x^{4\,n}\,\left(d\,c^2+2\,b\,e\,c\right)}{4\,n+1}+\frac{x\,x^n\,\left(e\,a^2+2\,b\,d\,a\right)}{n+1}+\frac{x\,x^{2\,n}\,\left(d\,b^2+2\,a\,e\,b+2\,a\,c\,d\right)}{2\,n+1}+\frac{x\,x^{3\,n}\,\left(e\,b^2+2\,c\,d\,b+2\,a\,c\,e\right)}{3\,n+1}+\frac{c^2\,e\,x\,x^{5\,n}}{5\,n+1}$$

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] a^2*d*x + (x*x^(4*n)*(c^2*d + 2*b*c*e))/(4*n + 1) + (x*x^n*(a^2*e + 2*a*b*d
))/(n + 1) + (x*x^(2*n)*(b^2*d + 2*a*b*e + 2*a*c*d))/(2*n + 1) + (x*x^(3*n)
*(b^2*e + 2*a*c*e + 2*b*c*d))/(3*n + 1) + (c^2*e*x*x^(5*n))/(5*n + 1)
```

3.68 $\int (d+ex^n) (a+bx^n+cx^{2n})^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 218

$$\int (d+ex^n) (a+bx^n+cx^{2n})^3 \, dx = a^3dx + \frac{a^2(3bd+ae)x^{1+n}}{1+n} + \frac{3a(b^2d+acd+abe)\,x^{1+2n}}{1+2n} + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)\,x^{1+3n}}{1+3n} + \frac{(3b^2cd+3ac^2d+b^3e+6abce)\,x^{1+4n}}{1+4n} + \frac{3c(bcd+b^2e+ace)\,x^{1+5n}}{1+5n} + \frac{c^2(cd+3be)x^{1+6n}}{1+6n} + \frac{c^3ex^{1+7n}}{1+7n}$$

```
[Out] a^3*d*x+a^2*(a*e+3*b*d)*x^(1+n)/(1+n)+3*a*(a*b*e+a*c*d+b^2*d)*x^(1+2*n)/(1+
2*n)+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)*x^(1+3*n)/(1+3*n)+(6*a*b*c*e+3*a
*c^2*d+b^3*e+3*b^2*c*d)*x^(1+4*n)/(1+4*n)+3*c*(a*c*e+b^2*e+b*c*d)*x^(1+5*n)
/(1+5*n)+c^2*(3*b*e+c*d)*x^(1+6*n)/(1+6*n)+c^3*e*x^(1+7*n)/(1+7*n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used

$= \{1446\}$

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx = a^3dx+\frac{x^{3n+1}(3a^2ce+3ab^2e+6abcd+b^3d)}{3n+1}+\frac{a^2x^{n+1}(ae+3bd)}{n+1}+\frac{3ax^{2n+1}(abe+acd+b^2d)}{2n+1}+\frac{3cx^{5n+1}(ace+b^2e+bcd)}{5n+1}+\frac{x^{4n+1}(6abce+3ac^2d+b^3e+3b^2cd)}{4n+1}+\frac{c^2x^{6n+1}(3be+cd)}{6n+1}+\frac{c^3ex^{7n+1}}{7n+1}$$

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a*c*d + a*b
*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x
^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(1 +
 4*n))/(1 + 4*n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n) + (c
^2*(c*d + 3*b*e)*x^(1 + 6*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n)
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \Big(a^3d+a^2(3bd+ae)x^n+3a\left(b^2d+acd+abe\right)x^{2n}+\left(b^3d+6abcd+3ab^2e+3a^2ce\right)x^{3n}+\left(3b^2cd+3ac^2d+b^3e+6abce\right)x^{4n}+3c\left(bcd+b^2e+ace\right)x^{5n}+c^2(cd+3be)x^{6n}+c^3ex^{7n}\Big)\,dx\\ &= a^3dx+\frac{a^2(3bd+ae)x^{1+n}}{1+n}+\frac{3a\left(b^2d+acd+abe\right)x^{1+2n}}{1+2n}+\frac{\left(b^3d+6abcd+3ab^2e+3a^2ce\right)x^{1+3n}}{1+3n}+\frac{\left(3b^2cd+3ac^2d+b^3e+6abce\right)x^{1+4n}}{1+4n}+\frac{3c\left(bcd+b^2e+ace\right)x^{1+5n}}{1+5n}+\frac{c^2(cd+3be)x^{1+6n}}{1+6n}+\frac{c^3ex^{1+7n}}{1+7n}\end{aligned}$$

Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^3 \, dx = x\left(a^3d+\frac{a^2(3bd+ae)x^n}{1+n}+\frac{3a(b^2d+acd+abe)\,x^{2n}}{1+2n} + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)\,x^{3n}}{1+3n} + \frac{(3b^2cd+3ac^2d+b^3e+6abce)\,x^{4n}}{1+4n} + \frac{3c(bcd+b^2e+ace)\,x^{5n}}{1+5n}+\frac{c^2(cd+3be)x^{6n}}{1+6n}+\frac{c^3ex^{7n}}{1+7n}\right)$$

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(a^3*d + (a^2*(3*b*d + a*e)*x^n)/(1 + n) + (3*a*(b^2*d + a*c*d + a*b*e)*x
^(2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^(3*n))/(
1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(4*n))/(1 + 4*n)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^(5*n))/(1 + 5*n) + (c^2*(c*d + 3*b*e)*x^(6
*n))/(1 + 6*n) + (c^3*e*x^(7*n))/(1 + 7*n))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

method	result
risch	$a^3dx+\frac{(6abce+3a\,c^2d+b^3e+3b^2cd)x\,x^{4n}}{1+4n}+\frac{(3a^2ce+3a\,b^2e+6abcd+b^3d)x\,x^{3n}}{1+3n}+\frac{a^2(ae+3bd)x\,x^n}{1+n}+\frac{c^2(3be+cd)x\,x^{6n}}{1+6n}$
norman	$a^3dx+\frac{(6abce+3a\,c^2d+b^3e+3b^2cd)x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{(3a^2ce+3a\,b^2e+6abcd+b^3d)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{a^2(ae+3bd)x\,\mathrm{e}^{n\ln(x)}}{1+n}+$
parallelrisch	Expression too large to display

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*d*x+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)/(1+4*n)*x*(x^n)^4+(3*a^2*c*e+
3*a*b^2*e+6*a*b*c*d+b^3*d)/(1+3*n)*x*(x^n)^3+a^2*(a*e+3*b*d)/(1+n)*x*x^n+c^
2*(3*b*e+c*d)/(1+6*n)*x*(x^n)^6+c^3*e/(1+7*n)*x*(x^n)^7+3*a*(a*b*e+a*c*d+b^
2*d)/(1+2*n)*x*(x^n)^2+3*c*(a*c*e+b^2*e+b*c*d)/(1+5*n)*x*(x^n)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. $2(218) = 436$.

Time = 0.37 (sec) , antiderivative size = 1209, normalized size of antiderivative = 5.55

$$\int (d + ex^n) \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((720*c^3*e*n^6 + 1764*c^3*e*n^5 + 1624*c^3*e*n^4 + 735*c^3*e*n^3 + 175*c^3
*e*n^2 + 21*c^3*e*n + c^3*e)*x*x^(7*n) + (840*(c^3*d + 3*b*c^2*e)*n^6 + 203
8*(c^3*d + 3*b*c^2*e)*n^5 + 1849*(c^3*d + 3*b*c^2*e)*n^4 + c^3*d + 3*b*c^2*
e + 820*(c^3*d + 3*b*c^2*e)*n^3 + 190*(c^3*d + 3*b*c^2*e)*n^2 + 22*(c^3*d +
 3*b*c^2*e)*n)*x*x^(6*n) + 3*(1008*(b*c^2*d + (b^2*c + a*c^2)*e)*n^6 + 2412
*(b*c^2*d + (b^2*c + a*c^2)*e)*n^5 + 2144*(b*c^2*d + (b^2*c + a*c^2)*e)*n^4
 + b*c^2*d + 925*(b*c^2*d + (b^2*c + a*c^2)*e)*n^3 + 207*(b*c^2*d + (b^2*c
+ a*c^2)*e)*n^2 + (b^2*c + a*c^2)*e + 23*(b*c^2*d + (b^2*c + a*c^2)*e)*n)*x
*x^(5*n) + (1260*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^6 + 2952*(3*(b
^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^5 + 2545*(3*(b^2*c + a*c^2)*d + (b^3
 + 6*a*b*c)*e)*n^4 + 1056*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^3 + 2
26*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^2 + 3*(b^2*c + a*c^2)*d + (b
^3 + 6*a*b*c)*e + 24*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n)*x*x^(4*n)
 + (1680*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^6 + 3796*((b^3 + 6*a*b
*c)*d + 3*(a*b^2 + a^2*c)*e)*n^5 + 3112*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2
*c)*e)*n^4 + 1219*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^3 + 247*((b^3
 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^2 + (b^3 + 6*a*b*c)*d + 3*(a*b^2 + a
^2*c)*e + 25*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n)*x*x^(3*n) + 3*(25
20*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^6 + 5274*(a^2*b*e + (a*b^2 + a^2*c)*d)*n
^5 + 3929*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^4 + a^2*b*e + 1420*(a^2*b*e + (a*
b^2 + a^2*c)*d)*n^3 + 270*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^2 + (a*b^2 + a^2*
c)*d + 26*(a^2*b*e + (a*b^2 + a^2*c)*d)*n)*x*x^(2*n) + (5040*(3*a^2*b*d + a
^3*e)*n^6 + 8028*(3*a^2*b*d + a^3*e)*n^5 + 5104*(3*a^2*b*d + a^3*e)*n^4 + 3
*a^2*b*d + a^3*e + 1665*(3*a^2*b*d + a^3*e)*n^3 + 295*(3*a^2*b*d + a^3*e)*n
^2 + 27*(3*a^2*b*d + a^3*e)*n)*x*x^n + (5040*a^3*d*n^7 + 13068*a^3*d*n^6 +
13132*a^3*d*n^5 + 6769*a^3*d*n^4 + 1960*a^3*d*n^3 + 322*a^3*d*n^2 + 28*a^3*
d*n + a^3*d)*x)/(5040*n^7 + 13068*n^6 + 13132*n^5 + 6769*n^4 + 1960*n^3 + 3
22*n^2 + 28*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9190 vs. $2(212) = 424$.

Time = 5.81 (sec) , antiderivative size = 9190, normalized size of antiderivative = 42.16

$$\int (d + ex^n) (a + bx^n + cx^{2n})^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**3,x)
[Out] Piecewise((a**3*d*x + a**3*e*log(x) + 3*a**2*b*d*log(x) - 3*a**2*b*e/x - 3*
a**2*c*d/x - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/x - 3*a*b**2*e/(2*x**2) - 3*a
*b*c*d/x**2 - 2*a*b*c*e/x**3 - a*c**2*d/x**3 - 3*a*c**2*e/(4*x**4) - b**3*d
/(2*x**2) - b**3*e/(3*x**3) - b**2*c*d/x**3 - 3*b**2*c*e/(4*x**4) - 3*b*c**
2*d/(4*x**4) - 3*b*c**2*e/(5*x**5) - c**3*d/(5*x**5) - c**3*e/(6*x**6), Eq(
n, -1)), (a**3*d*x + 2*a**3*e*sqrt(x) + 6*a**2*b*d*sqrt(x) + 3*a**2*b*e*log
(x) + 3*a**2*c*d*log(x) - 6*a**2*c*e/sqrt(x) + 3*a*b**2*d*log(x) - 6*a*b**2
*e/sqrt(x) - 12*a*b*c*d/sqrt(x) - 6*a*b*c*e/x - 3*a*c**2*d/x - 2*a*c**2*e/x
**(3/2) - 2*b**3*d/sqrt(x) - b**3*e/x - 3*b**2*c*d/x - 2*b**2*c*e/x**(3/2)
- 2*b*c**2*d/x**(3/2) - 3*b*c**2*e/(2*x**2) - c**3*d/(2*x**2) - 2*c**3*e/(5
*x**(5/2)), Eq(n, -1/2)), (a**3*d*x + 3*a**3*e*x**(2/3)/2 + 9*a**2*b*d*x**(
2/3)/2 + 9*a**2*b*e*x**(1/3) + 9*a**2*c*d*x**(1/3) + 3*a**2*c*e*log(x) + 9*
a*b**2*d*x**(1/3) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x) - 18*a*b*c*e/x**(1
/3) - 9*a*c**2*d/x**(1/3) - 9*a*c**2*e/(2*x**(2/3)) + b**3*d*log(x) - 3*b**
3*e/x**(1/3) - 9*b**2*c*d/x**(1/3) - 9*b**2*c*e/(2*x**(2/3)) - 9*b*c**2*d/(
2*x**(2/3)) - 3*b*c**2*e/x - c**3*d/x - 3*c**3*e/(4*x**(4/3)), Eq(n, -1/3))
, (a**3*d*x + 4*a**3*e*x**(3/4)/3 + 4*a**2*b*d*x**(3/4) + 6*a**2*b*e*sqrt(x
) + 6*a**2*c*d*sqrt(x) + 12*a**2*c*e*x**(1/4) + 6*a*b**2*d*sqrt(x) + 12*a*b
**2*e*x**(1/4) + 24*a*b*c*d*x**(1/4) + 6*a*b*c*e*log(x) + 3*a*c**2*d*log(x)
 - 12*a*c**2*e/x**(1/4) + 4*b**3*d*x**(1/4) + b**3*e*log(x) + 3*b**2*c*d*lo
g(x) - 12*b**2*c*e/x**(1/4) - 12*b*c**2*d/x**(1/4) - 6*b*c**2*e/sqrt(x) - 2
*c**3*d/sqrt(x) - 4*c**3*e/(3*x**(3/4)), Eq(n, -1/4)), (a**3*d*x + 5*a**3*e
*x**(4/5)/4 + 15*a**2*b*d*x**(4/5)/4 + 5*a**2*b*e*x**(3/5) + 5*a**2*c*d*x**
(3/5) + 15*a**2*c*e*x**(2/5)/2 + 5*a*b**2*d*x**(3/5) + 15*a*b**2*e*x**(2/5)
/2 + 15*a*b*c*d*x**(2/5) + 30*a*b*c*e*x**(1/5) + 15*a*c**2*d*x**(1/5) + 3*a
*c**2*e*log(x) + 5*b**3*d*x**(2/5)/2 + 5*b**3*e*x**(1/5) + 15*b**2*c*d*x**(
1/5) + 3*b**2*c*e*log(x) + 3*b*c**2*d*log(x) - 15*b*c**2*e/x**(1/5) - 5*c**
3*d/x**(1/5) - 5*c**3*e/(2*x**(2/5)), Eq(n, -1/5)), (a**3*d*x + 6*a**3*e*x*
*(5/6)/5 + 18*a**2*b*d*x**(5/6)/5 + 9*a**2*b*e*x**(2/3)/2 + 9*a**2*c*d*x**(
2/3)/2 + 6*a**2*c*e*sqrt(x) + 9*a*b**2*d*x**(2/3)/2 + 6*a*b**2*e*sqrt(x) +
12*a*b*c*d*sqrt(x) + 18*a*b*c*e*x**(1/3) + 9*a*c**2*d*x**(1/3) + 18*a*c**2*
e*x**(1/6) + 2*b**3*d*sqrt(x) + 3*b**3*e*x**(1/3) + 9*b**2*c*d*x**(1/3) + 1
8*b**2*c*e*x**(1/6) + 18*b*c**2*d*x**(1/6) + 3*b*c**2*e*log(x) + c**3*d*log
(x) - 6*c**3*e/x**(1/6), Eq(n, -1/6)), (a**3*d*x + 7*a**3*e*x**(6/7)/6 + 7*
a**2*b*d*x**(6/7)/2 + 21*a**2*b*e*x**(5/7)/5 + 21*a**2*c*d*x**(5/7)/5 + 21*
```

a**2*c*e*x**(4/7)/4 + 21*a*b**2*d*x**(5/7)/5 + 21*a*b**2*e*x**(4/7)/4 + 21*
a*b*c*d*x**(4/7)/2 + 14*a*b*c*e*x**(3/7) + 7*a*c**2*d*x**(3/7) + 21*a*c**2*
e*x**(2/7)/2 + 7*b**3*d*x**(4/7)/4 + 7*b**3*e*x**(3/7)/3 + 7*b**2*c*d*x**(3
/7) + 21*b**2*c*e*x**(2/7)/2 + 21*b*c**2*d*x**(2/7)/2 + 21*b*c**2*e*x**(1/7
) + 7*c**3*d*x**(1/7) + c**3*e*log(x), Eq(n, -1/7)), (5040*a**3*d*n**7*x/(5
040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*
n + 1) + 13068*a**3*d*n**6*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n*
*4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 13132*a**3*d*n**5*x/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6769
*a**3*d*n**4*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
 + 322*n**2 + 28*n + 1) + 1960*a**3*d*n**3*x/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 322*a**3*d*n**2*x/(
5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28
*n + 1) + 28*a**3*d*n*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
1960*n**3 + 322*n**2 + 28*n + 1) + a**3*d*x/(5040*n**7 + 13068*n**6 + 13132
*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5040*a**3*e*n**6*x*x
n/(5040*n7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
 + 28*n + 1) + 8028*a**3*e*n**5*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5
 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5104*a**3*e*n**4*x*x**n/(
5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28
*n + 1) + 1665*a**3*e*n**3*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 67
69*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 295*a**3*e*n**2*x*x**n/(5040*n
7 + 13068*n6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1
) + 27*a**3*e*n*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1
960*n**3 + 322*n**2 + 28*n + 1) + a**3*e*x*x**n/(5040*n**7 + 13068*n**6 + 1
3132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15120*a**2*b*d*n
6*x*xn/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 24084*a**2*b*d*n**5*x*x**n/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15312*a**2*b*d*
n**4*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 4995*a**2*b*d*n**3*x*x**n/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 885*a**2*b*d*n*
*2*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 32
2*n**2 + 28*n + 1) + 81*a**2*b*d*n*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n
5 + 6769*n4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a**2*b*d*x*x**n/(504
0*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
+ 1) + 7560*a**2*b*e*n**6*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15822*a**2*b*e*n**5*x*x**(2
*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
 + 28*n + 1) + 11787*a**2*b*e*n**4*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 4260*a**2*b*e*n**3
*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 810*a**2*b*e*n**2*x*x**(2*n)/(5040*n**7 + 13068*n**6
 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 78*a**2*b*e*
n*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +

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 322*n**2 + 28*n + 1) + 3*a**2*b*e*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7560*a**2*c*d*n**6
*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 15822*a**2*c*d*n**5*x*x**(2*n)/(5040*n**7 + 13068*n*
*6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 11787*a**2
*c*d*n**4*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 196
0*n**3 + 322*n**2 + 28*n + 1) + 4260*a**2*c*d*n**3*x*x**(2*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 81
0*a**2*c*d*n**2*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4
 + 1960*n**3 + 322*n**2 + 28*n + 1) + 78*a**2*c*d*n*x*x**(2*n)/(5040*n**7 +
 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3
*a**2*c*d*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 196
0*n**3 + 322*n**2 + 28*n + 1) + 5040*a**2*c*e*n**6*x*x**(3*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 11
388*a**2*c*e*n**5*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n*
*4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 9336*a**2*c*e*n**4*x*x**(3*n)/(5040
*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n +
 1) + 3657*a**2*c*e*n**3*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 741*a**2*c*e*n**2*x*x**(3*n)
/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 +
28*n + 1) + 75*a**2*c*e*n*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a**2*c*e*x*x**(3*n)/(5040
*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n +
 1) + 7560*a*b**2*d*n**6*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15822*a*b**2*d*n**5*x*x**(2*
n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
+ 28*n + 1) + 11787*a*b**2*d*n**4*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 4260*a*b**2*d*n**3*
x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 810*a*b**2*d*n**2*x*x**(2*n)/(5040*n**7 + 13068*n**6
+ 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 78*a*b**2*d*n
*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 3*a*b**2*d*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5040*a*b**2*e*n**6*
x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 11388*a*b**2*e*n**5*x*x**(3*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 9336*a*b**2
*e*n**4*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*
n**3 + 322*n**2 + 28*n + 1) + 3657*a*b**2*e*n**3*x*x**(3*n)/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 741*
a*b**2*e*n**2*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
 1960*n**3 + 322*n**2 + 28*n + 1) + 75*a*b**2*e*n*x*x**(3*n)/(5040*n**7 + 1
3068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a
*b**2*e*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*
n**3 + 322*n**2 + 28*n + 1) + 10080*a*b*c*d*n**6*x*x**(3*n)/(5040*n**7 + 13
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068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2277
6*a*b*c*d*n**5*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4
+ 1960*n**3 + 322*n**2 + 28*n + 1) + 18672*a*b*c*d*n**4*x*x**(3*n)/(5040*n*
*7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1)
 + 7314*a*b*c*d*n**3*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769
*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1482*a*b*c*d*n**2*x*x**(3*n)/(50
40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
 + 1) + 150*a*b*c*d*n*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 676
9*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6*a*b*c*d*x*x**(3*n)/(5040*n**7
 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) +
 7560*a*b*c*e*n**6*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n
4 + 1960*n3 + 322*n**2 + 28*n + 1) + 17712*a*b*c*e*n**5*x*x**(4*n)/(504
0*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
+ 1) + 15270*a*b*c*e*n**4*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6336*a*b*c*e*n**3*x*x**(4*n
)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 +
 28*n + 1) + 1356*a*b*c*e*n**2*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n
5 + 6769*n4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 144*a*b*c*e*n*x*x**(4*
n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
+ 28*n + 1) + 6*a*b*c*e*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6
769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3780*a*c**2*d*n**6*x*x**(4*n)
/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 +
28*n + 1) + 8856*a*c**2*d*n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n
5 + 6769*n4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7635*a*c**2*d*n**4*x*x
(4*n)/(5040*n7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*
n**2 + 28*n + 1) + 3168*a*c**2*d*n**3*x*x**(4*n)/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 678*a*c**2*d*n*
*2*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
+ 322*n**2 + 28*n + 1) + 72*a*c**2*d*n*x*x**(4*n)/(5040*n**7 + 13068*n**6 +
 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a*c**2*d*x*x
(4*n)/(5040*n7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*
n**2 + 28*n + 1) + 3024*a*c**2*e*n**6*x*x**(5*n)/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236*a*c**2*e*n
5*x*x(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
 + 322*n**2 + 28*n + 1) + 6432*a*c**2*e*n**4*x*x**(5*n)/(5040*n**7 + 13068*
n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2775*a*c
2*e*n3*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 19
60*n**3 + 322*n**2 + 28*n + 1) + 621*a*c**2*e*n**2*x*x**(5*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 69
*a*c**2*e*n*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1
960*n**3 + 322*n**2 + 28*n + 1) + 3*a*c**2*e*x*x**(5*n)/(5040*n**7 + 13068*
n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1680*b**
3*d*n**6*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960
*n**3 + 322*n**2 + 28*n + 1) + 3796*b**3*d*n**5*x*x**(3*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3112*

b**3*d*n**4*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1
960*n**3 + 322*n**2 + 28*n + 1) + 1219*b**3*d*n**3*x*x**(3*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 24
7*b**3*d*n**2*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
1960*n**3 + 322*n**2 + 28*n + 1) + 25*b**3*d*n*x*x**(3*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + b**3*
d*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 1260*b**3*e*n**6*x*x**(4*n)/(5040*n**7 + 13068*n**6
+ 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2952*b**3*e*
n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**
3 + 322*n**2 + 28*n + 1) + 2545*b**3*e*n**4*x*x**(4*n)/(5040*n**7 + 13068*n
6 + 13132*n5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1056*b**3
*e*n**3*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*
n**3 + 322*n**2 + 28*n + 1) + 226*b**3*e*n**2*x*x**(4*n)/(5040*n**7 + 13068
*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 24*b**3
*e*n*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**
3 + 322*n**2 + 28*n + 1) + b**3*e*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3780*b**2*c*d*n**6*
x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 8856*b**2*c*d*n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6
+ 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7635*b**2*c*
d*n**4*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n
3 + 322*n2 + 28*n + 1) + 3168*b**2*c*d*n**3*x*x**(4*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 678*b
2*c*d*n2*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
1960*n**3 + 322*n**2 + 28*n + 1) + 72*b**2*c*d*n*x*x**(4*n)/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b*
*2*c*d*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n
3 + 322*n2 + 28*n + 1) + 3024*b**2*c*e*n**6*x*x**(5*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236*
b**2*c*e*n**5*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
1960*n**3 + 322*n**2 + 28*n + 1) + 6432*b**2*c*e*n**4*x*x**(5*n)/(5040*n**
7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1)
+ 2775*b**2*c*e*n**3*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769
*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 621*b**2*c*e*n**2*x*x**(5*n)/(50
40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
+ 1) + 69*b**2*c*e*n*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 676
9*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b**2*c*e*x*x**(5*n)/(5040*n**
7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1)
+ 3024*b*c**2*d*n**6*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769
*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236*b*c**2*d*n**5*x*x**(5*n)/(5
040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*
n + 1) + 6432*b*c**2*d*n**4*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5
+ 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2775*b*c**2*d*n**3*x*x**(
5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**

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2 + 28*n + 1) + 621*b*c**2*d*n**2*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 69*b*c**2*d*n*x*x**
(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n*
*2 + 28*n + 1) + 3*b*c**2*d*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5
 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2520*b*c**2*e*n**6*x*x**(
6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 6114*b*c**2*e*n**5*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5547*b*c**2*e*n**4
*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 2460*b*c**2*e*n**3*x*x**(6*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 570*b*c**2*
e*n**2*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n
**3 + 322*n**2 + 28*n + 1) + 66*b*c**2*e*n*x*x**(6*n)/(5040*n**7 + 13068*n*
*6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b*c**2*e
*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 840*c**3*d*n**6*x*x**(6*n)/(5040*n**7 + 13068*n**6 +
 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2038*c**3*d*n*
*5*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
+ 322*n**2 + 28*n + 1) + 1849*c**3*d*n**4*x*x**(6*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 820*c**3*d*
n**3*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**
3 + 322*n**2 + 28*n + 1) + 190*c**3*d*n**2*x*x**(6*n)/(5040*n**7 + 13068*n*
*6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 22*c**3*d*
n*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
 322*n**2 + 28*n + 1) + c**3*d*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n
**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 720*c**3*e*n**6*x*x**(
7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 1764*c**3*e*n**5*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 13132
*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1624*c**3*e*n**4*x*x
**(7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*
n**2 + 28*n + 1) + 735*c**3*e*n**3*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 175*c**3*e*n**2*x*
x**(7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322
*n**2 + 28*n + 1) + 21*c**3*e*n*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 13132*
n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + c**3*e*x*x**(7*n)/(50
40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
 + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.77

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx = a^3dx + \frac{c^3ex^{7n+1}}{7n+1} + \frac{c^3dx^{6n+1}}{6n+1} + \frac{3bc^2ex^{6n+1}}{6n+1} + \frac{3bc^2dx^{5n+1}}{5n+1} + \frac{3b^2cex^{5n+1}}{5n+1} + \frac{3ac^2ex^{5n+1}}{5n+1} + \frac{3b^2cdx^{4n+1}}{4n+1} + \frac{3ac^2dx^{4n+1}}{4n+1} + \frac{b^3ex^{4n+1}}{4n+1} + \frac{6abcex^{4n+1}}{4n+1} + \frac{b^3dx^{3n+1}}{3n+1} + \frac{6abcdx^{3n+1}}{3n+1} + \frac{3ab^2ex^{3n+1}}{3n+1} + \frac{3a^2cex^{3n+1}}{3n+1} + \frac{3ab^2dx^{2n+1}}{2n+1} + \frac{3a^2cdx^{2n+1}}{2n+1} + \frac{3a^2bex^{2n+1}}{2n+1} + \frac{3a^2bdx^{n+1}}{n+1} + \frac{a^3ex^{n+1}}{n+1}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] a^3*d*x + c^3*e*x^(7*n + 1)/(7*n + 1) + c^3*d*x^(6*n + 1)/(6*n + 1) + 3*b*c
^2*e*x^(6*n + 1)/(6*n + 1) + 3*b*c^2*d*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*e*x^
(5*n + 1)/(5*n + 1) + 3*a*c^2*e*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*d*x^(4*n +
1)/(4*n + 1) + 3*a*c^2*d*x^(4*n + 1)/(4*n + 1) + b^3*e*x^(4*n + 1)/(4*n + 1
) + 6*a*b*c*e*x^(4*n + 1)/(4*n + 1) + b^3*d*x^(3*n + 1)/(3*n + 1) + 6*a*b*c
*d*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*e*x^(3*n + 1)/(3*n + 1) + 3*a^2*c*e*x^(3
*n + 1)/(3*n + 1) + 3*a*b^2*d*x^(2*n + 1)/(2*n + 1) + 3*a^2*c*d*x^(2*n + 1)
/(2*n + 1) + 3*a^2*b*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*d*x^(n + 1)/(n + 1)
+ a^3*e*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2064 vs. 2(218) = 436.

Time = 0.38 (sec) , antiderivative size = 2064, normalized size of antiderivative = 9.47

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] (5040*a^3*d*n^7*x + 720*c^3*e*n^6*x*x^(7*n) + 840*c^3*d*n^6*x*x^(6*n) + 252
0*b*c^2*e*n^6*x*x^(6*n) + 3024*b*c^2*d*n^6*x*x^(5*n) + 3024*b^2*c*e*n^6*x*x
^(5*n) + 3024*a*c^2*e*n^6*x*x^(5*n) + 3780*b^2*c*d*n^6*x*x^(4*n) + 3780*a*c
^2*d*n^6*x*x^(4*n) + 1260*b^3*e*n^6*x*x^(4*n) + 7560*a*b*c*e*n^6*x*x^(4*n)
```

+ 1680*b^3*d*n^6*x*x^(3*n) + 10080*a*b*c*d*n^6*x*x^(3*n) + 5040*a*b^2*e*n^6
*x*x^(3*n) + 5040*a^2*c*e*n^6*x*x^(3*n) + 7560*a*b^2*d*n^6*x*x^(2*n) + 7560
*a^2*c*d*n^6*x*x^(2*n) + 7560*a^2*b*e*n^6*x*x^(2*n) + 15120*a^2*b*d*n^6*x*x
^n + 5040*a^3*e*n^6*x*x^n + 13068*a^3*d*n^6*x + 1764*c^3*e*n^5*x*x^(7*n) +
2038*c^3*d*n^5*x*x^(6*n) + 6114*b*c^2*e*n^5*x*x^(6*n) + 7236*b*c^2*d*n^5*x*
x^(5*n) + 7236*b^2*c*e*n^5*x*x^(5*n) + 7236*a*c^2*e*n^5*x*x^(5*n) + 8856*b^
2*c*d*n^5*x*x^(4*n) + 8856*a*c^2*d*n^5*x*x^(4*n) + 2952*b^3*e*n^5*x*x^(4*n)
 + 17712*a*b*c*e*n^5*x*x^(4*n) + 3796*b^3*d*n^5*x*x^(3*n) + 22776*a*b*c*d*n
^5*x*x^(3*n) + 11388*a*b^2*e*n^5*x*x^(3*n) + 11388*a^2*c*e*n^5*x*x^(3*n) +
15822*a*b^2*d*n^5*x*x^(2*n) + 15822*a^2*c*d*n^5*x*x^(2*n) + 15822*a^2*b*e*n
^5*x*x^(2*n) + 24084*a^2*b*d*n^5*x*x^n + 8028*a^3*e*n^5*x*x^n + 13132*a^3*d
*n^5*x + 1624*c^3*e*n^4*x*x^(7*n) + 1849*c^3*d*n^4*x*x^(6*n) + 5547*b*c^2*e
*n^4*x*x^(6*n) + 6432*b*c^2*d*n^4*x*x^(5*n) + 6432*b^2*c*e*n^4*x*x^(5*n) +
6432*a*c^2*e*n^4*x*x^(5*n) + 7635*b^2*c*d*n^4*x*x^(4*n) + 7635*a*c^2*d*n^4*
x*x^(4*n) + 2545*b^3*e*n^4*x*x^(4*n) + 15270*a*b*c*e*n^4*x*x^(4*n) + 3112*b
^3*d*n^4*x*x^(3*n) + 18672*a*b*c*d*n^4*x*x^(3*n) + 9336*a*b^2*e*n^4*x*x^(3*
n) + 9336*a^2*c*e*n^4*x*x^(3*n) + 11787*a*b^2*d*n^4*x*x^(2*n) + 11787*a^2*c
*d*n^4*x*x^(2*n) + 11787*a^2*b*e*n^4*x*x^(2*n) + 15312*a^2*b*d*n^4*x*x^n +
5104*a^3*e*n^4*x*x^n + 6769*a^3*d*n^4*x + 735*c^3*e*n^3*x*x^(7*n) + 820*c^3
*d*n^3*x*x^(6*n) + 2460*b*c^2*e*n^3*x*x^(6*n) + 2775*b*c^2*d*n^3*x*x^(5*n)
 + 2775*b^2*c*e*n^3*x*x^(5*n) + 2775*a*c^2*e*n^3*x*x^(5*n) + 3168*b^2*c*d*n^
3*x*x^(4*n) + 3168*a*c^2*d*n^3*x*x^(4*n) + 1056*b^3*e*n^3*x*x^(4*n) + 6336*
a*b*c*e*n^3*x*x^(4*n) + 1219*b^3*d*n^3*x*x^(3*n) + 7314*a*b*c*d*n^3*x*x^(3*
n) + 3657*a*b^2*e*n^3*x*x^(3*n) + 3657*a^2*c*e*n^3*x*x^(3*n) + 4260*a*b^2*d
*n^3*x*x^(2*n) + 4260*a^2*c*d*n^3*x*x^(2*n) + 4260*a^2*b*e*n^3*x*x^(2*n) +
4995*a^2*b*d*n^3*x*x^n + 1665*a^3*e*n^3*x*x^n + 1960*a^3*d*n^3*x + 175*c^3*
e*n^2*x*x^(7*n) + 190*c^3*d*n^2*x*x^(6*n) + 570*b*c^2*e*n^2*x*x^(6*n) + 621
*b*c^2*d*n^2*x*x^(5*n) + 621*b^2*c*e*n^2*x*x^(5*n) + 621*a*c^2*e*n^2*x*x^(5
*n) + 678*b^2*c*d*n^2*x*x^(4*n) + 678*a*c^2*d*n^2*x*x^(4*n) + 226*b^3*e*n^2
*x*x^(4*n) + 1356*a*b*c*e*n^2*x*x^(4*n) + 247*b^3*d*n^2*x*x^(3*n) + 1482*a*
b*c*d*n^2*x*x^(3*n) + 741*a*b^2*e*n^2*x*x^(3*n) + 741*a^2*c*e*n^2*x*x^(3*n)
 + 810*a*b^2*d*n^2*x*x^(2*n) + 810*a^2*c*d*n^2*x*x^(2*n) + 810*a^2*b*e*n^2*
x*x^(2*n) + 885*a^2*b*d*n^2*x*x^n + 295*a^3*e*n^2*x*x^n + 322*a^3*d*n^2*x +
 21*c^3*e*n*x*x^(7*n) + 22*c^3*d*n*x*x^(6*n) + 66*b*c^2*e*n*x*x^(6*n) + 69*
b*c^2*d*n*x*x^(5*n) + 69*b^2*c*e*n*x*x^(5*n) + 69*a*c^2*e*n*x*x^(5*n) + 72*
b^2*c*d*n*x*x^(4*n) + 72*a*c^2*d*n*x*x^(4*n) + 24*b^3*e*n*x*x^(4*n) + 144*a
*b*c*e*n*x*x^(4*n) + 25*b^3*d*n*x*x^(3*n) + 150*a*b*c*d*n*x*x^(3*n) + 75*a*
b^2*e*n*x*x^(3*n) + 75*a^2*c*e*n*x*x^(3*n) + 78*a*b^2*d*n*x*x^(2*n) + 78*a^
2*c*d*n*x*x^(2*n) + 78*a^2*b*e*n*x*x^(2*n) + 81*a^2*b*d*n*x*x^n + 27*a^3*e*
n*x*x^n + 28*a^3*d*n*x + c^3*e*x*x^(7*n) + c^3*d*x*x^(6*n) + 3*b*c^2*e*x*x^
(6*n) + 3*b*c^2*d*x*x^(5*n) + 3*b^2*c*e*x*x^(5*n) + 3*a*c^2*e*x*x^(5*n) + 3
*b^2*c*d*x*x^(4*n) + 3*a*c^2*d*x*x^(4*n) + b^3*e*x*x^(4*n) + 6*a*b*c*e*x*x^
(4*n) + b^3*d*x*x^(3*n) + 6*a*b*c*d*x*x^(3*n) + 3*a*b^2*e*x*x^(3*n) + 3*a^2
*c*e*x*x^(3*n) + 3*a*b^2*d*x*x^(2*n) + 3*a^2*c*d*x*x^(2*n) + 3*a^2*b*e*x*x^
(2*n) + 3*a^2*b*d*x*x^n + a^3*e*x*x^n + a^3*d*x)/(5040*n^7 + 13068*n^6 + 13

```
132*n^5 + 6769*n^4 + 1960*n^3 + 322*n^2 + 28*n + 1)
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.04

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx = a^3\,d\,x+\frac{x\,x^n\,\left(e\,a^3+3\,b\,d\,a^2\right)}{n+1}+\frac{x\,x^{2\,n}\,\left(3\,e\,a^2\,b+3\,c\,d\,a^2+3\,d\,a\,b^2\right)}{2\,n+1}+\frac{x\,x^{5\,n}\,\left(3\,e\,b^2\,c+3\,d\,b\,c^2+3\,a\,e\,c^2\right)}{5\,n+1}+\frac{x\,x^{3\,n}\,\left(3\,c\,e\,a^2+3\,e\,a\,b^2+6\,c\,d\,a\,b+d\,b^3\right)}{3\,n+1}+\frac{x\,x^{4\,n}\,\left(e\,b^3+3\,d\,b^2\,c+6\,a\,e\,b\,c+3\,a\,d\,c^2\right)}{4\,n+1}+\frac{x\,x^{6\,n}\,\left(d\,c^3+3\,b\,e\,c^2\right)}{6\,n+1}+\frac{c^3\,e\,x\,x^{7\,n}}{7\,n+1}$$

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] a^3*d*x + (x*x^n*(a^3*e + 3*a^2*b*d))/(n + 1) + (x*x^(2*n)*(3*a*b^2*d + 3*a
^2*b*e + 3*a^2*c*d))/(2*n + 1) + (x*x^(5*n)*(3*a*c^2*e + 3*b*c^2*d + 3*b^2*
c*e))/(5*n + 1) + (x*x^(3*n)*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d))/(
3*n + 1) + (x*x^(4*n)*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e))/(4*n + 1
) + (x*x^(6*n)*(c^3*d + 3*b*c^2*e))/(6*n + 1) + (c^3*e*x*x^(7*n))/(7*n + 1)
```

3.69 $\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 308

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)} + \frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2c\ldots}{b-\sqrt{b\ldots}}\right.}{c^2\left(b-\sqrt{b^2-4ac}\right)} + \frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3-\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2c\ldots}{b+\sqrt{b\ldots}}\right.}{c^2\left(b+\sqrt{b^2-4ac}\right)}$$

```
[Out] e^2*(-b*e+3*c*d)*x/c^2+e^3*x^(1+n)/c/(1+n)+x*hypergeom([1, 1/n],[1+1/n],-2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2*e-3*b*c*d*e^2+b^2*e^3-a*c*e^3+(-b*
e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^2/(b-(-4*a
*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(3*c^2*d^2*e-3*b*c*d*e^2+b^2*e^3-a*c*e^3-(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*
e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used

$= \{1438, 1436, 251\}$

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{x\left(\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e\right) \text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1+\frac{1}{n}, -\frac{2c}{b-\sqrt{b}}\right.}{c^2\left(b-\sqrt{b^2-4ac}\right)}$$
$$+ \frac{x\left(-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e\right) \text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1+\frac{1}{n}, -\right.}{c^2\left(\sqrt{b^2-4ac}+b\right)}$$
$$+ \frac{e^2x(3cd-be)}{c^2} + \frac{e^3x^{n+1}}{c(n+1)}$$

```
[In] Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^(1 + n))/(c*(1 + n)) + ((3*c^2*d^2*e - 3
*b*c*d*e^2 + b^2*e^3 - a*c*e^3 + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b
*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c^2*(b - Sqrt[b^2 - 4*a*c])) + ((3*c
^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3 - ((2*c*d - b*e)*(c^2*d^2 + b^2*
e^2 - c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c^2*(b + Sqrt[b^2 - 4*a*
c]))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^n}{c}+\frac{c^2d^3-3acde^2+abe^3+\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3\right)x^n}{c^2\left(a+bx^n+cx^{2n}\right)}\right)dx$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\int \frac{c^2d^3-3acde^2+abe^3+\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3\right)x^n}{a+bx^n+cx^{2n}}\,dx}{c^2}$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2c^2}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2c^2}$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c^2\left(b+\sqrt{b^2-4ac}\right)}$$

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \frac{x\left(e^2(3cd-be)+\frac{ce^3x^n}{1+n}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}\right.}{c^2}$$

```
[In] Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(e^2*(3*c*d - b*e) + (c*e^3*x^n)/(1 + n) + ((3*c^2*d^2*e - 3*b*c*d*e^2 +
 b^2*e^3 - a*c*e^3 + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))
)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]) + ((3*c^2*d^2*e - 3*b*c*d*e
^2 + b^2*e^3 - a*c*e^3 + ((-2*c*d + b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*
a*e)))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])))/c^2
```

Maple [F]

$$\int \frac{(d+e\,x^n)^3}{a+b\,x^n+c\,x^{2n}}dx$$

[In] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)

[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)

Fricas [F]

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+bx^n+a}\,dx$$

[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c*x^(2*n) + b*x^n + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

[In] integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+bx^n+a}\,dx$$

[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] (c*e^3*x*x^n + (3*c*d*e^2*(n + 1) - b*e^3*(n + 1))*x)/(c^2*(n + 1)) - integrate(-(c^2*d^3 - (3*c*d*e^2 - b*e^3)*a + (3*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3)*x^n)/(c^3*x^(2*n) + b*c^2*x^n + a*c^2), x)

Giac [F]

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^3}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x)
[Out] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n)), x)
```

3.70 $\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 224

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \frac{e^2x}{c} + \frac{\left(2cde-be^2+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} + \frac{\left(2cde-be^2-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)}$$

```
[Out] e^2*x/c+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*
d*e-b*e^2+(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c/(b-(-4*
a*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)
))*(2*c*d*e-b*e^2+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/
c/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used

$= \{1438, 1436, 251\}$

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{x\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{x\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{e^2x}{c}$$

```
[In] Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (e^2*x)/c + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + ((2*c*d*e - b*e^2 - (2
*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b + Sqr
t[b^2 - 4*a*c]))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2}{c} + \frac{cd^2 - ae^2 + (2cde - be^2)\,x^n}{c\,(a + bx^n + cx^{2n})}\right) dx$$

$$= \frac{e^2x}{c} + \frac{\int \frac{cd^2-ae^2+(2cde-be^2)x^n}{a+bx^n+cx^{2n}}\,dx}{c}$$

$$= \frac{e^2x}{c} + \frac{\left(2cde - be^2 - \frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2c} + \frac{\left(2cde - be^2 + \frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2c}$$

$$= \frac{e^2x}{c} + \frac{\left(2cde - be^2 + \frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b - \sqrt{b^2-4ac}\right)} + \frac{\left(2cde - be^2 - \frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(b + \sqrt{b^2-4ac}\right)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \frac{x\left(e^2 + \frac{\left(2cde-be^2+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}} + \frac{\left(2cde-be^2-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{Hyp}}{b+\sqrt{b}}\right.}{c}$$

```
[In] Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(e^2 + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqr
t[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]) + ((2*c*d*e - b*e^2 - (2*c^2*d^2
 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c
])))/c
```

Maple [F]

$$\int \frac{(d+e\,x^n)^2}{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral((d + e*x**n)**2/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^n)/(c^2*x^(2*n) +
 b*c*x^n + a*c), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^2}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x)
[Out] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n)), x)
```

3.71 $\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx=\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b+\sqrt{b^2-4ac}}$$

```
[Out] x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(e+(-b*e+2*c*
d)/(-4*a*c+b^2)^(1/2))/(b-(-4*a*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],
-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b+(-4*
a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1436, 251}

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx=\frac{x\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{x\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b}$$

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]) + ((e -
```

```
 (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx\\&\quad+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx\\&=\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b+\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.87

$$\int\frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx=\frac{x\left(\left(bd+\sqrt{b^2-4ac}d-2ae\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\ldots}{2a\sqrt{b^2-4ac}}$$

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^2 - 4*a*c]*d +
 2*a*e)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c])
```

Maple [F]

$$\int \frac{d+e\,x^n}{a+b\,x^n+c\,x^{2n}}dx$$

[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)

[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)

Fricas [F]

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx = \int \frac{ex^n+d}{cx^{2\,n}+bx^n+a}\,dx$$

[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)

Sympy [F]

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx = \int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx$$

[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)

[Out] Integral((d + e*x**n)/(a + b*x**n + c*x**(2*n)), x)

Maxima [F]

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx = \int \frac{ex^n+d}{cx^{2\,n}+bx^n+a}\,dx$$

[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)

Giac [F]

$$\int \frac{d + ex^n}{a + bx^n + cx^{2n}}\, dx = \int \frac{ex^n + d}{cx^{2\,n} + bx^n + a}\, dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d + ex^n}{a + bx^n + cx^{2n}}\, dx = \int \frac{d + e\,x^n}{a + b\,x^n + c\,x^{2\,n}}\, dx$$

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n)),x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n)), x)
```

3.72 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 243

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx = -\frac{c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)} - \frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)} + \frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)}$$

```
[Out] e^2*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)-c*x*hyperg
eom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(e+(-b*e+2*c*d)/(-4*a
*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))-c*x*hypergeom([1,
 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))/(a*e^2-b*d*e+c*d^2)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used

= {1438, 251, 1436}

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$$
$$= -\frac{cx\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)}$$
$$-\frac{cx\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(\sqrt{b^2-4ac}+b\right)(ae^2-bde+cd^2)}$$
$$+\frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)}$$

```
[In] Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 -
4*a*c])*(c*d^2 - b*d*e + a*e^2))) - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c]
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/((b + Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)) + (e^2*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^n)}+\frac{cd-be-cex^n}{(cd^2-bde+ae^2)(a+bx^n+cx^{2n})}\right)dx\\
&= \frac{\int \frac{cd-be-cex^n}{a+bx^n+cx^{2n}}\,dx}{cd^2-bde+ae^2}+\frac{e^2\int \frac{1}{d+ex^n}\,dx}{cd^2-bde+ae^2}\\
&= \frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)}-\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2(cd^2-bde+ae^2)}\\
&\quad-\frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2(cd^2-bde+ae^2)}\\
&= -\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)}\\
&\quad-\frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.82

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$$
$$=\frac{x\left(-\frac{c\left(e+\frac{-2cd+be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}-\frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b+\sqrt{b^2-4ac}}+\frac{e^2 \cdots}{\cdots}\right.}{cd^2+e(-bd+ae)}$$

```
[In] Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (x*(-((c*(e + (-2*c*d + b*e)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]))
 - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c]) + (e
^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d))/(c*d^2 + e*(
-(b*d) + a*e))
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)\,(a+b\,x^n+c\,x^{2n})}dx$$

[In] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)

[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)

Fricas [F]

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)(ex^n+d)}\,dx$$

[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral(1/(b*e*x^(2*n) + a*d + (c*e*x^n + c*d)*x^(2*n) + (b*d + a*e)*x^n), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)(ex^n+d)}\,dx$$

[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)

Giac [F]

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(d+e\,x^n)\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

```
[In] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))), x)
```

3.73 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 368

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = -\frac{c\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2\cdots}{b-\sqrt{\cdots}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{c\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2\cdots}{b+\sqrt{\cdots}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}$$
$$+\frac{e^2(2cd-be)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(cd^2-bde+ae^2\right)^2}$$
$$+\frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(cd^2-bde+ae^2\right)}$$

```
[Out] e^2*(-b*e+2*c*d)*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^
2)^2+e^2*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)-c*x
*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d^2+b*e
^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*
e+c*d^2)^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*x*hypergeom([1, 1/n],[1+1/n],
-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*
c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c-b*(-4*
a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1438, 251, 1436}

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx =$$
$$-\frac{cx\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2}{b-\sqrt{}}\right.}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$
$$-\frac{cx\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{}{b+}\right.}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$
$$+\frac{e^2x(2cd-be)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^2}$$
$$+\frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4
*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
a*e^2)^2)) - (c*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - S
qrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2
 - b*d*e + a*e^2)^2) + (e^2*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hypergeomet
ric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2}{(cd^2 - bde + ae^2)(d + ex^n)^2} - \frac{e^2(-2cd + be)}{(cd^2 - bde + ae^2)^2 (d + ex^n)} + \frac{c^2d^2 - 2bcde + b^2e^2 - ace^2 - (2c^2de - bce^2)x^n}{(cd^2 - bde + ae^2)^2 (a + bx^n + cx^{2n})} \right) dx$$

$$= \frac{\int \frac{c^2d^2-2bcde+b^2e^2-ace^2-(2c^2de-bce^2)x^n}{a+bx^n+cx^{2n}}\, dx}{(cd^2 - bde + ae^2)^2} + \frac{(e^2(2cd - be)) \int \frac{1}{d+ex^n}\, dx}{(cd^2 - bde + ae^2)^2} + \frac{e^2 \int \frac{1}{(d+ex^n)^2}\, dx}{cd^2 - bde + ae^2}$$

$$= \frac{e^2(2cd - be)x\, {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d\,(cd^2 - bde + ae^2)^2} + \frac{e^2x\, {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\,(cd^2 - bde + ae^2)} - \frac{\left(c\left(2c^2d^2 + b\left(b - \sqrt{b^2 - 4ac}\right)e^2 - 2ce\left(bd - \sqrt{b^2 - 4ac}d + ae\right)\right)\right) \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\, dx}{2\sqrt{b^2 - 4ac}\,(cd^2 - bde + ae^2)^2} + \frac{\left(c\left(2c^2d^2 + b\left(b + \sqrt{b^2 - 4ac}\right)e^2 - 2ce\left(bd + \sqrt{b^2 - 4ac}d + ae\right)\right)\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\, dx}{2\sqrt{b^2 - 4ac}\,(cd^2 - bde + ae^2)^2}$$

$$= \frac{c\left(2c^2d^2 + b\left(b + \sqrt{b^2 - 4ac}\right)e^2 - 2ce\left(bd + \sqrt{b^2 - 4ac}d + ae\right)\right) x\, {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac}\left(b - \sqrt{b^2 - 4ac}\right)(cd^2 - bde + ae^2)^2} - \frac{c\left(2c^2d^2 + b\left(b - \sqrt{b^2 - 4ac}\right)e^2 - 2ce\left(bd - \sqrt{b^2 - 4ac}d + ae\right)\right) x\, {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac}\left(b + \sqrt{b^2 - 4ac}\right)(cd^2 - bde + ae^2)^2} + \frac{e^2(2cd - be)x\, {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d\,(cd^2 - bde + ae^2)^2} + \frac{e^2x\, {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\,(cd^2 - bde + ae^2)}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx$$

$$= \frac{x\left(\frac{c\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{-b^2+4ac+b\sqrt{b^2-4ac}}+\frac{c\left(-2c^2d^2+b\left(-b+\sqrt{b^2-4ac}\right)e^2+\right.}{}\right.}{}$$

```
[In] Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (x*((c*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 -
 4*a*c]*d + a*e))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])/(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (c*(-2*c^2*d^2
+ b*(-b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (e^2*(2*c*d - b*e)*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d + (e^2*(c*d^2 + e*(-(b*d) + a*e
))*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d^2))/(c*d^2 + e
*(-(b*d) + a*e))^2
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)^2\,(a+b\,x^n+c\,x^{2n})}dx$$

```
[In] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*e^2*x^(3*n) + a*d^2 + (c*e^2*x^(2*n) + 2*c*d*e*x^n + c*d^2)*x
^(2*n) + (2*b*d*e + a*e^2)*x^(2*n) + (b*d^2 + 2*a*d*e)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] e^2*x/(c*d^4*n - b*d^3*e*n + a*d^2*e^2*n + (c*d^3*e*n - b*d^2*e^2*n + a*d*e
^3*n)*x^n) + (c*d^2*e^2*(3*n - 1) - b*d*e^3*(2*n - 1) + a*e^4*(n - 1))*inte
grate(1/(c^2*d^6*n - 2*b*c*d^5*e*n + b^2*d^4*e^2*n + a^2*d^2*e^4*n + 2*(c*d
^4*e^2*n - b*d^3*e^3*n)*a + (c^2*d^5*e*n - 2*b*c*d^4*e^2*n + b^2*d^3*e^3*n
+ a^2*d*e^5*n + 2*(c*d^3*e^3*n - b*d^2*e^4*n)*a)*x^n), x) + integrate((c^2*
d^2 - 2*b*c*d*e + b^2*e^2 - a*c*e^2 - (2*c^2*d*e - b*c*e^2)*x^n)/(a^3*e^4 +
 2*(c*d^2*e^2 - b*d*e^3)*a^2 + (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*a + (c
^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + a^2*c*e^4 + 2*(c^2*d^2*e^2 - b*c*d
*e^3)*a)*x^(2*n) + (b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + a^2*b*e^4 + 2
*(b*c*d^2*e^2 - b^2*d*e^3)*a)*x^n), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(d+e\,x^n)^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))), x)
```

3.74 $\int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 552

$$\int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})}\,dx = -\frac{c\left(2c^3d^3-b^2\left(b+\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd+\sqrt{b^2-4ac}d+2ae\right)+ce^2\left(3b^2d+a\sqrt{b^2-4ac}e+3b\left(\sqrt{b^2\right.\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae\right.} - \frac{c\left(2c^3d^3-b^2\left(b-\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd-\sqrt{b^2-4ac}d+2ae\right)+ce^2\left(3b^2d-3b\sqrt{b^2-4ac}d+3abe-\right.\right.}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae\right.} + \frac{e^2(3c^2d^2+b^2e^2-ce(3bd+ae))\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(cd^2-bde+ae^2\right)^3} + \frac{e^2(2cd-be)x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(cd^2-bde+ae^2\right)^2} + \frac{e^2x\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^3\left(cd^2-bde+ae^2\right)}$$

```
[Out] e^2*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))*x*hypergeom([1, 1/n],[1+1/n],-e*x^n
/d)/d/(a*e^2-b*d*e+c*d^2)^3+e^2*(-b*e+2*c*d)*x*hypergeom([2, 1/n],[1+1/n],-
e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^2+e^2*x*hypergeom([3, 1/n],[1+1/n],-e*x^n/
d)/d^3/(a*e^2-b*d*e+c*d^2)-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*(2*c^3*d^3-b^2*e^3*(b-(-4*a*c+b^2)^(1/2))-3*c^2*d*e*(b*d+2*
a*e-d*(-4*a*c+b^2)^(1/2))+c*e^2*(3*b^2*d+3*a*b*e-3*b*d*(-4*a*c+b^2)^(1/2)-(
-4*a*c+b^2)^(1/2)*a*e))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/
2))-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c^3*
d^3-b^2*e^3*(b+(-4*a*c+b^2)^(1/2))-3*c^2*d*e*(b*d+2*a*e+d*(-4*a*c+b^2)^(1/2
))+c*e^2*(3*b^2*d+(-4*a*c+b^2)^(1/2)*a*e+3*b*(a*e+d*(-4*a*c+b^2)^(1/2))))/(
a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1438, 251, 1436}

$$\int \frac{1}{(d+ex^n)^3\,(a+bx^n+cx^{2n})}\,dx$$
$$= \frac{e^2x(-ce(ae+3bd)+b^2e^2+3c^2d^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)^3}$$
$$-\frac{cx\left(-3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(3b\left(d\sqrt{b^2-4ac}+ae\right)+ae\sqrt{b^2-4ac}+3b^2d\right)-b^2e^3\left(\sqrt{b^2-4\right.}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd}$$
$$-\frac{cx\left(-3c^2de\left(-d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)-b^2e^3\left(b-\right.}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+c}$$
$$+\frac{e^2x(2cd-be)\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\,(ae^2-bde+cd^2)^2}$$
$$+\frac{e^2x\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^3\,(ae^2-bde+cd^2)}$$

```
[In] Int[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d + Sqrt[b
^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt
[b^2 - 4*a*c]*d + a*e)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 -
 b*d*e + a*e^2)^3)) - (c*(2*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 3*c
^2*d*e*(b*d - Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2
- 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(3*c^2*d^2 + b^2*e^2 - c*e*
(3*b*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d
*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(2*c*d - b*e)*x*Hypergeometric2F1[2, n^(
-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hy
pergeometric2F1[3, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^3*(c*d^2 - b*d*e +
 a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^n)^3} - \frac{e^2(-2cd+be)}{(cd^2-bde+ae^2)^2(d+ex^n)^2} + \frac{e^2(3c^2d^2+b^2e^2-ce(3bd+ae))}{(cd^2-bde+ae^2)^3(d+ex^n)} + \frac{c^3d^3-3bc^2d^2e+3b^2cde^2-3ac^2de^2-b^3e^3+2abce^3-(3c^3d^2e-3bc^2de^2+b^2ce^3-ac^2e^3)x^n}{(cd^2-bde+ae^2)^3(a+bx^n+cx^{2n})}\right)dx$$

$$= \frac{\int \frac{c^3d^3-3bc^2d^2e+3b^2cde^2-3ac^2de^2-b^3e^3+2abce^3-(3c^3d^2e-3bc^2de^2+b^2ce^3-ac^2e^3)x^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^3} + \frac{(e^2(2cd-be))\int \frac{1}{(d+ex^n)^2}\,dx}{(cd^2-bde+ae^2)^2} + \frac{e^2\int \frac{1}{(d+ex^n)^3}\,dx}{cd^2-bde+ae^2} + \frac{(e^2(3c^2d^2+b^2e^2-ce(3bd+ae)))\int \frac{1}{d+ex^n}\,dx}{(cd^2-bde+ae^2)^3}$$

$$= \frac{e^2(3c^2d^2+b^2e^2-ce(3bd+ae))\,x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)^3} + \frac{e^2(2cd-be)x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\,(cd^2-bde+ae^2)^2} + \frac{e^2x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^3\,(cd^2-bde+ae^2)}$$

$$- \frac{\left(c\left(2c^3d^3-b^2\left(b-\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd-\sqrt{b^2-4ac}d+2ae\right)+ce^2\left(3b^2d-3b\sqrt{b^2-4ac}d\right.\right.\right.}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^3}$$

$$+ \frac{\left(c\left(2c^3d^3-b^2\left(b+\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd+\sqrt{b^2-4ac}d+2ae\right)+ce^2\left(3b^2d+a\sqrt{b^2-4ac}e+\right.\right.\right.}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^3}$$

$$= \frac{c\left(2c^3d^3 - b^2\left(b + \sqrt{b^2-4ac}\right)e^3 - 3c^2de\left(bd + \sqrt{b^2-4ac}d + 2ae\right) + ce^2\left(3b^2d + a\sqrt{b^2-4ac}e + 3b(\right.\right.}{\sqrt{b^2-4ac}\left(b - \sqrt{b^2-4ac}\right)(cd^2 - bde + ae^2)^3}$$

$$- \frac{c\left(2c^3d^3 - b^2\left(b - \sqrt{b^2-4ac}\right)e^3 - 3c^2de\left(bd - \sqrt{b^2-4ac}d + 2ae\right) + ce^2\left(3b^2d - 3b\sqrt{b^2-4ac}d + \right.\right.}{\sqrt{b^2-4ac}\left(b + \sqrt{b^2-4ac}\right)(cd^2 - bde + ae^2)}$$

$$+ \frac{e^2(3c^2d^2 + b^2e^2 - ce(3bd + ae))\, x\, {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d\,(cd^2 - bde + ae^2)^3}$$

$$+ \frac{e^2(2cd - be)x\, {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\,(cd^2 - bde + ae^2)^2} + \frac{e^2x\, {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^3\,(cd^2 - bde + ae^2)}$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d + ex^n)^3\,(a + bx^n + cx^{2n})}\,dx$$

$$= \frac{x\left(\frac{c\left(-2c^3d^3 + b^2\left(b + \sqrt{b^2-4ac}\right)e^3 + 3c^2de\left(bd + \sqrt{b^2-4ac}d + 2ae\right) - ce^2\left(3b^2d + a\sqrt{b^2-4ac}e + 3b\left(\sqrt{b^2-4ac}d + ae\right)\right)\right)\text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1 + \frac{1}{n}\right.}{b^2 - 4ac - b\sqrt{b^2-4ac}}\right.}{}$$

```
[In] Integrate[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (x*((c*(-2*c^3*d^3 + b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqr
t[b^2 - 4*a*c]*d + 2*a*e) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(S
qrt[b^2 - 4*a*c]*d + a*e)))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (c*(2*
c^3*d^3 + b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 -
 4*a*c]*d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a
*Sqrt[b^2 - 4*a*c]*e))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (e^2*(3*c^2
*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1)
, -((e*x^n)/d)])/d + (e^2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*Hypergeo
metric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d^2 + (e^2*(c*d^2 + e*(-(b*
d) + a*e))^2*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d^3))/
(c*d^2 + e*(-(b*d) + a*e))^3
```

Maple [F]

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{3} \left(a+b\,x^{n}+c\,x^{2 n}\right)}d x$$

```
[In] int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
[Out] int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} \, dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^3} \, dx$$

```
[In] integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(1/(b*e^3*x^(4*n) + a*d^3 + (3*b*d*e^2 + a*e^3)*x^(3*n) + (c*e^3*x^
(3*n) + 3*c*d*e^2*x^(2*n) + 3*c*d^2*e*x^n + c*d^3)*x^(2*n) + 3*(b*d^2*e + a
*d*e^2)*x^(2*n) + (b*d^3 + 3*a*d^2*e)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} \, dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^3} \, dx$$

```
[In] integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] ((12*n^2 - 7*n + 1)*c^2*d^4*e^2 - 2*(8*n^2 - 6*n + 1)*b*c*d^3*e^3 + (6*n^2
- 5*n + 1)*b^2*d^2*e^4 + (2*n^2 - 3*n + 1)*a^2*e^6 + 2*((3*n^2 - 5*n + 1)*c
*d^2*e^4 - (3*n^2 - 4*n + 1)*b*d*e^5)*a)*integrate(1/2/(c^3*d^9*n^2 - 3*b*c
^2*d^8*e*n^2 + 3*b^2*c*d^7*e^2*n^2 - b^3*d^6*e^3*n^2 + a^3*d^3*e^6*n^2 + 3*
(c*d^5*e^4*n^2 - b*d^4*e^5*n^2)*a^2 + 3*(c^2*d^7*e^2*n^2 - 2*b*c*d^6*e^3*n^
2 + b^2*d^5*e^4*n^2)*a + (c^3*d^8*e*n^2 - 3*b*c^2*d^7*e^2*n^2 + 3*b^2*c*d^6
```

```
*e^3*n^2 - b^3*d^5*e^4*n^2 + a^3*d^2*e^7*n^2 + 3*(c*d^4*e^5*n^2 - b*d^3*e^6
*n^2)*a^2 + 3*(c^2*d^6*e^3*n^2 - 2*b*c*d^5*e^4*n^2 + b^2*d^4*e^5*n^2)*a)*x^
n), x) + 1/2*((c*d^2*e^3*(6*n - 1) - b*d*e^4*(4*n - 1) + a*e^5*(2*n - 1))*x
*x^n + (c*d^3*e^2*(7*n - 1) - b*d^2*e^3*(5*n - 1) + a*d*e^4*(3*n - 1))*x)/(
c^2*d^8*n^2 - 2*b*c*d^7*e*n^2 + b^2*d^6*e^2*n^2 + a^2*d^4*e^4*n^2 + 2*(c*d^
6*e^2*n^2 - b*d^5*e^3*n^2)*a + (c^2*d^6*e^2*n^2 - 2*b*c*d^5*e^3*n^2 + b^2*d
^4*e^4*n^2 + a^2*d^2*e^6*n^2 + 2*(c*d^4*e^4*n^2 - b*d^3*e^5*n^2)*a)*x^(2*n)
 + 2*(c^2*d^7*e*n^2 - 2*b*c*d^6*e^2*n^2 + b^2*d^5*e^3*n^2 + a^2*d^3*e^5*n^2
 + 2*(c*d^5*e^3*n^2 - b*d^4*e^4*n^2)*a)*x^n) + integrate((c^3*d^3 - 3*b*c^2
*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3 - (3*c^2*d*e^2 - 2*b*c*e^3)*a - (3*c^3*d^2
*e - 3*b*c^2*d*e^2 + b^2*c*e^3 - a*c^2*e^3)*x^n)/(a^4*e^6 + 3*(c*d^2*e^4 -
b*d*e^5)*a^3 + 3*(c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e^4)*a^2 + (c^3*d^6
 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*a + (c^4*d^6 - 3*b*c^3*d^
5*e + 3*b^2*c^2*d^4*e^2 - b^3*c*d^3*e^3 + a^3*c*e^6 + 3*(c^2*d^2*e^4 - b*c*
d*e^5)*a^2 + 3*(c^3*d^4*e^2 - 2*b*c^2*d^3*e^3 + b^2*c*d^2*e^4)*a)*x^(2*n) +
 (b*c^3*d^6 - 3*b^2*c^2*d^5*e + 3*b^3*c*d^4*e^2 - b^4*d^3*e^3 + a^3*b*e^6 +
 3*(b*c*d^2*e^4 - b^2*d*e^5)*a^2 + 3*(b*c^2*d^4*e^2 - 2*b^2*c*d^3*e^3 + b^3
*d^2*e^4)*a)*x^n), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} \, dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^3} \, dx$$

```
[In] integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} \, dx = \int \frac{1}{(d+e\,x^n)^3\,(a+b\,x^n+c\,x^{2\,n})} \, dx$$

```
[In] int(1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))), x)
```

3.75 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 750

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$$
$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^n)}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$
$$+\frac{e^2\left(e+\frac{6cd-3be}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{\left((ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,(1-n)+\frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+4ac^2d(cd^2-3a}{\sqrt{b^2-4ac}}\right.}{ac\,(b^2-4ac)\left(b-\sqrt{b^2-4a\right.}$$
$$+\frac{e^2\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)}$$
$$+\frac{\left((ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,(1-n)-\frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+4ac^2d(cd^2-3a}{\sqrt{b^2-4ac}}\right.}{ac\,(b^2-4ac)\left(b+\sqrt{b^2-4a\right.}$$

```
[Out] x*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)-(a*b^2*e^3+2*a*
c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^n)/a/c/(-4*a*c+b^2)/n/(a+b*x^
n+c*x^(2*n))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))*(e+(-3*b*e+6*c*d)/(-4*a*c+b^2)^(1/2))/c/(b-(-4*a*c+b^2)^(1/2))+x*hyperg
eom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((a*b^2*e^3+2*a*c*e*(
-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*(1-n)+(b^2*c*d*(3*a*e^2*(1-3*n)-c*d^
2*(1-n))-a*b^3*e^3*(1-3*n)+4*a*c^2*d*(-3*a*e^2+c*d^2)*(1-2*n)+2*a*b*c*e*(a*
e^2*(2-5*n)+3*c*d^2*n))/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)/n/(b-(-4*a*c+b
^2)^(1/2))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
```

```
)*(e-3*(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))+x*hypergeo
m([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((a*b^2*e^3+2*a*c*e*(-a
*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*(1-n)+(-b^2*c*d*(3*a*e^2*(1-3*n)-c*d^2
*(1-n))+a*b^3*e^3*(1-3*n)-4*a*c^2*d*(-3*a*e^2+c*d^2)*(1-2*n)-2*a*b*c*e*(a*e
^2*(2-5*n)+3*c*d^2*n))/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)/n/(b+(-4*a*c+b^
2)^(1/2))
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 1444, 1436, 251}

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$$

$$= \frac{x(-(x^n(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)))-abe(ae^2+3cd^2)-2acd(cd^2-3ae^2)+b^2cd^3)}{acn\,(b^2-4ac)\,(a+bx^n+cx^{2n})}$$

$$+\frac{e^2x\left(\frac{6cd-3be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}$$

$$+\frac{e^2x\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}$$

$$+\frac{x\Big((1-n)\left(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)\right)+\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-\cdots\right.}{\sqrt{b^2-4ac}}}{acn\,(b^2-4ac)\left(b-\sqrt{b^2-4a\cdots\right.}}$$

$$+\frac{x\Big((1-n)\left(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)\right)-\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-\cdots\right.}{\sqrt{b^2-4ac}}}{acn\,(b^2-4ac)\left(\sqrt{b^2-4ac}+\cdots\right.}$$

```
[In] Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b^
2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n))/(a*c*(b^
2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (e^2*(e + (6*c*d - 3*b*e)/Sqrt[b^2
 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3*c*d^
2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) + (b^2*c*d*(3*a*e^2*(1 - 3*n)
 - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2)*(1 -
2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4*a*c])*x*Hyperg
eometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*
c*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*n) + (e^2*(e - (3*(2*c*d - b*e))/Sq
rt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(c*(b + Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*
(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) - (b^2*c*d*(3*a*e^2*(1
```

```
 - 3*n) - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2
)*(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4*a*c])*x
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)])/(a*c*(b^2 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^2d^3 - 3acde^2 + abe^3 + (3c^2d^2e - 3bcde^2 + b^2e^3 - ace^3)\,x^n}{c^2\,(a + bx^n + cx^{2n})^2} + \frac{e^2(3cd - be + cex^n)}{c^2\,(a + bx^n + cx^{2n})} \right) dx$$

$$= \frac{\int \frac{c^2d^3-3acde^2+abe^3+\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3\right)x^n}{(a+bx^n+cx^{2n})^2}\,dx}{c^2} + \frac{e^2\int \frac{3cd-be+cex^n}{a+bx^n+cx^{2n}}\,dx}{c^2}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^n)}{ac\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)}$$

$$+ \frac{\left(e^2\left(e+\frac{6cd-3be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2c}$$

$$+ \frac{\left(e^2\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2c}$$

$$- \frac{\int \frac{-abce(3cd^2+ae^2(1-4n))-2ac^2d(cd^2-3ae^2)(1-2n)-ab^3e^3n+b^2cd(cd^2(1-n)+3ae^2n)-c(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3a}{a+bx^n+cx^{2n}}}{ac^2\left(b^2-4ac\right)n}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^n)}{ac\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)}$$

$$+ \frac{e^2\left(e+\frac{6cd-3be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}$$

$$+ \frac{e^2\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)}$$

$$+ \frac{\left((ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,(1-n)-\frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+4ac^2d}{\sqrt{b^2-4a}}\right.}{2ac\left(b^2-4ac\right)n}$$

$$+ \frac{\left((ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,(1-n)+\frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+4ac^2d}{\sqrt{b^2-4a}}\right.}{2ac\left(b^2-4ac\right)n}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^n)}{ac\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)}$$

$$+ \frac{e^2\left(e+\frac{6cd-3be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}$$

$$+ \frac{\left((ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,(1-n)+\frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+4ac^2d}{\sqrt{b^2-4a}}\right.}{ac\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}$$

$$+ \frac{e^2\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)}$$

$$+ \frac{\left((ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,(1-n)-\frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+4ac^2d}{\sqrt{b^2-4a}}\right.}{ac\left(b^2-4ac\right)\left(b+\sqrt{b^2-4ac}\right)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5537 vs. 2(750) = 1500.

Time = 7.56 (sec) , antiderivative size = 5537, normalized size of antiderivative = 7.38

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2n})^2}dx$$

```
[In] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^2*x^(4*n) +
 b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b*c^2*d^3 + 2*a^2*c*e^3 - (6*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*a)*x*x^n
+ (b^2*c*d^3 + (6*c*d*e^2 - b*e^3)*a^2 - (2*c^2*d^3 + 3*b*c*d^2*e)*a)*x)/(a
^2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a^2*c^3*n)*x^(2*n) + (a*b^3*c*n
 - 4*a^2*b*c^2*n)*x^n) + integrate((b^2*c*d^3*(n - 1) - (6*c*d*e^2 - b*e^3)
*a^2 - (2*c^2*d^3*(2*n - 1) - 3*b*c*d^2*e)*a - (2*a^2*c*e^3*(n + 1) - b*c^2
*d^3*(n - 1) + (6*c^2*d^2*e*(n - 1) - 3*b*c*d*e^2*(n - 1) - b^2*e^3)*a)*x^n
)/(a^2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a^2*c^3*n)*x^(2*n) + (a*b^3
*c*n - 4*a^2*b*c^2*n)*x^n), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

```
[In] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2, x)
```

3.76 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 543

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$
$$-\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}$$
$$-\frac{\left((bcd^2-4acde+abe^2)\,(1-n)-\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4ac(cd^2-ae^2)(1-2n)+4abcden}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}}{a\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)n}$$
$$-\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$
$$-\frac{\left((bcd^2-4acde+abe^2)\,(1-n)+\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4ac(cd^2-ae^2)(1-2n)+4abcden}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}}{a\,(b^2-4ac)\left(b+\sqrt{b^2-4ac}\right)n}$$

```
[Out] x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^n)/a/
(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))-x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))*((a*b*e^2-4*a*c*d*e+b*c*d^2)*(1-n)+(-b^2*(a*e^2*(1-3*
n)-c*d^2*(1-n))-4*a*c*(-a*e^2+c*d^2)*(1-2*n)-4*a*b*c*d*e*n)/(-4*a*c+b^2)^(1
/2))/a/(-4*a*c+b^2)/n/(b-(-4*a*c+b^2)^(1/2))-x*hypergeom([1, 1/n],[1+1/n],-
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((a*b*e^2-4*a*c*d*e+b*c*d^2)*(1-n)+(b^2*(a*
e^2*(1-3*n)-c*d^2*(1-n))+4*a*c*(-a*e^2+c*d^2)*(1-2*n)+4*a*b*c*d*e*n)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b+(-4*a*c+b^2)^(1/2))-2*e^2*x*hypergeom([1,
 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c-b*(-4*a*c+b^2)^(1
/2))-2*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b
^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1450, 1444, 1436, 251, 1361}

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx =$$
$$-\frac{x\left((1-n)\left(abe^2-4acde+bcd^2\right)-\frac{b^2\left(ae^2(1-3n)-cd^2(1-n)\right)+4abcden+4ac(1-2n)\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1\right.}{an\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{x\left(\frac{b^2\left(ae^2(1-3n)-cd^2(1-n)\right)+4abcden+4ac(1-2n)\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}+(1-n)\left(abe^2-4acde+bcd^2\right)\right)\text{Hypergeometric2F1}\left(1\right.}{an\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{x(x^n(abe^2-4acde+bcd^2)-2abde-2a(cd^2-ae^2)+b^2d^2)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$
$$-\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}$$
$$-\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[In] Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^n))/(a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) - (2*e^2*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 -
 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) -
 (b^2*(a*e^2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) +
 4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*
a*c])*n) - (2*e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - (((b*c*d^2 - 4
*a*c*d*e + a*b*e^2)*(1 - n) + (b^2*(a*e^2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*
c*(c*d^2 - a*e^2)*(1 - 2*n) + 4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^
2 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{cd^2 - ae^2 + (2cde - be^2)\, x^n}{c\,(a + bx^n + cx^{2n})^2} + \frac{e^2}{c\,(a + bx^n + cx^{2n})} \right) dx$$

$$= \frac{\int \frac{cd^2 - ae^2 + (2cde - be^2)x^n}{(a+bx^n+cx^{2n})^2}\, dx}{c} + \frac{e^2 \int \frac{1}{a+bx^n+cx^{2n}}\, dx}{c}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)\,x^n)}{a\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})} + \frac{e^2 \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^n}\,dx}{\sqrt{b^2 - 4ac}} - \frac{e^2 \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^n}\,dx}{\sqrt{b^2 - 4ac}} - \frac{\int \frac{-2abcde - 2ac(cd^2-ae^2)(1-2n) + b^2(cd^2(1-n) + ae^2n) + c(bcd^2 - 4acde + abe^2)(1-n)x^n}{a + bx^n + cx^{2n}}\,dx}{ac\,(b^2 - 4ac)\,n}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)\,x^n)}{a\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})} - \frac{2e^2x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2-4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{2e^2x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2-4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} - \frac{\left((bcd^2 - 4acde + abe^2)\,(1 - n) - \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4ac(cd^2-ae^2)(1-2n) + 4abcden}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^{n}}}{2a\,(b^2 - 4ac)\,n} - \frac{\left((bcd^2 - 4acde + abe^2)\,(1 - n) + \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4ac(cd^2-ae^2)(1-2n) + 4abcden}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^{n}}}{2a\,(b^2 - 4ac)\,n}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)\,x^n)}{a\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})} - \frac{2e^2x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2-4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{\left((bcd^2 - 4acde + abe^2)\,(1 - n) - \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4ac(cd^2-ae^2)(1-2n) + 4abcden}{\sqrt{b^2-4ac}}\right) x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \right.}{a\,(b^2 - 4ac)\left(b - \sqrt{b^2 - 4ac}\right) n} - \frac{2e^2x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2-4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} - \frac{\left((bcd^2 - 4acde + abe^2)\,(1 - n) + \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4ac(cd^2-ae^2)(1-2n) + 4abcden}{\sqrt{b^2-4ac}}\right) x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \right.}{a\,(b^2 - 4ac)\left(b + \sqrt{b^2 - 4ac}\right) n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2980 vs. 2(543) = 1086.

Time = 3.84 (sec) , antiderivative size = 2980, normalized size of antiderivative = 5.49

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
[Out] -((x*(-(a*Sqrt[b^2 - 4*a*c]*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^n + a*b*e*(-2*
d + e*x^n) - 2*a*c*d*(d + 2*e*x^n))) + (a*b*c*d^2*(a + x^n*(b + c*x^n))*(Hy
```

pergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) - 2^(2 - n^(-1))*a^2*c*d*e*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + (a^2*b*e^2*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) - (a*b*c*d^2*n*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) + 2^(2 - n^(-1))*a^2*c*d*e*n*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) - (a^2*b*e^2*n*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) + (b^2*d^2*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^((1 + n)/n) - (a*c*d^2*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) - (a*b*d*e*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1

```
), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) + (a
^2*e^2*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt
[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1
), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) - (b
^2*d^2*n*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sq
rt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(
-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^((1 + n)/
n) + 2^((-1 + n)/n)*a*c*d^2*n*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^
2 - 4*a*c] - ((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (
-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*
x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*H
ypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))
^n^(-1))))/(a^2*(b^2 - 4*a*c)^(3/2)*n*(a + x^n*(b + c*x^n))))
```

Maple [F]

$$\int \frac{(d+e\,x^n)^2}{(a+b\,x^n+c\,x^{2n})^2}dx$$

[In] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)

[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)

Fricas [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{(ex^n+d)^2}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")

[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b
x^n + a^2 + 2(b*c*x^n + a*c)*x^(2*n)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b*c*d^2 - (4*c*d*e - b*e^2)*a)*x*x^n + (b^2*d^2 + 2*a^2*e^2 - 2*(c*d^2 +
b*d*e)*a)*x)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (
a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(-(b^2*d^2*(n - 1) - 2*a^2*e^2 - 2*(
c*d^2*(2*n - 1) - b*d*e)*a + (b*c*d^2*(n - 1) - (4*c*d*e*(n - 1) - b*e^2*(n
 - 1))*a)*x^n)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) +
 (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(d+e\,x^n)^2}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

```
[In] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2, x)
```

3.77 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 362

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$
$$-\frac{c\left(2a\left(2cd(1-2n)+\sqrt{b^2-4ac}e(1-n)\right)-b^2(d-dn)-b\left(\sqrt{b^2-4ac}d(1-n)-2aen\right)\right)x\,\text{Hypergeomet}}{a\,(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)n}$$
$$-\frac{c\left(2a\left(cd(2-4n)-\sqrt{b^2-4ac}e(1-n)\right)-b^2d(1-n)+b\left(\sqrt{b^2-4ac}d(1-n)+2aen\right)\right)x\,\text{Hypergeometri}}{a\,(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)n}$$

```
[Out] x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2
*n))-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^2*
d*(1-n)+b*(2*a*e*n+d*(1-n)*(-4*a*c+b^2)^(1/2))+2*a*(c*d*(2-4*n)-e*(1-n)*(-4
*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*x*hyp
ergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(-b^2*(-d*n+d)-b*(
-2*a*e*n+d*(1-n)*(-4*a*c+b^2)^(1/2))+2*a*(2*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^2
)^(1/2)))/a/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

= {1444, 1436, 251}

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx =$$
$$-\frac{cx\left(-(1-n)\sqrt{b^2-4ac}(bd-2ae)+2aben+2acd(2-4n)+b^2(-d)(1-n)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n}\right.}{an\left(b^2-4ac\right)\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}$$
$$-\frac{cx\left((1-n)\sqrt{b^2-4ac}(bd-2ae)+2aben+4acd(1-2n)+b^2(-d)(1-n)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},\right.}{an\left(b^2-4ac\right)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$
$$+\frac{x(cx^n(bd-2ae)-abe-2acd+b^2d)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4*a*c)*n*(a +
 b*x^n + c*x^(2*n))) - (c*(2*a*c*d*(2 - 4*n) - b^2*d*(1 - n) - Sqrt[b^2 - 4
*a*c]*(b*d - 2*a*e)*(1 - n) + 2*a*b*e*n)*x*Hypergeometric2F1[1, n^(-1), 1 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c
 - b*Sqrt[b^2 - 4*a*c])*n) - (c*(4*a*c*d*(1 - 2*n) - b^2*d*(1 - n) + Sqrt[b
^2 - 4*a*c]*(b*d - 2*a*e)*(1 - n) + 2*a*b*e*n)*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
```

```
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)n(a+bx^n+cx^{2n})} - \frac{\int \frac{-abe-2acd(1-2n)+b^2(d-dn)+c(bd-2ae)(1-n)x^n}{a+bx^n+cx^{2n}}\,dx}{a(b^2-4ac)n}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)n(a+bx^n+cx^{2n})} - \frac{\left(c\left(2acd(2-4n)-b^2d(1-n)+\sqrt{b^2-4ac}(bd-2ae)(1-n)+2aben\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2a(b^2-4ac)^{3/2}n} - \frac{\left(c\left((bd-2ae)(1-n)-\frac{4acd(1-2n)+2aben-b^2(d-dn)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2a(b^2-4ac)n}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)n(a+bx^n+cx^{2n})} - \frac{c\left((bd-2ae)(1-n)-\frac{4acd(1-2n)+2aben-b^2(d-dn)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)n} - \frac{c\left(2acd(2-4n)-b^2d(1-n)+\sqrt{b^2-4ac}(bd-2ae)(1-n)+2aben\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-}\cdots}\right)}{a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)n}$$

Mathematica [A] (verified)

Time = 4.18 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.67

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \frac{cx\left(\frac{4(b^2-4ac)(b^2d(-1+n)x^n(b+cx^n)-2a^2c(2dn+ex^n)+a(-2c^2d(-1+2n)x^{2n}+bcx^n(3d-4dn+ex^n)+b^2(dn+ex^n)))}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(a+x^n(b+cx^n))} + \frac{2^{-1/n}\left(4ac\left(\sqrt{b^2-4ac}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (c*x*((4*(b^2 - 4*a*c)*(b^2*d*(-1 + n)*x^n*(b + c*x^n) - 2*a^2*c*(2*d*n + e
*x^n) + a*(-2*c^2*d*(-1 + 2*n)*x^(2*n) + b*c*x^n*(3*d - 4*d*n + e*x^n) + b^
2*(d*n + e*x^n))))/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(b^2 - 4*a*c + b*Sq
rt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n))) + ((4*a*c*(Sqrt[b^2 - 4*a*c]*d*(1 -
 2*n) + 2*a*e*(-1 + n)) + b^3*d*(-1 + n) + b^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e
)*(-1 + n) + 2*a*b*(-2*c*d*(-1 + n) + Sqrt[b^2 - 4*a*c]*e*n))*Hypergeometri
```

```
c2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*(-b^2 + 4*a*c + b*Sqrt[b^2
- 4*a*c])*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + ((b*Sqrt[b^
2 - 4*a*c]*d*(-1 + n) - 2*a*Sqrt[b^2 - 4*a*c]*e*(-1 + n) - 2*a*b*e*n + 4*a*
c*d*(-1 + 2*n) + b^2*(d - d*n))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n
)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^n))^n^(-1))))/(a*(-b^2 + 4*a*c)*n)
```

Maple [F]

$$\int \frac{d+e\,x^{n}}{\left(a+b\,x^{n}+c\,x^{2n}\right)^{2}}dx$$

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*
x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b*c*d - 2*a*c*e)*x*x^n + (b^2*d - (2*c*d + b*e)*a)*x)/(a^2*b^2*n - 4*a^3*
c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) + in
tegrate((b^2*d*(n - 1) - (2*c*d*(2*n - 1) - b*e)*a + (b*c*d*(n - 1) - 2*a*c
*e*(n - 1))*x^n)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n)
 + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^2, x)
```

3.78 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 726

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2} dx = \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^n)}{a(b^2-4ac)(cd^2-bde+ae^2)n(a+bx^n+cx^{2n})}$$
$$-\frac{ce^2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^2}$$
$$-\frac{c\left(\frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)-b^3e(1-n)}{\sqrt{b^2-4ac}}+(bcd-b^2e+2ace)(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1-\right.}{a(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)n}$$
$$-\frac{ce^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^2}$$
$$+\frac{c\left(bc\left(2ae(2-3n)-\sqrt{b^2-4ac}d(1-n)\right)-2ac\left(2cd(1-2n)+\sqrt{b^2-4ac}e(1-n)\right)-b^3e(1-n)+b^2(c\right.}{a(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-}$$
$$+\frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^2}$$

```
[Out] x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4*a*c
+b^2)/(a*e^2-b*d*e+c*d^2)/n/(a+b*x^n+c*x^(2*n))+e^4*x*hypergeom([1, 1/n],[1
+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^2-c*x*hypergeom([1, 1/n],[1+1/n],-2*c
*x^n/(b-(-4*a*c+b^2)^(1/2)))*((2*a*c*e-b^2*e+b*c*d)*(1-n)+(2*a*b*c*e*(2-3*n
)-4*a*c^2*d*(1-2*n)+b^2*c*d*(1-n)-b^3*e*(1-n))/(-4*a*c+b^2)^(1/2))/a/(-4*a*
c+b^2)/(a*e^2-b*d*e+c*d^2)/n/(b-(-4*a*c+b^2)^(1/2))-c*e^2*x*hypergeom([1, 1
/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*e^2*x*hypergeom
([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)
```

^(1/2)))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*x*hyperge
om([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n
)*(c*d+e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e*(2-3*n)-d*(1-n)*(-4*a*c+b^2)^(1/2))
-2*a*c*(2*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(a*e^2-b*
d*e+c*d^2)/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 251, 1444, 1436}

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx$$

$$= -\frac{ce^2x\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$

$$-\frac{ce^2x\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$

$$-\frac{cx\left((1-n)\left(2ace+b^2(-e)+bcd\right)+\frac{2abce(2-3n)-4ac^2d(1-2n)+b^3(-e)(1-n)+b^2cd(1-n)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\ldots\right.}{an\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)\left(ae^2-bde+cd^2\right)}$$

$$+\frac{x\left(cx^n\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d-b^3e+b^2cd\right)}{an\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)\left(a+bx^n+cx^{2n}\right)}$$

$$+\frac{cx\left(b^2(1-n)\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(2ae(2-3n)-d(1-n)\sqrt{b^2-4ac}\right)-2ac\left(e(1-n)\sqrt{b^2-4ac}+2c\ldots\right.}{an\left(b^2-4ac\right)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae\ldots\right.}$$

$$+\frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^2}$$

[In] Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]

[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))) -
(c*e^2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2
- 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) - (c*((2*a*b*c*e*(2 - 3*n) - 4*a*c^2*d
*(1 - 2*n) + b^2*c*d*(1 - n) - b^3*e*(1 - n))/Sqrt[b^2 - 4*a*c] + (b*c*d -
b^2*e + 2*a*c*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(c*
d^2 - b*d*e + a*e^2)*n) - (c*e^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*x*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) + (c*(b*c*(2

```
*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sq
rt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e
)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2
 - b*d*e + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e
*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2 (d+ex^n)} + \frac{cd-be-cex^n}{(cd^2-bde+ae^2)(a+bx^n+cx^{2n})^2} - \frac{e^2(-cd+be+cex^n)}{(cd^2-bde+ae^2)^2 (a+bx^n+cx^{2n})} \right) dx$$

$$=-\frac{e^2\int \frac{-cd+be+cex^n}{a+bx^n+cx^{2n}}\,dx}{\left(cd^2-bde+ae^2\right)^2}+\frac{e^4\int \frac{1}{d+ex^n}\,dx}{\left(cd^2-bde+ae^2\right)^2}+\frac{\int \frac{cd-be-cex^n}{(a+bx^n+cx^{2n})^2}\,dx}{cd^2-bde+ae^2}$$

$$=\frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})}$$
$$+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,\left(cd^2-bde+ae^2\right)^2}-\frac{\left(ce^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{\left(ce^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{\int \frac{abce-2ac(cd-be)(1-2n)+b^2(cd-be)(1-n)+c(bcd-b^2e+2ace)(1-n)x^n}{a+bx^n+cx^{2n}}\,dx}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n}$$

$$=\frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})}$$
$$-\frac{ce^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{ce^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{\left(c\left(\frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)-b^3e(1-n)}{\sqrt{b^2-4ac}}+(bcd-b^2e+2ace)\,(1-n)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n}$$
$$-\frac{\left(c\left((bcd-b^2e+2ace)\,(1-n)-\frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)-b^3(e-en)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n}$$

$$=\frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})}$$
$$-\frac{ce^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{c\left(\frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)-b^3e(1-n)}{\sqrt{b^2-4ac}}+(bcd-b^2e+2ace)\,(1-n)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)\,n}$$
$$-\frac{ce^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{c\left((bcd-b^2e+2ace)\,(1-n)-\frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)-b^3(e-en)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\,(b^2-4ac)\left(b+\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)\,n}$$
$$+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,\left(cd^2-bde+ae^2\right)^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11767 vs. 2(726) = 1452.

Time = 6.91 (sec) , antiderivative size = 11767, normalized size of antiderivative = 16.21

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx=\text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)\left(a+b\,x^n+c\,x^{2n}\right)^2}dx$$

```
[In] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx=\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^2(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*e*x^(3*n) + a^2*d + (c^2*e*x^n + c^2*d)*x^(4*n) + 2*(b*c*e*
x^(2*n) + a*c*d + (b*c*d + a*c*e)*x^n)*x^(2*n) + (b^2*d + 2*a*b*e)*x^(2*n)
+ (2*a*b*d + a^2*e)*x^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^2(ex^n+d)}\,dx$$

[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")

[Out] e^4*integrate(1/(c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + a^2*d*e^4 + 2*(c*d^3 *e^2 - b*d^2*e^3)*a + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + a^2*e^5 + 2*(c*d^2*e^3 - b*d*e^4)*a)*x^n), x) - ((b*c^2*d - b^2*c*e + 2*a*c^2*e)*x*x^ n + (b^2*c*d - b^3*e - (2*c^2*d - 3*b*c*e)*a)*x)/(4*a^4*c*e^2*n + (4*c^2*d^ 2*n - 4*b*c*d*e*n - b^2*e^2*n)*a^3 - (b^2*c*d^2*n - b^3*d*e*n)*a^2 + (4*a^3 *c^2*e^2*n + (4*c^3*d^2*n - 4*b*c^2*d*e*n - b^2*c*e^2*n)*a^2 - (b^2*c^2*d^2 *n - b^3*c*d*e*n)*a)*x^(2*n) + (4*a^3*b*c*e^2*n + (4*b*c^2*d^2*n - 4*b^2*c* d*e*n - b^3*e^2*n)*a^2 - (b^3*c*d^2*n - b^4*d*e*n)*a)*x^n) - integrate((b^2 *c^2*d^3*(n - 1) - 2*b^3*c*d^2*e*(n - 1) + b^4*d*e^2*(n - 1) + (b*c*e^3*(8* n - 3) - 2*c^2*d*e^2*(4*n - 1))*a^2 + (b*c^2*d^2*e*(8*n - 5) - 2*c^3*d^3*(2 *n - 1) - b^3*e^3*(2*n - 1) - 2*b^2*c*d*e^2*(n - 1))*a + (2*a^2*c^2*e^3*(3* n - 1) + b*c^3*d^3*(n - 1) - 2*b^2*c^2*d^2*e*(n - 1) + b^3*c*d*e^2*(n - 1) - (b^2*c*e^3*(2*n - 1) - 2*c^3*d^2*e*(n - 1) + b*c^2*d*e^2*(n - 1))*a)*x^n) /(4*a^5*c*e^4*n + (8*c^2*d^2*e^2*n - 8*b*c*d*e^3*n - b^2*e^4*n)*a^4 + 2*(2* c^3*d^4*n - 4*b*c^2*d^3*e*n + b^2*c*d^2*e^2*n + b^3*d*e^3*n)*a^3 - (b^2*c^2 *d^4*n - 2*b^3*c*d^3*e*n + b^4*d^2*e^2*n)*a^2 + (4*a^4*c^2*e^4*n + (8*c^3*d ^2*e^2*n - 8*b*c^2*d*e^3*n - b^2*c*e^4*n)*a^3 + 2*(2*c^4*d^4*n - 4*b*c^3*d^ 3*e*n + b^2*c^2*d^2*e^2*n + b^3*c*d*e^3*n)*a^2 - (b^2*c^3*d^4*n - 2*b^3*c^2 *d^3*e*n + b^4*c*d^2*e^2*n)*a)*x^(2*n) + (4*a^4*b*c*e^4*n + (8*b*c^2*d^2*e^ 2*n - 8*b^2*c*d*e^3*n - b^3*e^4*n)*a^3 + 2*(2*b*c^3*d^4*n - 4*b^2*c^2*d^3*e *n + b^3*c*d^2*e^2*n + b^4*d*e^3*n)*a^2 - (b^3*c^2*d^4*n - 2*b^4*c*d^3*e*n + b^5*d^2*e^2*n)*a)*x^n), x)

Giac [F]

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^2(ex^n+d)}\,dx$$

[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})^2}\,dx=\int \frac{1}{(d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^2}\,dx$$

```
[In] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x)
[Out] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2), x)
```

3.79 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 1129

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx = -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3}{a(b^2-4ac)(cd^2-bde+ae^2)^2n(a+bx^n+cx^{2n})}$$

$$-\frac{2ce^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+\frac{c\left(4ac^2\left(e\left(ae(1-2n)+\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-2n)\right)-b^2c\left(e\left(ae(5-7n)+2\sqrt{b^2-4ac}d(1-n)\right)\right.}{}$$

$$-\frac{2ce^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\right.}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+\frac{c\left(4ac^2\left(e\left(ae(1-2n)-\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-2n)\right)-b^2c\left(e\left(ae(5-7n)-2\sqrt{b^2-4ac}d(1-n)\right)\right.}{}$$

$$+\frac{2e^4(2cd-be)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^3}$$

$$+\frac{e^4x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2(cd^2-bde+ae^2)^2}$$

```
[Out] -x*(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^
2+c*d^2)+c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))*x^n)/a/(-
4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))+2*e^4*(-b*e+2*c*d)*x
*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^3+e^4*x*hyperge
om([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^2-2*c*e^2*x*hypergeom
```

```
([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2+b*e^2*(b-(-4*
a*c+b^2)^(1/2))-c*e*(3*b*d+a*e-2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)
^3/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-2*c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2
*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-c*e*
(3*b*d+a*e+2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c-b*(-4*
a*c+b^2)^(1/2))+c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/
2)))*(b^4*e^2*(1-n)-b^3*e*(1-n)*(2*c*d+e*(-4*a*c+b^2)^(1/2))-b^2*c*(-c*d^2*
(1-n)+e*(a*e*(5-7*n)-2*d*(1-n)*(-4*a*c+b^2)^(1/2)))+b*c*(3*a*e^2*(1-n)*(-4*
a*c+b^2)^(1/2)+c*d*(4*a*e*(2-3*n)-d*(1-n)*(-4*a*c+b^2)^(1/2)))+4*a*c^2*(-c*
d^2*(1-2*n)+e*(a*e*(1-2*n)-d*(1-n)*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*
e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))+c*x*hypergeom([1, 1/n
],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(b^4*e^2*(1-n)-b^3*e*(1-n)*(2*c*
d-e*(-4*a*c+b^2)^(1/2))+b*c*(-3*a*e^2*(1-n)*(-4*a*c+b^2)^(1/2)+c*d*(4*a*e*(
2-3*n)+d*(1-n)*(-4*a*c+b^2)^(1/2)))+4*a*c^2*(-c*d^2*(1-2*n)+e*(a*e*(1-2*n)+
d*(1-n)*(-4*a*c+b^2)^(1/2)))-b^2*c*(-c*d^2*(1-n)+e*(a*e*(5-7*n)+2*d*(1-n)*(
-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c-b*(-
4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 1129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 251, 1444, 1436}

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2} \, dx$$

$$= \frac{2(2cd-be)x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^4}{d\left(cd^2-bed+ae^2\right)^3}$$

$$+ \frac{x \text{ Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^4}{d^2\left(cd^2-bed+ae^2\right)^2}$$

$$- \frac{2c\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{}{b}\right.}{\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}$$

$$- \frac{2c\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{}{b}\right.}{\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}$$

$$+ \frac{c\left(e^2(1-n)b^4-e\left(2cd-\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(5-7n)+2\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-n\right.\right.}{}$$

$$+ \frac{c\left(e^2(1-n)b^4-e\left(2cd+\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(5-7n)-2\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-n\right.\right.}{}$$

$$- \frac{x\left(c\left(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de\right)x^n-b^4e^2-6abc^2de+2b^3cde-b^2c\left(cd^2-4ae^2\right)+2ac\right.}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2 n\left(bx^n+cx^{2n}+a\right)}$$

```
[In] Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
```

```
[Out] -((x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a
*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2
- 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n +
 c*x^(2*n)))) - (2*c*e^2*(3*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(
3*b*d + 2*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a
*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) + Sqrt[b^2
- 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) + 2*Sqrt[b
^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n) + Sqrt[
b^2 - 4*a*c]*d*(1 - n)) - 3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 -
 n) - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b
^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (2*c*e^2*(
3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d - 2*Sqrt[b^2 - 4*a*c
]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^
2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^
2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c
*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) +
3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d + Sqrt[
b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 -
4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (2*e^4*(2*c*d - b*e)*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) +
(e^4*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2
- b*d*e + a*e^2)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2(d+ex^n)^2} - \frac{2e^4(-2cd+be)}{(cd^2-bde+ae^2)^3(d+ex^n)} + \frac{c^2d^2-2bcde+b^2e^2-ace^2-(2c^2de-bce^2)x^n}{(cd^2-bde+ae^2)^2(a+bx^n+cx^{2n})^2} + \frac{e^2(3c^2d^2-5bcde+2b^2e^2-ace^2+(-4c^2de+2bce^2)x^n)}{(cd^2-bde+ae^2)^3(a+bx^n+cx^{2n})}\right)dx$$

$$= \frac{e^2\int \frac{3c^2d^2-5bcde+2b^2e^2-ace^2+(-4c^2de+2bce^2)x^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^3} + \frac{(2e^4(2cd-be))\int \frac{1}{d+ex^n}\,dx}{(cd^2-bde+ae^2)^3} + \frac{\int \frac{c^2d^2-2bcde+b^2e^2-ace^2-(2c^2de-bce^2)x^n}{(a+bx^n+cx^{2n})^2}\,dx}{(cd^2-bde+ae^2)^2} + \frac{e^4\int \frac{1}{(d+ex^n)^2}\,dx}{(cd^2-bde+ae^2)^2}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2b^2cde-4ac^2de-b^3e^2-b}{a(b^2-4ac)(cd^2-bde+ae^2)^2n(a+bx^n+cx^{2n})} + \frac{2e^4(2cd-be)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^3} + \frac{e^4x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2-bde+ae^2)^2} - \frac{\left(ce^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}(cd^2-bde+ae^2)^3} + \frac{\left(ce^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}(cd^2-bde+ae^2)^3} - \frac{\int \frac{-b^2c(ae^2(4-5n)-cd^2(1-n))+2abc^2de(3-4n)-2ac^2(cd^2-ae^2)(1-2n)-2b^3cde(1-n)+b^4e^2(1-n)-c(2b^2cde-4ac^2de-b^3e^2-bc}{a+bx^n+cx^{2n}}}{a(b^2-4ac)(cd^2-bde+ae^2)^2n}$$

$$= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2-4ae^2) + 2ac^2(cd^2-ae^2) + c(2b^2cde - 4ac^2de - b^3e^2 - bc}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+bx^n+cx^{2n})}$$

$$+ \frac{2ce^2\left(3c^2d^2 + b\left(b+\sqrt{b^2-4ac}\right)e^2 - ce\left(3bd + 2\sqrt{b^2-4ac}d + ae\right)\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$- \frac{2ce^2\left(3c^2d^2 + b\left(b-\sqrt{b^2-4ac}\right)e^2 - ce\left(3bd - 2\sqrt{b^2-4ac}d + ae\right)\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+ \frac{2e^4(2cd-be)x\,{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)^3} + \frac{e^4x\,{}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\,(cd^2-bde+ae^2)^2}$$

$$- \frac{\left(c\left(4ac^2\left(e\left(ae(1-2n) + \sqrt{b^2-4ac}d(1-n)\right) - cd^2(1-2n)\right) - b^2c\left(e\left(ae(5-7n) + 2\sqrt{b^2-4ac}d\right.\right.\right.\right.}{}$$

$$+ \frac{\left(c\left(4ac^2\left(e\left(ae(1-2n) - \sqrt{b^2-4ac}d(1-n)\right) - cd^2(1-2n)\right) - b^2c\left(e\left(ae(5-7n) - 2\sqrt{b^2-4ac}d\right.\right.\right.\right.}{}$$

$$= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2-4ae^2) + 2ac^2(cd^2-ae^2) + c(2b^2cde - 4ac^2de - b^3e^2 - bc}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+bx^n+cx^{2n})}$$

$$+ \frac{2ce^2\left(3c^2d^2 + b\left(b+\sqrt{b^2-4ac}\right)e^2 - ce\left(3bd + 2\sqrt{b^2-4ac}d + ae\right)\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$- \frac{c\left(4ac^2\left(e\left(ae(1-2n) + \sqrt{b^2-4ac}d(1-n)\right) - cd^2(1-2n)\right) - b^2c\left(e\left(ae(5-7n) + 2\sqrt{b^2-4ac}d(\right.\right.\right.}{}$$

$$- \frac{2ce^2\left(3c^2d^2 + b\left(b-\sqrt{b^2-4ac}\right)e^2 - ce\left(3bd - 2\sqrt{b^2-4ac}d + ae\right)\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+ \frac{c\left(4ac^2\left(e\left(ae(1-2n) - \sqrt{b^2-4ac}d(1-n)\right) - cd^2(1-2n)\right) - b^2c\left(e\left(ae(5-7n) - 2\sqrt{b^2-4ac}d(\right.\right.\right.}{}$$

$$+ \frac{2e^4(2cd-be)x\,{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)^3} + \frac{e^4x\,{}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\,(cd^2-bde+ae^2)^2}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 16855 vs. 2(1129) = 2258.

Time = 7.87 (sec) , antiderivative size = 16855, normalized size of antiderivative = 14.93

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)^2(a+b\,x^n+c\,x^{2n})^2}dx$$

```
[In] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*e^2*x^(4*n) + a^2*d^2 + (c^2*e^2*x^(2*n) + 2*c^2*d*e*x^n +
c^2*d^2)*x^(4*n) + 2*(b^2*d*e + a*b*e^2)*x^(3*n) + 2*(b*c*e^2*x^(3*n) + a*c
*d^2 + (2*b*c*d*e + a*c*e^2)*x^(2*n) + (b*c*d^2 + 2*a*c*d*e)*x^n)*x^(2*n) +
 (b^2*d^2 + 4*a*b*d*e + a^2*e^2)*x^(2*n) + 2*(a*b*d^2 + a^2*d*e)*x^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] (c*d^2*e^4*(5*n - 1) - b*d*e^5*(3*n - 1) + a*e^6*(n - 1))*integrate(1/(c^3*
d^8*n - 3*b*c^2*d^7*e*n + 3*b^2*c*d^6*e^2*n - b^3*d^5*e^3*n + a^3*d^2*e^6*n
 + 3*(c*d^4*e^4*n - b*d^3*e^5*n)*a^2 + 3*(c^2*d^6*e^2*n - 2*b*c*d^5*e^3*n +
 b^2*d^4*e^4*n)*a + (c^3*d^7*e*n - 3*b*c^2*d^6*e^2*n + 3*b^2*c*d^5*e^3*n -
b^3*d^4*e^4*n + a^3*d*e^7*n + 3*(c*d^3*e^5*n - b*d^2*e^6*n)*a^2 + 3*(c^2*d^
5*e^3*n - 2*b*c*d^4*e^4*n + b^2*d^3*e^5*n)*a)*x^n), x) - ((b*c^3*d^3*e - 2*
b^2*c^2*d^2*e^2 + b^3*c*d*e^3 - 4*a^2*c^2*e^4 + (4*c^3*d^2*e^2 - 3*b*c^2*d*
e^3 + b^2*c*e^4)*a)*x*x^(2*n) + (b*c^3*d^4 - b^2*c^2*d^3*e - b^3*c*d^2*e^2
+ b^4*d*e^3 + 2*(c^2*d*e^3 - 2*b*c*e^4)*a^2 + (2*c^3*d^3*e + 3*b*c^2*d^2*e^
2 - 4*b^2*c*d*e^3 + b^3*e^4)*a)*x*x^n + (b^2*c^2*d^4 - 2*b^3*c*d^3*e + b^4*
d^2*e^2 - 4*a^3*c*e^4 + (2*c^2*d^2*e^2 + b^2*e^4)*a^2 - 2*(c^3*d^4 - 3*b*c^
2*d^3*e + 2*b^2*c*d^2*e^2)*a)*x)/(4*a^5*c*d^2*e^4*n + (8*c^2*d^4*e^2*n - 8*
b*c*d^3*e^3*n - b^2*d^2*e^4*n)*a^4 + 2*(2*c^3*d^6*n - 4*b*c^2*d^5*e*n + b^2
*c*d^4*e^2*n + b^3*d^3*e^3*n)*a^3 - (b^2*c^2*d^6*n - 2*b^3*c*d^5*e*n + b^4*
d^4*e^2*n)*a^2 + (4*a^4*c^2*d*e^5*n + (8*c^3*d^3*e^3*n - 8*b*c^2*d^2*e^4*n
- b^2*c*d*e^5*n)*a^3 + 2*(2*c^4*d^5*e*n - 4*b*c^3*d^4*e^2*n + b^2*c^2*d^3*e
^3*n + b^3*c*d^2*e^4*n)*a^2 - (b^2*c^3*d^5*e*n - 2*b^3*c^2*d^4*e^2*n + b^4*
c*d^3*e^3*n)*a)*x^(3*n) + (4*(c^2*d^2*e^4*n + b*c*d*e^5*n)*a^4 + (8*c^3*d^4
*e^2*n - 9*b^2*c*d^2*e^4*n - b^3*d*e^5*n)*a^3 + 2*(2*c^4*d^6*n - 2*b*c^3*d^
5*e*n - 3*b^2*c^2*d^4*e^2*n + 2*b^3*c*d^3*e^3*n + b^4*d^2*e^4*n)*a^2 - (b^2
*c^3*d^6*n - b^3*c^2*d^5*e*n - b^4*c*d^4*e^2*n + b^5*d^3*e^3*n)*a)*x^(2*n)
+ (4*a^5*c*d*e^5*n + (8*c^2*d^3*e^3*n - 4*b*c*d^2*e^4*n - b^2*d*e^5*n)*a^4
+ (4*c^3*d^5*e*n - 6*b^2*c*d^3*e^3*n + b^3*d^2*e^4*n)*a^3 + (4*b*c^3*d^6*n
- 9*b^2*c^2*d^5*e*n + 4*b^3*c*d^4*e^2*n + b^4*d^3*e^3*n)*a^2 - (b^3*c^2*d^6
*n - 2*b^4*c*d^5*e*n + b^5*d^4*e^2*n)*a)*x^n) + integrate(-(2*a^3*c^2*e^4*(
4*n - 1) + b^2*c^3*d^4*(n - 1) - 3*b^3*c^2*d^3*e*(n - 1) + 3*b^4*c*d^2*e^2*
(n - 1) - b^5*d*e^3*(n - 1) - 2*(b^2*c*e^4*(7*n - 2) - 2*b*c^2*d*e^3*(6*n -
 1) + 6*c^3*d^2*e^2*n)*a^2 + (b^4*e^4*(3*n - 1) + 4*b*c^3*d^3*e*(3*n - 2) -
 2*c^4*d^4*(2*n - 1) - 2*b^3*c*d*e^3*(n + 1) - 9*b^2*c^2*d^2*e^2*(n - 1))*a
 + (b*c^4*d^4*(n - 1) - 3*b^2*c^3*d^3*e*(n - 1) + 3*b^3*c^2*d^2*e^2*(n - 1)
 - b^4*c*d*e^3*(n - 1) - (b*c^2*e^4*(11*n - 3) - 4*c^3*d*e^3*(5*n - 1))*a^2
 - (b^2*c^2*d*e^3*(3*n + 1) - b^3*c*e^4*(3*n - 1) - 4*c^4*d^3*e*(n - 1) + 6
*b*c^3*d^2*e^2*(n - 1))*a)*x^n)/(4*a^6*c*e^6*n + (12*c^2*d^2*e^4*n - 12*b*c
*d*e^5*n - b^2*e^6*n)*a^5 + 3*(4*c^3*d^4*e^2*n - 8*b*c^2*d^3*e^3*n + 3*b^2*
c*d^2*e^4*n + b^3*d*e^5*n)*a^4 + (4*c^4*d^6*n - 12*b*c^3*d^5*e*n + 9*b^2*c^
2*d^4*e^2*n + 2*b^3*c*d^3*e^3*n - 3*b^4*d^2*e^4*n)*a^3 - (b^2*c^3*d^6*n - 3
*b^3*c^2*d^5*e*n + 3*b^4*c*d^4*e^2*n - b^5*d^3*e^3*n)*a^2 + (4*a^5*c^2*e^6*
```

```
n + (12*c^3*d^2*e^4*n - 12*b*c^2*d*e^5*n - b^2*c*e^6*n)*a^4 + 3*(4*c^4*d^4*
e^2*n - 8*b*c^3*d^3*e^3*n + 3*b^2*c^2*d^2*e^4*n + b^3*c*d*e^5*n)*a^3 + (4*c
^5*d^6*n - 12*b*c^4*d^5*e*n + 9*b^2*c^3*d^4*e^2*n + 2*b^3*c^2*d^3*e^3*n - 3
*b^4*c*d^2*e^4*n)*a^2 - (b^2*c^4*d^6*n - 3*b^3*c^3*d^5*e*n + 3*b^4*c^2*d^4*
e^2*n - b^5*c*d^3*e^3*n)*a)*x^(2*n) + (4*a^5*b*c*e^6*n + (12*b*c^2*d^2*e^4*
n - 12*b^2*c*d*e^5*n - b^3*e^6*n)*a^4 + 3*(4*b*c^3*d^4*e^2*n - 8*b^2*c^2*d^
3*e^3*n + 3*b^3*c*d^2*e^4*n + b^4*d*e^5*n)*a^3 + (4*b*c^4*d^6*n - 12*b^2*c^
3*d^5*e*n + 9*b^3*c^2*d^4*e^2*n + 2*b^4*c*d^3*e^3*n - 3*b^5*d^2*e^4*n)*a^2
- (b^3*c^3*d^6*n - 3*b^4*c^2*d^5*e*n + 3*b^5*c*d^4*e^2*n - b^6*d^3*e^3*n)*a
)*x^n), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(d+e\,x^n)^2\,(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

```
[In] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x)
[Out] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2), x)
```

3.80 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 1707

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx$$
$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^n)}{2ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$
$$+\frac{e^2x(3b^2cd-6ac^2d-b^3e+abce+c(3bcd-b^2e-2ace)\,x^n)}{ac^2\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$
$$-\frac{x(ab^2c^2d(3ae^2(1-9n)-5cd^2(1-3n))+4a^2c^3d(cd^2-3ae^2)\,(1-4n)-2ab^5e^3n+2a^2bc^2e(3cd^2(2-3n}{}$$
$$+\frac{e^2\left(bc\left(2ae(2-5n)+3\sqrt{b^2-4ac}d(1-n)\right)-2ac\left(6cd(1-2n)+\sqrt{b^2-4ac}e(1-n)\right)-b^3e(1-n)+b^2(}{ac\,(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4}\right.}$$
$$+\frac{\left((1-n)\,(4a^2c^2e(3cd^2-ae^2)\,(1-3n)-2ab^4e^3n-2abc^2d(cd^2(2-7n)+3ae^2n)+b^3cd(cd^2(1-2n)+6\right.}{}$$
$$+\frac{e^2\left(bc\left(2ae(2-5n)-3\sqrt{b^2-4ac}d(1-n)\right)-2ac\left(6cd(1-2n)-\sqrt{b^2-4ac}e(1-n)\right)-b^3e(1-n)+b^2(}{ac\,(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4}\right.}$$
$$+\frac{\left((1-n)\,(4a^2c^2e(3cd^2-ae^2)\,(1-3n)-2ab^4e^3n-2abc^2d(cd^2(2-7n)+3ae^2n)+b^3cd(cd^2(1-2n)+6\right.}{}$$

```
[Out] 1/2*x*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)-(a*b^2*e^3+
2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^n)/a/c/(-4*a*c+b^2)/n/(a+
b*x^n+c*x^(2*n))^2+e^2*x*(3*b^2*c*d-6*a*c^2*d-b^3*e+a*b*c*e+c*(-2*a*c*e-b^2
*e+3*b*c*d)*x^n)/a/c^2/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))-1/2*x*(a*b^2*c^2*
```

```
d*(3*a*e^2*(1-9*n)-5*c*d^2*(1-3*n))+4*a^2*c^3*d*(-3*a*e^2+c*d^2)*(1-4*n)-2*
a*b^5*e^3*n+2*a^2*b*c^2*e*(3*c*d^2*(2-3*n)-5*a*e^2*n)-3*a*b^3*c*e*(-3*a*e^2
*n+c*d^2)+b^4*c*d*(c*d^2*(1-2*n)+6*a*e^2*n)+c*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)
*(1-3*n)-2*a*b^4*e^3*n-2*a*b*c^2*d*(c*d^2*(2-7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2
*(1-2*n)+6*a*e^2*n)-a*b^2*c*e*(3*c*d^2-a*e^2*(1+2*n))))*x^n)/a^2/c^2/(-4*a*c
+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))+1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(
b-(-4*a*c+b^2)^(1/2)))*((1-n)*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)*(1-3*n)-2*a*b^4
*e^3*n-2*a*b*c^2*d*(c*d^2*(2-7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2*(1-2*n)+6*a*e^2
*n)-a*b^2*c*e*(3*c*d^2-a*e^2*(1+2*n)))+(-2*a*b^5*e^3*(1-n)*n+b^4*c*d*(1-n)*
(c*d^2*(1-2*n)+6*a*e^2*n)+8*a^2*c^3*d*(-3*a*e^2+c*d^2)*(8*n^2-6*n+1)-6*a*b^
2*c^2*d*(c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2-10*n+1))+4*a^2*b*c^2*e*(3*c*d^2*
(-3*n^2-n+1)+a*e^2*(19*n^2-11*n+1))-a*b^3*c*e*(3*c*d^2*(1-n)+a*e^2*(30*n^2-
19*n+1)))/(-4*a*c+b^2)^(1/2))/a^2/c/(-4*a*c+b^2)^2/n^2/(b-(-4*a*c+b^2)^(1/2
))+1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((1-n)
*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)*(1-3*n)-2*a*b^4*e^3*n-2*a*b*c^2*d*(c*d^2*(2-
7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2*(1-2*n)+6*a*e^2*n)-a*b^2*c*e*(3*c*d^2-a*e^2*
(1+2*n)))+(2*a*b^5*e^3*(1-n)*n-b^4*c*d*(1-n)*(c*d^2*(1-2*n)+6*a*e^2*n)-8*a^
2*c^3*d*(-3*a*e^2+c*d^2)*(8*n^2-6*n+1)+6*a*b^2*c^2*d*(c*d^2*(3*n^2-4*n+1)-a
*e^2*(15*n^2-10*n+1))-4*a^2*b*c^2*e*(3*c*d^2*(-3*n^2-n+1)+a*e^2*(19*n^2-11*
n+1))+a*b^3*c*e*(3*c*d^2*(1-n)+a*e^2*(30*n^2-19*n+1)))/(-4*a*c+b^2)^(1/2))/
a^2/c/(-4*a*c+b^2)^2/n^2/(b+(-4*a*c+b^2)^(1/2))+e^2*x*hypergeom([1, 1/n],[1
+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(3*c*d+e*(-4
*a*c+b^2)^(1/2))+b*c*(2*a*e*(2-5*n)-3*d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(6*
c*d*(1-2*n)-e*(1-n)*(-4*a*c+b^2)^(1/2)))/a/c/(-4*a*c+b^2)/n/(b^2-4*a*c+b*(-
4*a*c+b^2)^(1/2))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(3*c*d-e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e*(2
-5*n)+3*d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(6*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^
2)^(1/2)))/a/c/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 3.24 (sec) , antiderivative size = 1707, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used

$= \{1450, 1444, 1436, 251\}$

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx$$

$$= \frac{\left(-e(1-n)b^3+\left(3cd-\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-5n)+3\sqrt{b^2-4ac}d(1-n)\right)b-2ac(6cd(1-2n\right.}{ac\,(b^2-4ac)\left(b^2-\sqrt{b^2-4ac}b-4\right.}$$

$$+\frac{\left(-e(1-n)b^3+\left(3cd+\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-5n)-3\sqrt{b^2-4ac}d(1-n)\right)b-2ac(6cd(1-\right.}{ac\,(b^2-4ac)\left(b^2+\sqrt{b^2-4ac}b-\right.}$$

$$+\frac{x(c(-eb^2+3cdb-2ace)\,x^n-6ac^2d+3b^2cd-b^3e+abce)\,e^2}{ac^2\,(b^2-4ac)\,n\,(bx^n+cx^{2n}+a)}$$

$$+\frac{\left((1-n)\left(-2ae^3nb^4+cd(c(1-2n)d^2+6ae^2n)\,b^3-ace(3cd^2-ae^2(2n+1))\,b^2-2ac^2d(c(2-7n)d^2+3\right.\right.}{}$$

$$+\frac{\left((1-n)\left(-2ae^3nb^4+cd(c(1-2n)d^2+6ae^2n)\,b^3-ace(3cd^2-ae^2(2n+1))\,b^2-2ac^2d(c(2-7n)d^2+3\right.\right.}{}$$

$$-\frac{x(c(-2ae^3nb^4+cd(c(1-2n)d^2+6ae^2n)\,b^3-ace(3cd^2-ae^2(2n+1))\,b^2-2ac^2d(c(2-7n)d^2+3ae^2n)}{}$$

$$+\frac{x(-((ab^2e^3+2ac(3cd^2-ae^2)\,e-bcd(cd^2+3ae^2))\,x^n)+b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2))}{2ac\,(b^2-4ac)\,n\,(bx^n+cx^{2n}+a)^2}$$

```
[In] Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b^
2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n))/(2*a*c*(
b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(3*b^2*c*d - 6*a*c^2*d -
 b^3*e + a*b*c*e + c*(3*b*c*d - b^2*e - 2*a*c*e)*x^n))/(a*c^2*(b^2 - 4*a*c)
*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*d*(3*a*e^2*(1 - 9*n) - 5*c*d^2*
(1 - 3*n)) + 4*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 4*n) - 2*a*b^5*e^3*n + 2*a^
2*b*c^2*e*(3*c*d^2*(2 - 3*n) - 5*a*e^2*n) - 3*a*b^3*c*e*(c*d^2 - 3*a*e^2*n)
 + b^4*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) + c*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)
*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^
3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))
)*x^n))/(2*a^2*c^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (e^2*(b*c
*(2*a*e*(2 - 5*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n)
 + Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d - Sqrt[b^2 - 4
*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c
])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n
- 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*
a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) - (2*a*b^5*e^3*(1 - n)*n
- b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*
```

```
e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1
- 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e^2*(1 - 11*
n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30*n^2)))/Sqr
t[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/(2*a^2*c*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*n^2)
+ (e^2*(b*c*(2*a*e*(2 - 5*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*
d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d + S
qrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a
*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1
- 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) + (2*a*b^5*e^3
*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c
*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2)
- a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e
^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30
*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*c*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*
a*c])*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^2d^3-3acde^2+abe^3+(3c^2d^2e-3bcde^2+b^2e^3-ace^3)\,x^n}{c^2\,(a+bx^n+cx^{2n})^3} + \frac{e^2(3cd-be+cex^n)}{c^2\,(a+bx^n+cx^{2n})^2}\right)dx$$

$$= \frac{\int \frac{c^2d^3-3acde^2+abe^3+(3c^2d^2e-3bcde^2+b^2e^3-ace^3)x^n}{(a+bx^n+cx^{2n})^3}\,dx}{c^2} + \frac{e^2\int \frac{3cd-be+cex^n}{(a+bx^n+cx^{2n})^2}\,dx}{c^2}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^n)}{2ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2} + \frac{e^2x(3b^2cd-6ac^2d-b^3e+abce+c(3bcd-b^2e-2ace)\,x^n)}{ac^2\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})} - \frac{\int \frac{-abce(3cd^2+ae^2(1-8n))-2ac^2d(cd^2-3ae^2)(1-4n)-2ab^3e^3n+b^2cd(cd^2(1-2n)+6ae^2n)-c(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+\ldots}{(a+bx^n+cx^{2n})^2}}{2ac^2\,(b^2-4ac)\,n} - \frac{e^2\int \frac{-abce-2ac(3cd-be)(1-2n)+b^2(3cd-be)(1-n)+c(3bcd-b^2e-2ace)(1-n)x^n}{a+bx^n+cx^{2n}}\,dx}{ac^2\,(b^2-4ac)\,n}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^n)}{2ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2} + \frac{e^2x(3b^2cd-6ac^2d-b^3e+abce+c(3bcd-b^2e-2ace)\,x^n)}{ac^2\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})} - \frac{x(ab^2c^2d(3ae^2(1-9n)-5cd^2(1-3n))+4a^2c^3d(cd^2-3ae^2)\,(1-4n)-2ab^5e^3n+2a^2bc^2e(3cd^2\ldots}{} + \frac{\int \frac{-2ab^5e^3(1-n)n+b^4cd(1-n)(cd^2(1-2n)+6ae^2n)+2a^2bc^2e(3cd^2(2-5n)-ae^2(7-16n)n)-ab^3ce(3cd^2(1-n)-2ae^2(5-8n)n)+4a^2\ldots}{}}{} - \frac{\left(e^2\left(\frac{2abce(2-5n)-12ac^2d(1-2n)+3b^2cd(1-n)-b^3e(1-n)}{\sqrt{b^2-4ac}}+(3bcd-b^2e-2ace)\,(1-n)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\ldots}{2ac\,(b^2-4ac)\,n} + \frac{\left(e^2\left(bc\left(2ae(2-5n)-3\sqrt{b^2-4ac}d(1-n)\right)-2ac\left(6cd(1-2n)-\sqrt{b^2-4ac}e(1-n)\right)+b^2\left(3cd\ldots\right.\right.\right.}{2ac\,(b^2-4ac)^{3/2}\,n}$$

$$= \text{Too large to display}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13018 vs. 2(1707) = 3414.

Time = 7.82 (sec) , antiderivative size = 13018, normalized size of antiderivative = 7.63

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2n})^3}dx$$

```
[In] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^3*x^(6*n) +
 b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*
x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*c^2*d^3*(2*n - 1) + 4*a^3*c^2*e^3*(n + 1) + (12*c^3*d^2*e*(3*n -
1) + b^2*c*e^3*(2*n - 1) - 18*b*c^2*d*e^2*n)*a^2 - (2*b*c^3*d^3*(7*n - 2) -
 3*b^2*c^2*d^2*e)*a)*x*x^(3*n) + (2*b^4*c*d^3*(2*n - 1) + 2*(b*c*e^3*(3*n +
 2) + 6*c^2*d*e^2)*a^3 - (3*b^2*c*d*e^2*(9*n + 1) - 6*b*c^2*d^2*e*(9*n - 4)
 - 4*c^3*d^3*(4*n - 1) - b^3*e^3*(3*n - 1))*a^2 - (b^2*c^2*d^3*(29*n - 9) -
 6*b^3*c*d^2*e)*a)*x*x^(2*n) + (b^5*d^3*(2*n - 1) - 4*a^4*c*e^3*(n - 1) + (
b^2*e^3*(10*n - 1) + 12*c^2*d^2*e*(5*n - 1) - 6*b*c*d*e^2*(5*n - 2))*a^3 +
(3*b^2*c*d^2*e*(4*n - 3) - 3*b^3*d*e^2*(2*n + 1) - 2*b*c^2*d^3*n)*a^2 - (4*
b^3*c*d^3*(3*n - 1) - 3*b^4*d^2*e)*a)*x*x^n + (a*b^4*d^3*(3*n - 1) - 6*(2*c
*d*e^2*(2*n - 1) - b*e^3*n)*a^4 + (4*c^2*d^3*(6*n - 1) + 6*b*c*d^2*e*(5*n -
 2) - 3*b^2*d*e^2*(n + 1))*a^3 - (b^2*c*d^3*(21*n - 5) + 3*b^3*d^2*e*(n - 1
))*a^2)*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n
^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3
*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 +
 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*
n^2)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*d^3 + 6*(2*c*d*e^2*(2*n -
1) - b*e^3*n)*a^3 + (4*(8*n^2 - 6*n + 1)*c^2*d^3 - 6*b*c*d^2*e*(5*n - 2) +
3*b^2*d*e^2*(n + 1))*a^2 - ((16*n^2 - 21*n + 5)*b^2*c*d^3 - 3*b^3*d^2*e*(n
- 1))*a + ((2*n^2 - 3*n + 1)*b^3*c*d^3 + 4*(n^2 - 1)*a^3*c*e^3 + (12*(3*n^2
 - 4*n + 1)*c^2*d^2*e - 18*(n^2 - n)*b*c*d*e^2 + (2*n^2 - 3*n + 1)*b^2*e^3)
*a^2 - (2*(7*n^2 - 9*n + 2)*b*c^2*d^3 - 3*b^2*c*d^2*e*(n - 1))*a)*x^n)/(a^3
*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^
2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b
*c^2*n^2)*x^n), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

```
[In] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3, x)
```

3.81 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 1191

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3} dx = \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)x^n)}{2a(b^2-4ac)n(a+bx^n+cx^{2n})^2}$$

$$+\frac{e^2x(b^2-2ac+bcx^n)}{ac(b^2-4ac)n(a+bx^n+cx^{2n})}$$

$$+\frac{x(2ab^3cde-ab^2c(ae^2(1-9n)-5cd^2(1-3n))-4a^2c^2(cd^2-ae^2)(1-4n)-4a^2bc^2de(2-3n)-b^4(cd^2}{2a^2c(b^2-4ac)}$$

$$-\frac{e^2\left(4ac(1-2n)-b^2(1-n)-b\sqrt{b^2-4ac}(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)n}$$

$$-\frac{\Big((1-n)(2ab^2cde-8a^2c^2de(1-3n)+2abc(cd^2(2-7n)+ae^2n)-b^3(cd^2(1-2n)+2ae^2n))+\frac{2ab^3cde(1}{}}{}$$

$$-\frac{e^2\left(4ac(1-2n)-b^2(1-n)+b\sqrt{b^2-4ac}(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)n}$$

$$-\frac{\Big((1-n)(2ab^2cde-8a^2c^2de(1-3n)+2abc(cd^2(2-7n)+ae^2n)-b^3(cd^2(1-2n)+2ae^2n))-\frac{2ab^3cde(1}{}}{}$$

```
[Out] 1/2*x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^n
)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^2+e^2*x*(b^2-2*a*c+b*c*x^n)/a/c/(-4*
a*c+b^2)/n/(a+b*x^n+c*x^(2*n))+1/2*x*(2*a*b^3*c*d*e-a*b^2*c*(a*e^2*(1-9*n)-
5*c*d^2*(1-3*n))-4*a^2*c^2*(-a*e^2+c*d^2)*(1-4*n)-4*a^2*b*c^2*d*e*(2-3*n)-b
^4*(c*d^2*(1-2*n)+2*a*e^2*n)+c*(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*a*b*c
*(c*d^2*(2-7*n)+a*e^2*n)-b^3*(c*d^2*(1-2*n)+2*a*e^2*n))*x^n)/a^2/c/(-4*a*c+
b^2)^2/n^2/(a+b*x^n+c*x^(2*n))-1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b
```

```
-(-4*a*c+b^2)^(1/2)))*((1-n)*(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*a*b*c*(
c*d^2*(2-7*n)+a*e^2*n)-b^3*(c*d^2*(1-2*n)+2*a*e^2*n))+(2*a*b^3*c*d*e*(1-n)-
b^4*(1-n)*(c*d^2*(1-2*n)+2*a*e^2*n)-8*a^2*b*c^2*d*e*(-3*n^2-n+1)-8*a^2*c^2*
(-a*e^2+c*d^2)*(8*n^2-6*n+1)+2*a*b^2*c*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2
-10*n+1)))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/n^2/(b-(-4*a*c+b^2)^(1/2)
)-1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((1-n)*
(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*a*b*c*(c*d^2*(2-7*n)+a*e^2*n)-b^3*(c
*d^2*(1-2*n)+2*a*e^2*n))+(-2*a*b^3*c*d*e*(1-n)+b^4*(1-n)*(c*d^2*(1-2*n)+2*a
*e^2*n)+8*a^2*b*c^2*d*e*(-3*n^2-n+1)+8*a^2*c^2*(-a*e^2+c*d^2)*(8*n^2-6*n+1)
-2*a*b^2*c*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2-10*n+1)))/(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^2/n^2/(b+(-4*a*c+b^2)^(1/2))-e^2*x*hypergeom([1, 1/n],[
1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(4*a*c*(1-2*n)-b^2*(1-n)-b*(1-n)*(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-e^2*x*h
ypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(1-2*n)-b
^2*(1-n)+b*(1-n)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b^2-4*a*c+b*(-4*a*c+
b^2)^(1/2))
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1450, 1444, 1436, 251, 1359}

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx =$$

$$-\frac{\left(-((1-n)b^2)-\sqrt{b^2-4ac}(1-n)b+4ac(1-2n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)e^2}{a\left(b^2-4ac\right)\left(b^2-\sqrt{b^2-4ac}b-4ac\right)n}$$

$$-\frac{\left(-((1-n)b^2)+\sqrt{b^2-4ac}(1-n)b+4ac(1-2n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)e^2}{a\left(b^2-4ac\right)\left(b^2+\sqrt{b^2-4ac}b-4ac\right)n}$$

$$+\frac{x(bcx^n+b^2-2ac)\,e^2}{ac\left(b^2-4ac\right)n\left(bx^n+cx^{2n}+a\right)}$$

$$-\frac{\left((1-n)\left(-((c(1-2n)d^2+2ae^2n)\,b^3)+2acdeb^2+2ac(c(2-7n)d^2+ae^2n)\,b-8a^2c^2de(1-3n)\right)+\ldots\right.}{\ldots}$$

$$-\frac{\left((1-n)\left(-((c(1-2n)d^2+2ae^2n)\,b^3)+2acdeb^2+2ac(c(2-7n)d^2+ae^2n)\,b-8a^2c^2de(1-3n)\right)-\ldots\right.}{\ldots}$$

$$+\frac{x(c(-((c(1-2n)d^2+2ae^2n)\,b^3)+2acdeb^2+2ac(c(2-7n)d^2+ae^2n)\,b-8a^2c^2de(1-3n))\,x^n+2ab^3\ldots}{2a^2c\,(b^2-\ldots}$$

$$+\frac{x((bcd^2-4aced+abe^2)\,x^n+b^2d^2-2abde-2a(cd^2-ae^2))}{2a\left(b^2-4ac\right)n\left(bx^n+cx^{2n}+a\right)^2}$$

```
[In] Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2 -
2*a*c + b*c*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (x*(2*a*b
^3*c*d*e - a*b^2*c*(a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) - 4*a^2*c^2*(c*d^2
 - a*e^2)*(1 - 4*n) - 4*a^2*b*c^2*d*e*(2 - 3*n) - b^4*(c*d^2*(1 - 2*n) + 2*
a*e^2*n) + c*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 -
 7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n))*x^n))/(2*a^2*c*(b^2 -
 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) - (e^2*(4*a*c*(1 - 2*n) - b^2*(1 - n
) - b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2
*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n)) + (
2*a*b^3*c*d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2*n) - 8*a^2*b
*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 2*
a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)))/Sqrt[b^2 -
 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*n^2) - (e^2*(4
*a*c*(1 - 2*n) - b^2*(1 - n) + b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 -
4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d*e -
8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*
(1 - 2*n) + 2*a*e^2*n)) - (2*a*b^3*c*d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 -
2*n) + 2*a*e^2*n) - 8*a^2*b*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*
e^2)*(1 - 6*n + 8*n^2) + 2*a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 -
10*n + 15*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b
^2 - 4*a*c])*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1359

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c +
n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n)
)^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p, -1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{cd^2-ae^2+(2cde-be^2)\,x^n}{c\,(a+bx^n+cx^{2n})^3}+\frac{e^2}{c\,(a+bx^n+cx^{2n})^2}\right)dx\\
&= \frac{\int \frac{cd^2-ae^2+(2cde-be^2)x^n}{(a+bx^n+cx^{2n})^3}\,dx}{c}+\frac{e^2\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx}{c}\\
&= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}\\
&\quad+\frac{e^2x(b^2-2ac+bcx^n)}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}\\
&\quad-\frac{\int \frac{-2abcde-2ac(cd^2-ae^2)(1-4n)+b^2(cd^2(1-2n)+2ae^2n)+c(bcd^2-4acde+abe^2)(1-3n)x^n}{(a+bx^n+cx^{2n})^2}\,dx}{2ac\,(b^2-4ac)\,n}\\
&\quad-\frac{e^2\int \frac{b^2-2ac-(b^2-4ac)n+bc(1-n)x^n}{a+bx^n+cx^{2n}}\,dx}{ac\,(b^2-4ac)\,n}
\end{aligned}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)\,x^n)}{2a\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})^2}$$

$$+ \frac{e^2x(b^2 - 2ac + bcx^n)}{ac\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})}$$

$$+ \frac{x(2ab^3cde - ab^2c(ae^2(1 - 9n) - 5cd^2(1 - 3n)) - 4a^2c^2(cd^2 - ae^2)\,(1 - 4n) - 4a^2bc^2de(2 - 3n) \cdots}{2a^2c\,(b \cdots}$$

$$+ \frac{\int \frac{4a^2bc^2de(2-5n)-2ab^3cde(1-n)+b^4(1-n)\left(cd^2(1-2n)+2ae^2n\right)+4a^2c^2\left(cd^2-ae^2\right)\left(1-6n+8n^2\right)-ab^2c\left(cd^2\left(5-21n+16n^2\right)-ae^2(1 \cdots}{a+bx^n+cx^{2n}}}{2a^2c\,(b^2 - 4ac)^2\,n^{\cdots}}$$

$$+ \frac{\left(e^2\left(4ac(1 - 2n) - b^2(1 - n) - b\sqrt{b^2 - 4ac}(1 - n)\right)\right)\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^n}\,dx}{2a\,(b^2 - 4ac)^{3/2}\,n}$$

$$- \frac{\left(e^2\left(4ac(1 - 2n) - b^2(1 - n) + b\sqrt{b^2 - 4ac}(1 - n)\right)\right)\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^n}\,dx}{2a\,(b^2 - 4ac)^{3/2}\,n}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)\,x^n)}{2a\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})^2}$$

$$+ \frac{e^2x(b^2 - 2ac + bcx^n)}{ac\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})}$$

$$+ \frac{x(2ab^3cde - ab^2c(ae^2(1 - 9n) - 5cd^2(1 - 3n)) - 4a^2c^2(cd^2 - ae^2)\,(1 - 4n) - 4a^2bc^2de(2 - 3n) \cdots}{2a^2c\,(b \cdots}$$

$$+ \frac{e^2\left(4ac(1 - 2n) - b^2(1 - n) - b\sqrt{b^2 - 4ac}(1 - n)\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2-4ac}}\right)}{a\,(b^2 - 4ac)^{3/2}\left(b - \sqrt{b^2 - 4ac}\right)n}$$

$$- \frac{e^2\left(4ac(1 - 2n) - b^2(1 - n) + b\sqrt{b^2 - 4ac}(1 - n)\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2-4ac}}\right)}{a\,(b^2 - 4ac)^{3/2}\left(b + \sqrt{b^2 - 4ac}\right)n}$$

$$- \frac{\left((1 - n)\,(2ab^2cde - 8a^2c^2de(1 - 3n) + 2abc(cd^2(2 - 7n) + ae^2n) - b^3(cd^2(1 - 2n) + 2ae^2n)) - \cdots\right.}{\cdots}$$

$$- \frac{\left((1 - n)\,(2ab^2cde - 8a^2c^2de(1 - 3n) + 2abc(cd^2(2 - 7n) + ae^2n) - b^3(cd^2(1 - 2n) + 2ae^2n)) + \cdots\right.}{\cdots}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)\,x^n)}{2a\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})^2}$$

$$+ \frac{e^2x(b^2 - 2ac + bcx^n)}{ac\,(b^2 - 4ac)\,n\,(a + bx^n + cx^{2n})}$$

$$+ \frac{x(2ab^3cde - ab^2c(ae^2(1 - 9n) - 5cd^2(1 - 3n)) - 4a^2c^2(cd^2 - ae^2)\,(1 - 4n) - 4a^2bc^2de(2 - 3n)}{2a^2c\,(}$$

$$+ \frac{e^2\left(4ac(1 - 2n) - b^2(1 - n) - b\sqrt{b^2 - 4ac}(1 - n)\right) x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a\,(b^2 - 4ac)^{3/2}\left(b - \sqrt{b^2 - 4ac}\right)n}$$

$$- \frac{\left((1 - n)\,(2ab^2cde - 8a^2c^2de(1 - 3n) + 2abc(cd^2(2 - 7n) + ae^2n) - b^3(cd^2(1 - 2n) + 2ae^2n)) \right.}{}$$

$$- \frac{e^2\left(4ac(1 - 2n) - b^2(1 - n) + b\sqrt{b^2 - 4ac}(1 - n)\right) x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\,(b^2 - 4ac)^{3/2}\left(b + \sqrt{b^2 - 4ac}\right)n}$$

$$- \frac{\left((1 - n)\,(2ab^2cde - 8a^2c^2de(1 - 3n) + 2abc(cd^2(2 - 7n) + ae^2n) - b^3(cd^2(1 - 2n) + 2ae^2n)) \right.}{}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10910 vs. 2(1191) = 2382.

Time = 7.52 (sec) , antiderivative size = 10910, normalized size of antiderivative = 9.16

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(d + e\,x^n)^2}{(a + b\,x^n + c\,x^{2n})^3}dx$$

```
[In] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b
^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x
^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*c^2*d^2*(2*n - 1) + 2*(4*c^3*d*e*(3*n - 1) - 3*b*c^2*e^2*n)*a^2 -
 2*(b*c^3*d^2*(7*n - 2) - b^2*c^2*d*e)*a)*x*x^(3*n) + (2*b^4*c*d^2*(2*n - 1
) + 4*a^3*c^2*e^2 - (b^2*c*e^2*(9*n + 1) - 4*b*c^2*d*e*(9*n - 4) - 4*c^3*d^
2*(4*n - 1))*a^2 - (b^2*c^2*d^2*(29*n - 9) - 4*b^3*c*d*e)*a)*x*x^(2*n) + (b
^5*d^2*(2*n - 1) + 2*(4*c^2*d*e*(5*n - 1) - b*c*e^2*(5*n - 2))*a^3 + (2*b^2
*c*d*e*(4*n - 3) - b^3*e^2*(2*n + 1) - 2*b*c^2*d^2*n)*a^2 - 2*(2*b^3*c*d^2*
(3*n - 1) - b^4*d*e)*a)*x*x^n + (a*b^4*d^2*(3*n - 1) - 4*a^4*c*e^2*(2*n - 1
) + (4*c^2*d^2*(6*n - 1) + 4*b*c*d*e*(5*n - 2) - b^2*e^2*(n + 1))*a^3 - (b^
2*c*d^2*(21*n - 5) + 2*b^3*d*e*(n - 1))*a^2)*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*
n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^
2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*
n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*
n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*((2*n^2 - 3
*n + 1)*b^4*d^2 + 4*a^3*c*e^2*(2*n - 1) + (4*(8*n^2 - 6*n + 1)*c^2*d^2 - 4*
```

```
b*c*d*e*(5*n - 2) + b^2*e^2*(n + 1))*a^2 - ((16*n^2 - 21*n + 5)*b^2*c*d^2 -
 2*b^3*d*e*(n - 1))*a + ((2*n^2 - 3*n + 1)*b^3*c*d^2 + 2*(4*(3*n^2 - 4*n +
1)*c^2*d*e - 3*(n^2 - n)*b*c*e^2)*a^2 - 2*((7*n^2 - 9*n + 2)*b*c^2*d^2 - b^
2*c*d*e*(n - 1))*a)*x^n)/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 +
(a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2
 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx=\int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx=\int \frac{(d+e\,x^n)^2}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

```
[In] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3, x)
```

3.82 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 713

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$

$$+\frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-3n)-2a^2bce(2-3n)-b^4d(1-2n)+c(ab^2e+2abcd(2-7n)-}{2a^2\,(b^2-4ac)^2\,n^2\,(a+bx^n+cx^{2n})}$$

$$+\frac{c\left(ab^2\left(\sqrt{b^2-4ac}e+6cd(1-3n)\right)(1-n)+b^3\left(ae-\sqrt{b^2-4ac}d(1-2n)\right)(1-n)-b^4d(1-3n+2n^2)\right.}{}$$

$$-\frac{c\left(ab^2\left(\sqrt{b^2-4ac}e-6cd(1-3n)\right)(1-n)-b^3\left(ae+\sqrt{b^2-4ac}d(1-2n)\right)(1-n)+b^4d(1-3n+2n^2)\right.}{}$$

```
[Out] 1/2*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*
x^(2*n))^2+1/2*x*(a*b^3*e-4*a^2*c^2*d*(1-4*n)+5*a*b^2*c*d*(1-3*n)-2*a^2*b*c
*e*(2-3*n)-b^4*d*(1-2*n)+c*(a*b^2*e+2*a*b*c*d*(2-7*n)-4*a^2*c*e*(1-3*n)-b^3
*d*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))+1/2*c*x*hyperge
om([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(-b^4*d*(2*n^2-3*n+1)+
a*b^2*(1-n)*(6*c*d*(1-3*n)+e*(-4*a*c+b^2)^(1/2))+b^3*(1-n)*(a*e-d*(1-2*n)*(
-4*a*c+b^2)^(1/2))-4*a^2*c*(2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4*a*c+b^2
)^(1/2))-2*a*b*c*(2*a*e*(-3*n^2-n+1)-d*(7*n^2-9*n+2)*(-4*a*c+b^2)^(1/2)))/a
^2/(-4*a*c+b^2)^2/n^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-1/2*c*x*hypergeom([1
, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^4*d*(2*n^2-3*n+1)+a*b^2*
(1-n)*(-6*c*d*(1-3*n)+e*(-4*a*c+b^2)^(1/2))-b^3*(1-n)*(a*e+d*(1-2*n)*(-4*a*
c+b^2)^(1/2))-4*a^2*c*(-2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4*a*c+b^2)^(1
/2))+2*a*b*c*(2*a*e*(-3*n^2-n+1)+d*(7*n^2-9*n+2)*(-4*a*c+b^2)^(1/2)))/a^2/(
-4*a*c+b^2)^2/n^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1444, 1436, 251}

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \frac{x(cx^n(-4a^2ce(1-3n)+ab^2e+2abcd(2-7n)+b^3(-d)(1-2n))-2a^2bce(2-3n)-4a^2c^2d(1-4n)+}{2a^2n^2(b^2-4ac)^2(a+bx^n+cx^{2n})}$$

$$+\frac{cx\left(-4a^2c\left(e(3n^2-4n+1)\sqrt{b^2-4ac}+2cd(8n^2-6n+1)\right)-2abc\left(2ae(-3n^2-n+1)-d(7n^2-9n\right.}{}$$

$$-\frac{cx\left(-4a^2c\left(e(3n^2-4n+1)\sqrt{b^2-4ac}-2cd(8n^2-6n+1)\right)+2abc\left(d(7n^2-9n+2)\sqrt{b^2-4ac}+2ae(\right.}{}$$

$$+\frac{x(cx^n(bd-2ae)-abe-2acd+b^2d)}{2an(b^2-4ac)(a+bx^n+cx^{2n})^2}$$

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a
 + b*x^n + c*x^(2*n))^2) + (x*(a*b^3*e - 4*a^2*c^2*d*(1 - 4*n) + 5*a*b^2*c*
d*(1 - 3*n) - 2*a^2*b*c*e*(2 - 3*n) - b^4*d*(1 - 2*n) + c*(a*b^2*e + 2*a*b*
c*d*(2 - 7*n) - 4*a^2*c*e*(1 - 3*n) - b^3*d*(1 - 2*n))*x^n))/(2*a^2*(b^2 -
4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e + 6*
c*d*(1 - 3*n))*(1 - n) + b^3*(a*e - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n)
- b^4*d*(1 - 3*n + 2*n^2) - 2*a*b*c*(2*a*e*(1 - n - 3*n^2) - Sqrt[b^2 - 4*a
*c]*d*(2 - 9*n + 7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) +
 2*c*d*(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*n^2) - (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e - 6*c*d*(1 - 3*n))*(1
 - n) - b^3*(a*e + Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^4*d*(1 - 3*n
+ 2*n^2) + 2*a*b*c*(2*a*e*(1 - n - 3*n^2) + Sqrt[b^2 - 4*a*c]*d*(2 - 9*n +
7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n +
 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n
^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

integral

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{-abe-2acd(1-4n)+b^2(d-2dn)+c(bd-2ae)(1-3n)x^n}{\left(a+bx^n+cx^{2n}\right)^2}\,dx}{2a\left(b^2-4ac\right)n}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)^2} + \frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-3n)-2a^2bce(2-3n)-b^4d(1-2n)+c(ab^2e+2abcd(2-7n)-}{2a^2\left(b^2-4ac\right)^2n^2\left(a+bx^n+cx^{2n}\right)} + \frac{\int \frac{2a^2bce(2-5n)-ab^3e(1-n)+b^4d(1-3n+2n^2)+4a^2c^2d(1-6n+8n^2)-ab^2cd(5-21n+16n^2)-c(ab^2e+2abcd(2-7n)-4a^2ce(1-3n)-b^3d(1-2n}{a+bx^n+cx^{2n}}}{2a^2\left(b^2-4ac\right)^2n^2}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)^2} + \frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-3n)-2a^2bce(2-3n)-b^4d(1-2n)+c(ab^2e+2abcd(2-7n)-}{2a^2\left(b^2-4ac\right)^2n^2\left(a+bx^n+cx^{2n}\right)} - \frac{\left(c\left(ab^2\left(\sqrt{b^2-4ac}e-6cd(1-3n)\right)(1-n)-b^3\left(ae+\sqrt{b^2-4ac}d(1-2n)\right)(1-n)+b^4d(1-3n+2n^2\right)}{} - \frac{\left(c\left(ab^2\left(\sqrt{b^2-4ac}e+6cd(1-3n)\right)(1-n)+b^3\left(ae-\sqrt{b^2-4ac}d(1-2n)\right)(1-n)-b^4d(1-3n+2n^2\right)}{}$$

$$= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{x(ab^3e - 4a^2c^2d(1 - 4n) + 5ab^2cd(1 - 3n) - 2a^2bce(2 - 3n) - b^4d(1 - 2n) + c(ab^2e + 2abcd(2 - 7n) -}{2a^2(b^2 - 4ac)^2 n^2(a + bx^n + cx^{2n})}$$

$$- \frac{c\left(ab^2\left(\sqrt{b^2 - 4ac}e + 6cd(1 - 3n)\right)(1 - n) + b^3\left(ae - \sqrt{b^2 - 4ac}d(1 - 2n)\right)(1 - n) - b^4d(1 - 3n + 2n^2)}{}$$

$$- \frac{c\left(ab^2\left(\sqrt{b^2 - 4ac}e - 6cd(1 - 3n)\right)(1 - n) - b^3\left(ae + \sqrt{b^2 - 4ac}d(1 - 2n)\right)(1 - n) + b^4d(1 - 3n + 2n^2)}{}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8593 vs. $2(713) = 1426$.

Time = 6.73 (sec) , antiderivative size = 8593, normalized size of antiderivative = 12.05

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^3} dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{d + e\,x^n}{(a + b\,x^n + c\,x^{2n})^3} dx$$

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^3} dx = \int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^3} dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
[Out] integral((e*x^n + d)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b
*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n
 + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a^2*c^3*e*(3*n - 1) + b^3*c^2*d*(2*n - 1) - (2*b*c^3*d*(7*n - 2) -
b^2*c^2*e)*a)*x*x^(3*n) + (2*b^4*c*d*(2*n - 1) + 2*(b*c^2*e*(9*n - 4) + 2*c
^3*d*(4*n - 1))*a^2 - (b^2*c^2*d*(29*n - 9) - 2*b^3*c*e)*a)*x*x^(2*n) + (4*
a^3*c^2*e*(5*n - 1) + b^5*d*(2*n - 1) + (b^2*c*e*(4*n - 3) - 2*b*c^2*d*n)*a
^2 - (4*b^3*c*d*(3*n - 1) - b^4*e)*a)*x*x^n + (a*b^4*d*(3*n - 1) + 2*(2*c^2
*d*(6*n - 1) + b*c*e*(5*n - 2))*a^3 - (b^2*c*d*(21*n - 5) + b^3*e*(n - 1))*
a^2)*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2
- 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^
3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32
*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2
)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*d + 2*(2*(8*n^2 - 6*n + 1)*c^
2*d - b*c*e*(5*n - 2))*a^2 - ((16*n^2 - 21*n + 5)*b^2*c*d - b^3*e*(n - 1))*
a + ((2*n^2 - 3*n + 1)*b^3*c*d + 4*(3*n^2 - 4*n + 1)*a^2*c^2*e - (2*(7*n^2
- 9*n + 2)*b*c^2*d - b^2*c*e*(n - 1))*a)*x^n)/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^
2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x
^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^3, x)
```

3.83 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 1708

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3} \, dx = \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})^2}$$

$$+ \frac{e^2x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+bx^n+cx^{2n})}$$

$$+ \frac{x(2a^2bc^2e(4-11n)-3ab^3ce(2-5n)-4a^2c^3d(1-4n)+5ab^2c^2d(1-3n)-b^4cd(1-2n)+b^5(e-2en)}{2a^2\,(b^2-4ac)^2\,(cd^2-bde+ae^2)}$$

$$- \frac{ce^4\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+ \frac{ce^2\left(bc\left(2ae(2-3n)+\sqrt{b^2-4ac}d(1-n)\right)-2ac\left(2cd(1-2n)-\sqrt{b^2-4ac}e(1-n)\right)-b^3e(1-n)+b^2(}{a\,(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-}$$

$$- \frac{c\left(ab^2c\left(\sqrt{b^2-4ac}e(5-14n)-6cd(1-3n)\right)(1-n)+b^3c\left(ae(7-18n)+\sqrt{b^2-4ac}d(1-2n)\right)(1-n)}{}$$

$$- \frac{ce^4\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+ \frac{ce^2\left(bc\left(2ae(2-3n)-\sqrt{b^2-4ac}d(1-n)\right)-2ac\left(2cd(1-2n)+\sqrt{b^2-4ac}e(1-n)\right)-b^3e(1-n)+b^2(}{a\,(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-}$$

$$+ \frac{c\left(ab^2c\left(\sqrt{b^2-4ac}e(5-14n)+6cd(1-3n)\right)(1-n)-b^3c\left(ae(7-18n)-\sqrt{b^2-4ac}d(1-2n)\right)(1-n)}{}$$

$$+ \frac{e^6x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)^3}$$

```
[Out] 1/2*x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/n/(a+b*x^n+c*x^(2*n))^2+e^2*x*(b^2*c*d-2*a*c^
2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4*a*c+b^2)/(a*e^2-b*d*
e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))+1/2*x*(2*a^2*b*c^2*e*(4-11*n)-3*a*b^3*c*e*
(2-5*n)-4*a^2*c^3*d*(1-4*n)+5*a*b^2*c^2*d*(1-3*n)-b^4*c*d*(1-2*n)+b^5*(-2*e
*n+e)-c*(a*b^2*c*e*(5-14*n)-2*a*b*c^2*d*(2-7*n)-4*a^2*c^2*e*(1-3*n)+b^3*c*d
*(1-2*n)-b^4*e*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/n^2/(a+
b*x^n+c*x^(2*n))+e^6*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+
c*d^2)^3-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c+b*(-4*a
*c+b^2)^(1/2))-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c-b
*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(c*d-e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e
*(2-3*n)+d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(2*c*d*(1-2*n)-e*(1-n)*(-4*a*c+b
^2)^(1/2)))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c-b*(-4*a*c+b^2
)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(-b^3*e*(1-n)+b^2*(1-n)*(c*d+e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e*(2-3*n)-d*(
1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(2*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^2)^(1/2)))/
a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))+1/2
*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^5*e*(2*
n^2-3*n+1)-b^4*(2*n^2-3*n+1)*(c*d+e*(-4*a*c+b^2)^(1/2))+a*b^2*c*(1-n)*(6*c*
d*(1-3*n)+e*(5-14*n)*(-4*a*c+b^2)^(1/2))-b^3*c*(1-n)*(a*e*(7-18*n)-d*(1-2*n
)*(-4*a*c+b^2)^(1/2))-4*a^2*c^2*(2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4*a*
c+b^2)^(1/2))-2*a*b*c^2*(-2*a*e*(13*n^2-13*n+3)+d*(7*n^2-9*n+2)*(-4*a*c+b^2
)^(1/2)))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/n^2/(b^2-4*a*c+b*(-4*a*c+b
^2)^(1/2))-1/2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))*(-b^5*e*(2*n^2-3*n+1)+b^4*(2*n^2-3*n+1)*(c*d-e*(-4*a*c+b^2)^(1/2))+a*b^
2*c*(1-n)*(-6*c*d*(1-3*n)+e*(5-14*n)*(-4*a*c+b^2)^(1/2))+b^3*c*(1-n)*(a*e*(
7-18*n)+d*(1-2*n)*(-4*a*c+b^2)^(1/2))-4*a^2*c^2*(-2*c*d*(8*n^2-6*n+1)+e*(3*
n^2-4*n+1)*(-4*a*c+b^2)^(1/2))-2*a*b*c^2*(2*a*e*(13*n^2-13*n+3)+d*(7*n^2-9*
n+2)*(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/n^2/(b^2-4
*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 3.23 (sec) , antiderivative size = 1708, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used

$= \{1450,\ 251,\ 1444,\ 1436\}$

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^3}\,dx = \frac{x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^6}{d\left(cd^2-bed+ae^2\right)^3}$$

$$-\frac{c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)e^4}{\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}$$

$$-\frac{c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)e^4}{\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}$$

$$+\frac{c\left(-e(1-n)b^3+\left(cd-\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-3n)+\sqrt{b^2-4ac}d(1-n)\right)b-2ac\left(2cd(1-2\right.\right.}{a\left(b^2-4ac\right)\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2\right.}$$

$$+\frac{c\left(-e(1-n)b^3+\left(cd+\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-3n)-\sqrt{b^2-4ac}d(1-n)\right)b-2ac\left(2cd(1-2\right.\right.}{a\left(b^2-4ac\right)\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2\right.}$$

$$+\frac{x\left(c\left(-eb^2+cdb+2ace\right)x^n-2ac^2d+b^2cd-b^3e+3abce\right)e^2}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2n\left(bx^n+cx^{2n}+a\right)}$$

$$-\frac{c\left(-e(2n^2-3n+1)\,b^5+\left(cd-\sqrt{b^2-4ac}e\right)(2n^2-3n+1)\,b^4+c\left(ae(7-18n)+\sqrt{b^2-4ac}d(1-2n)\right)(\right.}{}$$

$$+\frac{c\left(e(2n^2-3n+1)\,b^5-\left(cd+\sqrt{b^2-4ac}e\right)(2n^2-3n+1)\,b^4-c\left(ae(7-18n)-\sqrt{b^2-4ac}d(1-2n)\right)(1\right.}{}$$

$$+\frac{x\left(-c\left(-e(1-2n)b^4+cd(1-2n)b^3+ace(5-14n)b^2-2ac^2d(2-7n)b-4a^2c^2e(1-3n)\right)x^n+2a^2bc^2e(\right.}{2a^2\left(b^2-4ac\right)^2\left(cd^2-bed+ae\right.}$$

$$+\frac{x\left(c\left(-eb^2+cdb+2ace\right)x^n-2ac^2d+b^2cd-b^3e+3abce\right)}{2a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)n\left(bx^n+cx^{2n}+a\right)^2}$$

```
[In] Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))^2
) + (e^2*x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*
a*c*e)*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^
(2*n))) + (x*(2*a^2*b*c^2*e*(4 - 11*n) - 3*a*b^3*c*e*(2 - 5*n) - 4*a^2*c^3*
d*(1 - 4*n) + 5*a*b^2*c^2*d*(1 - 3*n) - b^4*c*d*(1 - 2*n) + b^5*(e - 2*e*n)
 - c*(a*b^2*c*e*(5 - 14*n) - 2*a*b*c^2*d*(2 - 7*n) - 4*a^2*c^2*e*(1 - 3*n)
+ b^3*c*d*(1 - 2*n) - b^4*e*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2
- b*d*e + a*e^2)*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2
*a*c*(2*c*d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*
```

```
(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b
*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (c*(a*b^2*c*(Sqrt[b^2 -
4*a*c]*e*(5 - 14*n) - 6*c*d*(1 - 3*n))*(1 - n) + b^3*c*(a*e*(7 - 18*n) + Sq
rt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^5*e*(1 - 3*n + 2*n^2) + b^4*(c*d -
 Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqrt[b^2 - 4*a*c]*e*(1
 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a*c]*d
*(2 - 9*n + 7*n^2) + 2*a*e*(3 - 13*n + 13*n^2)))*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^
2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n^2) - (c*e^4
*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*
c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 -
4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*(1 - n)) -
 b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*
a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (c*
(a*b^2*c*(Sqrt[b^2 - 4*a*c]*e*(5 - 14*n) + 6*c*d*(1 - 3*n))*(1 - n) - b^3*c
*(a*e*(7 - 18*n) - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^5*e*(1 - 3*n
+ 2*n^2) - b^4*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(S
qrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2
*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) - 2*a*e*(3 - 13*n + 13*n^2)))*x*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
 a*e^2)*n^2) + (e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)
])/(d*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
```

```
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^6}{(cd^2-bde+ae^2)^3(d+ex^n)} + \frac{cd-be-cex^n}{(cd^2-bde+ae^2)(a+bx^n+cx^{2n})^3} - \frac{e^2(-cd+be+cex^n)}{(cd^2-bde+ae^2)^2(a+bx^n+cx^{2n})^2} - \frac{e^4(-cd+be+cex^n)}{(cd^2-bde+ae^2)^3(a+bx^n+cx^{2n})}\right) dx$$

$$= -\frac{e^4\int \frac{-cd+be+cex^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^3} + \frac{e^6\int \frac{1}{d+ex^n}\,dx}{(cd^2-bde+ae^2)^3} - \frac{e^2\int \frac{-cd+be+cex^n}{(a+bx^n+cx^{2n})^2}\,dx}{(cd^2-bde+ae^2)^2} + \frac{\int \frac{cd-be-cex^n}{(a+bx^n+cx^{2n})^3}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^n)}{2a(b^2-4ac)(cd^2-bde+ae^2)n(a+bx^n+cx^{2n})^2} + \frac{e^2x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^n)}{a(b^2-4ac)(cd^2-bde+ae^2)^2n(a+bx^n+cx^{2n})} + \frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^3} - \frac{\left(ce^4\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2(cd^2-bde+ae^2)^3} - \frac{\left(ce^4\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2(cd^2-bde+ae^2)^3} + \frac{e^2\int \frac{-abce(3-4n)+2ac^2d(1-2n)-b^2cd(1-n)+b^3(e-en)-c(bcd-b^2e+2ace)(1-n)x^n}{a+bx^n+cx^{2n}}\,dx}{a(b^2-4ac)(cd^2-bde+ae^2)^2n} - \frac{\int \frac{abce-2ac(cd-be)(1-4n)+b^2(cd-be)(1-2n)+c(bcd-b^2e+2ace)(1-3n)x^n}{(a+bx^n+cx^{2n})^2}\,dx}{2a(b^2-4ac)(cd^2-bde+ae^2)n}$$

$$= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)\,x^n)}{2a\,(b^2 - 4ac)\,(cd^2 - bde + ae^2)\,n\,(a + bx^n + cx^{2n})^2}$$
$$+ \frac{e^2x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)\,x^n)}{a\,(b^2 - 4ac)\,(cd^2 - bde + ae^2)^2\,n\,(a + bx^n + cx^{2n})}$$
$$+ \frac{x(2a^2bc^2e(4 - 11n) - 3ab^3ce(2 - 5n) - 4a^2c^3d(1 - 4n) + 5ab^2c^2d(1 - 3n) - b^4cd(1 - 2n) + b^5}{2a^2\,(b^2 - 4ac)^2\,(cd^2 - b}$$
$$- \frac{ce^4\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b - \sqrt{b^2 - 4ac}\right)(cd^2 - bde + ae^2)^3}$$
$$- \frac{ce^4\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b + \sqrt{b^2 - 4ac}\right)(cd^2 - bde + ae^2)^3} + \frac{e^6x\,{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d\,(cd^2 - bde + ae^2)^3}$$
$$+ \frac{\int \frac{b^4cd(1-3n+2n^2)-b^5e(1-3n+2n^2)+2ab^3ce(3-11n+8n^2)+4a^2c^3d(1-6n+8n^2)-ab^2c^2d(5-21n+16n^2)-2a^2bc^2e(4-17n+16}{a+bx^n+cx^{2n}}}{2a^2\,(b^2 - 4ac)^2\,(cd^2 - bde + ae}$$
$$- \frac{\left(ce^2\left((bcd - b^2e + 2ace)\,(1 - n) - \frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)-b^3(e-en)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,c}{2a\,(b^2 - 4ac)\,(cd^2 - bde + ae^2)^2\,n}$$
$$+ \frac{\left(e^2\left(-\frac{1}{2}c(bcd - b^2e + 2ace)\,(1 - n) + \frac{bc(bcd-b^2e+2ace)(1-n)+2c(-abce(3-4n)+2ac^2d(1-2n)-b^2cd(1-n)+b^3(e}{2\sqrt{b^2-4ac}}\right.\right.}{a\,(b^2 - 4ac)\,(cd^2 - bde + ae^2)^2\,n}$$

= Too large to display

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 43535 vs. 2(1708) = 3416.

Time = 7.87 (sec) , antiderivative size = 43535, normalized size of antiderivative = 25.49

$$\int \frac{1}{(d + ex^n)\,(a + bx^n + cx^{2n})^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{1}{(d+e\,x^{n})\,(a+b\,x^{n}+c\,x^{2n})^{3}}dx$$

```
[In] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^{n})\,(a+bx^{n}+cx^{2n})^{3}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^{n}+a)^{3}(ex^{n}+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*e*x^(4*n) + a^3*d + (c^3*e*x^n + c^3*d)*x^(6*n) + 3*(b*c^2*
e*x^(2*n) + a*c^2*d + (b*c^2*d + a*c^2*e)*x^n)*x^(4*n) + (b^3*d + 3*a*b^2*e
)*x^(3*n) + 3*(b^2*c*e*x^(3*n) + a^2*c*d + (b^2*c*d + 2*a*b*c*e)*x^(2*n) +
(2*a*b*c*d + a^2*c*e)*x^n)*x^(2*n) + 3*(a*b^2*d + a^2*b*e)*x^(2*n) + (3*a^2
*b*d + a^3*e)*x^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^{n})\,(a+bx^{n}+cx^{2n})^{3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^{n})\,(a+bx^{n}+cx^{2n})^{3}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^{n}+a)^{3}(ex^{n}+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] e^6*integrate(1/(c^3*d^7 - 3*b*c^2*d^6*e + 3*b^2*c*d^5*e^2 - b^3*d^4*e^3 +
a^3*d*e^6 + 3*(c*d^3*e^4 - b*d^2*e^5)*a^2 + 3*(c^2*d^5*e^2 - 2*b*c*d^4*e^3
+ b^2*d^3*e^4)*a + (c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - b^3*d^3
*e^4 + a^3*e^7 + 3*(c*d^2*e^5 - b*d*e^6)*a^2 + 3*(c^2*d^4*e^3 - 2*b*c*d^3*e
```

^4 + b^2*d^2*e^5)*a)*x^n), x) - 1/2*((4*a^3*c^4*e^3*(7*n - 1) - b^3*c^4*d^3
*(2*n - 1) + 2*b^4*c^3*d^2*e*(2*n - 1) - b^5*c^2*d*e^2*(2*n - 1) - (b^2*c^3
e^3(26*n - 5) - 4*c^5*d^2*e*(3*n - 1) - 10*b*c^4*d*e^2*n)*a^2 - (b^2*c^4*
d^2*e*(28*n - 9) - 2*b*c^5*d^3*(7*n - 2) - 2*b^3*c^3*d*e^2*(5*n - 2) - b^4*
c^2*e^3*(4*n - 1))*a)*x*x^(3*n) - (2*b^4*c^3*d^3*(2*n - 1) - 4*b^5*c^2*d^2*
e*(2*n - 1) + 2*b^6*c*d*e^2*(2*n - 1) - 2*(b*c^3*e^3*(37*n - 6) - 2*c^4*d*e
^2*(8*n - 1))*a^3 - (2*b*c^4*d^2*e*(25*n - 8) + 3*b^2*c^3*d*e^2*(5*n + 1) -
11*b^3*c^2*e^3*(5*n - 1) - 4*c^5*d^3*(4*n - 1))*a^2 - (b^2*c^4*d^3*(29*n -
9) - 2*b^3*c^3*d^2*e*(29*n - 10) + 3*b^4*c^2*d*e^2*(7*n - 3) + 2*b^5*c*e^3
*(4*n - 1))*a)*x*x^(2*n) + (4*a^4*c^3*e^3*(9*n - 1) - b^5*c^2*d^3*(2*n - 1)
+ 2*b^6*c*d^2*e*(2*n - 1) - b^7*d*e^2*(2*n - 1) + (b^2*c^2*e^3*(14*n - 3)
- 2*b*c^3*d*e^2*(13*n - 2) + 4*c^4*d^2*e*(5*n - 1))*a^3 - (b^4*c*e^3*(24*n
- 5) - b^3*c^2*d*e^2*(20*n - 1) - 2*b*c^4*d^3*n + 3*b^2*c^3*d^2*e)*a^2 - (3
*b^4*c^2*d^2*e*(8*n - 3) - b^6*e^3*(4*n - 1) - 4*b^3*c^3*d^3*(3*n - 1) - 4*
b^5*c*d*e^2*(2*n - 1))*a)*x*x^n + (2*(b*c^2*e^3*(29*n - 4) - 2*c^3*d*e^2*(1
0*n - 1))*a^4 + (2*b*c^3*d^2*e*(29*n - 6) - 4*c^4*d^3*(6*n - 1) - 6*b^3*c*e
^3*(6*n - 1) - b^2*c^2*d*e^2*(n - 3))*a^3 - (b^3*c^2*d^2*e*(43*n - 11) - b^
2*c^3*d^3*(21*n - 5) - b^4*c*d*e^2*(17*n - 5) - b^5*e^3*(5*n - 1))*a^2 - (b
^4*c^2*d^3*(3*n - 1) - 2*b^5*c*d^2*e*(3*n - 1) + b^6*d*e^2*(3*n - 1))*a)*x)
/(16*a^8*c^2*e^4*n^2 + 8*(4*c^3*d^2*e^2*n^2 - 4*b*c^2*d*e^3*n^2 - b^2*c*e^4
*n^2)*a^7 + (16*c^4*d^4*n^2 - 32*b*c^3*d^3*e*n^2 + 16*b^3*c*d*e^3*n^2 + b^4
*e^4*n^2)*a^6 - 2*(4*b^2*c^3*d^4*n^2 - 8*b^3*c^2*d^3*e*n^2 + 3*b^4*c*d^2*e^
2*n^2 + b^5*d*e^3*n^2)*a^5 + (b^4*c^2*d^4*n^2 - 2*b^5*c*d^3*e*n^2 + b^6*d^2
*e^2*n^2)*a^4 + (16*a^6*c^4*e^4*n^2 + 8*(4*c^5*d^2*e^2*n^2 - 4*b*c^4*d*e^3*
n^2 - b^2*c^3*e^4*n^2)*a^5 + (16*c^6*d^4*n^2 - 32*b*c^5*d^3*e*n^2 + 16*b^3*
c^3*d*e^3*n^2 + b^4*c^2*e^4*n^2)*a^4 - 2*(4*b^2*c^5*d^4*n^2 - 8*b^3*c^4*d^3
*e*n^2 + 3*b^4*c^3*d^2*e^2*n^2 + b^5*c^2*d*e^3*n^2)*a^3 + (b^4*c^4*d^4*n^2
- 2*b^5*c^3*d^3*e*n^2 + b^6*c^2*d^2*e^2*n^2)*a^2)*x^(4*n) + 2*(16*a^6*b*c^3
*e^4*n^2 + 8*(4*b*c^4*d^2*e^2*n^2 - 4*b^2*c^3*d*e^3*n^2 - b^3*c^2*e^4*n^2)*
a^5 + (16*b*c^5*d^4*n^2 - 32*b^2*c^4*d^3*e*n^2 + 16*b^4*c^2*d*e^3*n^2 + b^5
*c*e^4*n^2)*a^4 - 2*(4*b^3*c^4*d^4*n^2 - 8*b^4*c^3*d^3*e*n^2 + 3*b^5*c^2*d^
2*e^2*n^2 + b^6*c*d*e^3*n^2)*a^3 + (b^5*c^3*d^4*n^2 - 2*b^6*c^2*d^3*e*n^2 +
b^7*c*d^2*e^2*n^2)*a^2)*x^(3*n) + (32*a^7*c^3*e^4*n^2 + 64*(c^4*d^2*e^2*n^
2 - b*c^3*d*e^3*n^2)*a^6 + 2*(16*c^5*d^4*n^2 - 32*b*c^4*d^3*e*n^2 + 16*b^2*
c^3*d^2*e^2*n^2 - 3*b^4*c*e^4*n^2)*a^5 - (12*b^4*c^2*d^2*e^2*n^2 - 12*b^5*c
*d*e^3*n^2 - b^6*e^4*n^2)*a^4 - 2*(3*b^4*c^3*d^4*n^2 - 6*b^5*c^2*d^3*e*n^2
+ 2*b^6*c*d^2*e^2*n^2 + b^7*d*e^3*n^2)*a^3 + (b^6*c^2*d^4*n^2 - 2*b^7*c*d^3
*e*n^2 + b^8*d^2*e^2*n^2)*a^2)*x^(2*n) + 2*(16*a^7*b*c^2*e^4*n^2 + 8*(4*b*c
^3*d^2*e^2*n^2 - 4*b^2*c^2*d*e^3*n^2 - b^3*c*e^4*n^2)*a^6 + (16*b*c^4*d^4*n
^2 - 32*b^2*c^3*d^3*e*n^2 + 16*b^4*c*d*e^3*n^2 + b^5*e^4*n^2)*a^5 - 2*(4*b^
3*c^3*d^4*n^2 - 8*b^4*c^2*d^3*e*n^2 + 3*b^5*c*d^2*e^2*n^2 + b^6*d*e^3*n^2)*
a^4 + (b^5*c^2*d^4*n^2 - 2*b^6*c*d^3*e*n^2 + b^7*d^2*e^2*n^2)*a^3)*x^n) - i
ntegrate(-1/2*((2*n^2 - 3*n + 1)*b^4*c^3*d^5 - 3*(2*n^2 - 3*n + 1)*b^5*c^2*
d^4*e + 3*(2*n^2 - 3*n + 1)*b^6*c*d^3*e^2 - (2*n^2 - 3*n + 1)*b^7*d^2*e^3 +
2*(2*(24*n^2 - 10*n + 1)*c^3*d*e^4 - (48*n^2 - 29*n + 4)*b*c^2*e^5)*a^4 +

(8*(12*n^2 - 8*n + 1)*c^4*d^3*e^2 - 12*(16*n^2 - 13*n + 2)*b*c^3*d^2*e^3 + (48*n^2 - 59*n + 11)*b^2*c^2*d*e^4 + 6*(8*n^2 - 6*n + 1)*b^3*c*e^5)*a^3 + (4*(8*n^2 - 6*n + 1)*c^5*d^5 - 2*(48*n^2 - 41*n + 8)*b*c^4*d^4*e + 2*(24*n^2 - 19*n + 5)*b^2*c^3*d^3*e^2 + 2*(32*n^2 - 39*n + 7)*b^3*c^2*d^2*e^3 - (42* n^2 - 53*n + 11)*b^4*c*d*e^4 - (6*n^2 - 5*n + 1)*b^5*e^5)*a^2 - ((16*n^2 - 21*n + 5)*b^2*c^4*d^5 - 16*(3*n^2 - 4*n + 1)*b^3*c^3*d^4*e + 3*(14*n^2 - 19 *n + 5)*b^4*c^2*d^3*e^2 - 2*(2*n^2 - 3*n + 1)*b^5*c*d^2*e^3 - 2*(3*n^2 - 4* n + 1)*b^6*d*e^4)*a + ((2*n^2 - 3*n + 1)*b^3*c^4*d^5 - 3*(2*n^2 - 3*n + 1)* b^4*c^3*d^4*e + 3*(2*n^2 - 3*n + 1)*b^5*c^2*d^3*e^2 - (2*n^2 - 3*n + 1)*b^6 *c*d^2*e^3 - 4*(15*n^2 - 8*n + 1)*a^4*c^3*e^5 - (8*(5*n^2 - 6*n + 1)*c^4*d^ 2*e^3 - 2*(9*n^2 - 11*n + 2)*b*c^3*d*e^4 - (42*n^2 - 31*n + 5)*b^2*c^2*e^5) *a^3 - (4*(3*n^2 - 4*n + 1)*c^5*d^4*e + 12*(n^2 - n)*b*c^4*d^3*e^2 - 2*(32* n^2 - 39*n + 7)*b^2*c^3*d^2*e^3 + 9*(4*n^2 - 5*n + 1)*b^3*c^2*d*e^4 + (6*n^ 2 - 5*n + 1)*b^4*c*e^5)*a^2 - (2*(7*n^2 - 9*n + 2)*b*c^5*d^5 - (42*n^2 - 55 *n + 13)*b^2*c^4*d^4*e + 12*(3*n^2 - 4*n + 1)*b^3*c^3*d^3*e^2 - (2*n^2 - 3* n + 1)*b^4*c^2*d^2*e^3 - 2*(3*n^2 - 4*n + 1)*b^5*c*d*e^4)*a)*x^n)/(16*a^8*c ^2*e^6*n^2 + 8*(6*c^3*d^2*e^4*n^2 - 6*b*c^2*d*e^5*n^2 - b^2*c*e^6*n^2)*a^7 + (48*c^4*d^4*e^2*n^2 - 96*b*c^3*d^3*e^3*n^2 + 24*b^2*c^2*d^2*e^4*n^2 + 24* b^3*c*d*e^5*n^2 + b^4*e^6*n^2)*a^6 + (16*c^5*d^6*n^2 - 48*b*c^4*d^5*e*n^2 + 24*b^2*c^3*d^4*e^2*n^2 + 32*b^3*c^2*d^3*e^3*n^2 - 21*b^4*c*d^2*e^4*n^2 - 3 *b^5*d*e^5*n^2)*a^5 - (8*b^2*c^4*d^6*n^2 - 24*b^3*c^3*d^5*e*n^2 + 21*b^4*c^ 2*d^4*e^2*n^2 - 2*b^5*c*d^3*e^3*n^2 - 3*b^6*d^2*e^4*n^2)*a^4 + (b^4*c^3*d^6 *n^2 - 3*b^5*c^2*d^5*e*n^2 + 3*b^6*c*d^4*e^2*n^2 - b^7*d^3*e^3*n^2)*a^3 + (16*a^7*c^3*e^6*n^2 + 8*(6*c^4*d^2*e^4*n^2 - 6*b*c^3*d*e^5*n^2 - b^2*c^2*e^6 *n^2)*a^6 + (48*c^5*d^4*e^2*n^2 - 96*b*c^4*d^3*e^3*n^2 + 24*b^2*c^3*d^2*e^4 *n^2 + 24*b^3*c^2*d*e^5*n^2 + b^4*c*e^6*n^2)*a^5 + (16*c^6*d^6*n^2 - 48*b*c ^5*d^5*e*n^2 + 24*b^2*c^4*d^4*e^2*n^2 + 32*b^3*c^3*d^3*e^3*n^2 - 21*b^4*c^2 *d^2*e^4*n^2 - 3*b^5*c*d*e^5*n^2)*a^4 - (8*b^2*c^5*d^6*n^2 - 24*b^3*c^4*d^5 *e*n^2 + 21*b^4*c^3*d^4*e^2*n^2 - 2*b^5*c^2*d^3*e^3*n^2 - 3*b^6*c*d^2*e^4*n ^2)*a^3 + (b^4*c^4*d^6*n^2 - 3*b^5*c^3*d^5*e*n^2 + 3*b^6*c^2*d^4*e^2*n^2 - b^7*c*d^3*e^3*n^2)*a^2)*x^(2*n) + (16*a^7*b*c^2*e^6*n^2 + 8*(6*b*c^3*d^2*e^ 4*n^2 - 6*b^2*c^2*d*e^5*n^2 - b^3*c*e^6*n^2)*a^6 + (48*b*c^4*d^4*e^2*n^2 - 96*b^2*c^3*d^3*e^3*n^2 + 24*b^3*c^2*d^2*e^4*n^2 + 24*b^4*c*d*e^5*n^2 + b^5* e^6*n^2)*a^5 + (16*b*c^5*d^6*n^2 - 48*b^2*c^4*d^5*e*n^2 + 24*b^3*c^3*d^4*e^ 2*n^2 + 32*b^4*c^2*d^3*e^3*n^2 - 21*b^5*c*d^2*e^4*n^2 - 3*b^6*d*e^5*n^2)*a^ 4 - (8*b^3*c^4*d^6*n^2 - 24*b^4*c^3*d^5*e*n^2 + 21*b^5*c^2*d^4*e^2*n^2 - 2* b^6*c*d^3*e^3*n^2 - 3*b^7*d^2*e^4*n^2)*a^3 + (b^5*c^3*d^6*n^2 - 3*b^6*c^2*d ^5*e*n^2 + 3*b^7*c*d^4*e^2*n^2 - b^8*d^3*e^3*n^2)*a^2)*x^n), x)

Giac [F]

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^3}\,dx=\int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^3(ex^n+d)}\,dx$$

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^3}\,dx=\int \frac{1}{(d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^3}\,dx$$

```
[In] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x)
[Out] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3), x)
```

3.84 $\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 2446

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3}\,dx =$$
$$-\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-}{2a(b^2-4ac)(cd^2-bde+ae^2)^2 n(a+bx^n+cx^{2n})^2}$$
$$-\frac{e^2x(5b^3cde-14abc^2de-2b^4e^2-b^2c(3cd^2-7ae^2)+2ac^2(3cd^2-ae^2)+c(5b^2cde-8ac^2de-2b^3e^2-b}{a(b^2-4ac)(cd^2-bde+ae^2)^3 n(a+bx^n+cx^{2n})}$$
$$-\frac{x(ab^2c^2(ae^2(13-37n)-5cd^2(1-3n))-b^4c(ae^2(7-17n)-cd^2(1-2n))-4a^2bc^3de(4-11n)+6ab}{}$$
$$-\frac{ce^4\left(10c^2d^2+3b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd+3\sqrt{b^2-4ac}d+ae\right)\right)x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n}\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^4}$$
$$+\frac{ce^2\left(4ac^2\left(e\left(ae(1-2n)+2\sqrt{b^2-4ac}d(1-n)\right)-3cd^2(1-2n)\right)-b^2c\left(e\left(ae(9-13n)+5\sqrt{b^2-4ac}d(1\right.\right.\right.}{}$$
$$+\frac{c\left((2abc^2(ae^2(4-13n)-cd^2(2-7n))-b^3c(2ae^2(3-8n)-cd^2(1-2n))+2ab^2c^2de(5-14n)-8a^2c\right.}{}$$
$$-\frac{ce^4\left(10c^2d^2+3b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd-3\sqrt{b^2-4ac}d+ae\right)\right)x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n}\right.}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^4}$$
$$+\frac{ce^2\left(4ac^2\left(e\left(ae(1-2n)-2\sqrt{b^2-4ac}d(1-n)\right)-3cd^2(1-2n)\right)-b^2c\left(e\left(ae(9-13n)-5\sqrt{b^2-4ac}d(1\right.\right.\right.}{}$$
$$+\frac{c\left((2abc^2(ae^2(4-13n)-cd^2(2-7n))-b^3c(2ae^2(3-8n)-cd^2(1-2n))+2ab^2c^2de(5-14n)-8a^2c\right.}{}$$
$$+\frac{3e^6(2cd-be)x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^4}$$
$$+\frac{e^6x\,\mathrm{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2(cd^2-bde+ae^2)^3}$$

```
[Out] -1/2*x*(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-
a*e^2+c*d^2)+c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))*x^n)/
a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))^2-e^2*x*(5*b^3*c
*d*e-14*a*b*c^2*d*e-2*b^4*e^2-b^2*c*(-7*a*e^2+3*c*d^2)+2*a*c^2*(-a*e^2+3*c*
d^2)+c*(5*b^2*c*d*e-8*a*c^2*d*e-2*b^3*e^2-b*c*(-5*a*e^2+3*c*d^2))*x^n)/a/(-
4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^3/n/(a+b*x^n+c*x^(2*n))-1/2*x*(a*b^2*c^2*(a*
e^2*(13-37*n)-5*c*d^2*(1-3*n))-b^4*c*(a*e^2*(7-17*n)-c*d^2*(1-2*n))-4*a^2*b
*c^3*d*e*(4-11*n)+6*a*b^3*c^2*d*e*(2-5*n)+4*a^2*c^3*(-a*e^2+c*d^2)*(1-4*n)-
```

2*b^5*c*d*e*(1-2*n)+b^6*e^2*(1-2*n)+c*(2*a*b*c^2*(a*e^2*(4-13*n)-c*d^2*(2-7
*n))-b^3*c*(2*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*d*e*(5-14*n)-8*a^2*c
^3*d*e*(1-3*n)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2
/(a*e^2-b*d*e+c*d^2)^2/n^2/(a+b*x^n+c*x^(2*n))+3*e^6*(-b*e+2*c*d)*x*hyperge
om([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^4+e^6*x*hypergeom([2, 1
/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^3+1/2*c*x*hypergeom([1, 1/n],
[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((2*a*b*c^2*(a*e^2*(4-13*n)-c*d^2*
(2-7*n))-b^3*c*(2*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*d*e*(5-14*n)-8*a
^2*c^3*d*e*(1-3*n)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*(1-n)+(-b^4*c*(4*a*
e^2*(2-5*n)-c*d^2*(1-2*n))*(1-n)-2*b^5*c*d*e*(2*n^2-3*n+1)+b^6*e^2*(2*n^2-3
*n+1)+8*a^2*c^3*(-a*e^2+c*d^2)*(8*n^2-6*n+1)-8*a^2*b*c^3*d*e*(13*n^2-13*n+3
)+2*a*b^3*c^2*d*e*(18*n^2-25*n+7)-2*a*b^2*c^2*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*
(35*n^2-38*n+9)))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2
)^2/n^2/(b-(-4*a*c+b^2)^(1/2))+1/2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))*((2*a*b*c^2*(a*e^2*(4-13*n)-c*d^2*(2-7*n))-b^3*c*(2
*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*d*e*(5-14*n)-8*a^2*c^3*d*e*(1-3*n
)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*(1-n)+(b^4*c*(4*a*e^2*(2-5*n)-c*d^2*
(1-2*n))*(1-n)+2*b^5*c*d*e*(2*n^2-3*n+1)-b^6*e^2*(2*n^2-3*n+1)-8*a^2*c^3*(-
a*e^2+c*d^2)*(8*n^2-6*n+1)+8*a^2*b*c^3*d*e*(13*n^2-13*n+3)-2*a*b^3*c^2*d*e*
(18*n^2-25*n+7)+2*a*b^2*c^2*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*(35*n^2-38*n+9)))/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/n^2/(b+(-4*a*c
+b^2)^(1/2))-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1
/2)))*(10*c^2*d^2+3*b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(5*b*d+a*e-3*d*(-4*a
*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^4/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*e^
4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(10*c^2*d^2
+3*b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c*e*(5*b*d+a*e+3*d*(-4*a*c+b^2)^(1/2)))/(
a*e^2-b*d*e+c*d^2)^4/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1,
 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*b^4*e^2*(1-n)-b^3*e*(1-n)
*(5*c*d+2*e*(-4*a*c+b^2)^(1/2))-b^2*c*(-3*c*d^2*(1-n)+e*(a*e*(9-13*n)-5*d*(
1-n)*(-4*a*c+b^2)^(1/2)))+b*c*(5*a*e^2*(1-n)*(-4*a*c+b^2)^(1/2)+c*d*(4*a*e*
(5-8*n)-3*d*(1-n)*(-4*a*c+b^2)^(1/2)))+4*a*c^2*(-3*c*d^2*(1-2*n)+e*(a*e*(1-
2*n)-2*d*(1-n)*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^3/n
/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x
^n/(b-(-4*a*c+b^2)^(1/2)))*(2*b^4*e^2*(1-n)-b^3*e*(1-n)*(5*c*d-2*e*(-4*a*c+
b^2)^(1/2))+4*a*c^2*(-3*c*d^2*(1-2*n)+e*(a*e*(1-2*n)+2*d*(1-n)*(-4*a*c+b^2)
^(1/2)))+b*c*(-5*a*e^2*(1-n)*(-4*a*c+b^2)^(1/2)+c*d*(4*a*e*(5-8*n)+3*d*(1-n
)*(-4*a*c+b^2)^(1/2)))-b^2*c*(-3*c*d^2*(1-n)+e*(a*e*(9-13*n)+5*d*(1-n)*(-4*
a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^3/n/(b^2-4*a*c-b*(-4*a
*c+b^2)^(1/2))

Rubi [A] (verified)

Time = 5.09 (sec) , antiderivative size = 2446, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1450, 251, 1444, 1436}

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^3}\,dx$$

$$= \frac{3(2cd-be)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^6}{d\left(cd^2-bed+ae^2\right)^4}$$

$$+\frac{x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^6}{d^2\left(cd^2-bed+ae^2\right)^3}$$

$$-\frac{c\left(10c^2d^2+3b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd+3\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\right.}{\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^4}$$

$$-\frac{c\left(10c^2d^2+3b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd-3\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\right.}{\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^4}$$

$$+\frac{c\left(2e^2(1-n)b^4-e\left(5cd-2\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(9-13n)+5\sqrt{b^2-4ac}d(1-n)\right)-3cd^2(1\right.\right.}{}$$

$$+\frac{c\left(2e^2(1-n)b^4-e\left(5cd+2\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(9-13n)-5\sqrt{b^2-4ac}d(1-n)\right)-3cd^2(1\right.\right.}{}$$

$$-\frac{x(c(-2e^2b^3+5cdeb^2-c(3cd^2-5ae^2)b-8ac^2de)x^n-2b^4e^2-14abc^2de+5b^3cde-b^2c(3cd^2-7ae^2)}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^3n\left(bx^n+cx^{2n}+a\right)}$$

$$+\frac{c\left(\left(e^2(1-2n)b^5-2cde(1-2n)b^4-c(2ae^2(3-8n)-cd^2(1-2n))b^3+2ac^2de(5-14n)b^2+2ac^2(ae^2\right.\right.}{}$$

$$+\frac{c\left(\left(e^2(1-2n)b^5-2cde(1-2n)b^4-c(2ae^2(3-8n)-cd^2(1-2n))b^3+2ac^2de(5-14n)b^2+2ac^2(ae^2\right.\right.}{}$$

$$-\frac{x(c(e^2(1-2n)b^5-2cde(1-2n)b^4-c(2ae^2(3-8n)-cd^2(1-2n))b^3+2ac^2de(5-14n)b^2+2ac^2(ae^2}{}$$

$$-\frac{x(c(-e^2b^3+2cdeb^2-c(cd^2-3ae^2)b-4ac^2de)x^n-b^4e^2-6abc^2de+2b^3cde-b^2c(cd^2-4ae^2)+2ac}{2a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2n\left(bx^n+cx^{2n}+a\right)^2}$$

```
[In] Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] -1/2*(x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d
^2 - 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^
n + c*x^(2*n))^2) - (e^2*x*(5*b^3*c*d*e - 14*a*b*c^2*d*e - 2*b^4*e^2 - b^2*
c*(3*c*d^2 - 7*a*e^2) + 2*a*c^2*(3*c*d^2 - a*e^2) + c*(5*b^2*c*d*e - 8*a*c^
```

```
2*d*e - 2*b^3*e^2 - b*c*(3*c*d^2 - 5*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^3*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*(a*e^2*(13 -
37*n) - 5*c*d^2*(1 - 3*n)) - b^4*c*(a*e^2*(7 - 17*n) - c*d^2*(1 - 2*n)) - 4
*a^2*b*c^3*d*e*(4 - 11*n) + 6*a*b^3*c^2*d*e*(2 - 5*n) + 4*a^2*c^3*(c*d^2 -
a*e^2)*(1 - 4*n) - 2*b^5*c*d*e*(1 - 2*n) + b^6*e^2*(1 - 2*n) + c*(2*a*b*c^2
*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1
 - 2*n)) + 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d
*e*(1 - 2*n) + b^5*e^2*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d
*e + a*e^2)^2*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(10*c^2*d^2 + 3*b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d + 3*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/
((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4
*a*c^2*(e*(a*e*(1 - 2*n) + 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - 2*
n)) - b^2*c*(e*(a*e*(9 - 13*n) + 5*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(
1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 5*a*
Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^3*e*(5*c*d - 2*Sqrt[
b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 -
4*a*c])*(c*d^2 - b*d*e + a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - 13*n) - c
*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2
*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^5*e^2
*(1 - 2*n))*(1 - n) - (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*n))*(1 - n)
+ 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^2) - 8*a^2*c^3*(c*
d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*e*(3 - 13*n + 13*n^2) - 2*a*
b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n + 3*n^2) -
a*e^2*(9 - 38*n + 35*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*
(b - Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) - (c*e^4*(10*c^2*d^2
 + 3*b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d - 3*Sqrt[b^2 - 4*a*c]*d +
 a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^4)
 + (c*e^2*(4*a*c^2*(e*(a*e*(1 - 2*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c
*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(9 - 13*n) - 5*Sqrt[b^2 - 4*a*c]*d*(1 - n))
 - 3*c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1
- n)) + 5*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^3*e*(5*c
*d + 2*Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4
 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) +
 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*
n) + b^5*e^2*(1 - 2*n))*(1 - n) + (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*
n))*(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^2) - 8
*a^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*e*(3 - 13*n + 13
*n^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n
 + 3*n^2) - a*e^2*(9 - 38*n + 35*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric
```

```
2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2
 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) + (3*e^6
*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d
*(c*d^2 - b*d*e + a*e^2)^4) + (e^6*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1
), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^6}{(cd^2-bde+ae^2)^3 (d+ex^n)^2} - \frac{3e^6(-2cd+be)}{(cd^2-bde+ae^2)^4 (d+ex^n)} + \frac{c^2d^2-2bcde+b^2e^2-ace^2-(2c^2de-bce^2)x^n}{(cd^2-bde+ae^2)^2 (a+bx^n+cx^{2n})^3} + \frac{e^2(3c^2d^2-5bcde+2b^2e^2-ace^2+(-4c^2de+2bce^2)x^n)}{(cd^2-bde+ae^2)^3 (a+bx^n+cx^{2n})^2} + \frac{e^4(5c^2d^2-8bcde+3b^2e^2-ace^2+(-6c^2de+3bce^2)x^n)}{(cd^2-bde+ae^2)^4 (a+bx^n+cx^{2n})} \right) dx$$

$$= \frac{e^4 \int \frac{5c^2d^2-8bcde+3b^2e^2-ace^2+(-6c^2de+3bce^2)x^n}{a+bx^n+cx^{2n}} dx}{(cd^2-bde+ae^2)^4} + \frac{(3e^6(2cd-be)) \int \frac{1}{d+ex^n} dx}{(cd^2-bde+ae^2)^4} + \frac{e^2 \int \frac{3c^2d^2-5bcde+2b^2e^2-ace^2+(-4c^2de+2bce^2)x^n}{(a+bx^n+cx^{2n})^2} dx}{(cd^2-bde+ae^2)^3} + \frac{e^6 \int \frac{1}{(d+ex^n)^2} dx}{(cd^2-bde+ae^2)^3} + \frac{\int \frac{c^2d^2-2bcde+b^2e^2-ace^2-(2c^2de-bce^2)x^n}{(a+bx^n+cx^{2n})^3} dx}{(cd^2-bde+ae^2)^2}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2b^2cde-4ac^2de-b^3e^2-bc}{2a(b^2-4ac)(cd^2-bde+ae^2)^2 n (a+bx^n+cx^{2n})^2} - \frac{e^2x(5b^3cde-14abc^2de-2b^4e^2-b^2c(3cd^2-7ae^2)+2ac^2(3cd^2-ae^2)+c(5b^2cde-8ac^2de-2b}{a(b^2-4ac)(cd^2-bde+ae^2)^3 n (a+bx^n+cx^{2n})} + \frac{3e^6(2cd-be)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^4} + \frac{e^6x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2-bde+ae^2)^3} - \frac{\left(ce^4\left(10c^2d^2+3b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd-3\sqrt{b^2-4ac}d+ae\right)\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n} dx}{2\sqrt{b^2-4ac}(cd^2-bde+ae^2)^4} + \frac{\left(ce^4\left(10c^2d^2+3b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd+3\sqrt{b^2-4ac}d+ae\right)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n} dx}{2\sqrt{b^2-4ac}(cd^2-bde+ae^2)^4} - \frac{e^2 \int \frac{-b^2c(ae^2(7-9n)-3cd^2(1-n))+2abc^2de(7-10n)-2ac^2(3cd^2-ae^2)(1-2n)-5b^3cde(1-n)+2b^4e^2(1-n)-c(5b^2cde-8ac^2de-2b}{a+bx^n+cx^{2n}}}{a(b^2-4ac)(cd^2-bde+ae^2)^3 n} - \frac{\int \frac{abce(2cd-be)-2ac(c^2d^2+b^2e^2-ce(2bd+ae))(1-4n)+b^2(c^2d^2+b^2e^2-ce(2bd+ae))(1-2n)-c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3ae}{(a+bx^n+cx^{2n})^2}}{2a(b^2-4ac)(cd^2-bde+ae^2)^2 n}$$

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Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 56566 vs. 2(2446) = 4892.

Time = 9.30 (sec) , antiderivative size = 56566, normalized size of antiderivative = 23.13

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{1}{(d+e\,x^n)^2 (a+b\,x^n+c\,x^{2n})^3} dx$$

```
[In] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)^2} dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*e^2*x^(5*n) + a^3*d^2 + (c^3*e^2*x^(2*n) + 2*c^3*d*e*x^n +
c^3*d^2)*x^(6*n) + 3*(b*c^2*e^2*x^(3*n) + a*c^2*d^2 + (2*b*c^2*d*e + a*c^2*
e^2)*x^(2*n) + (b*c^2*d^2 + 2*a*c^2*d*e)*x^n)*x^(4*n) + (2*b^3*d*e + 3*a*b^
2*e^2)*x^(4*n) + (b^3*d^2 + 6*a*b^2*d*e + 3*a^2*b*e^2)*x^(3*n) + 3*(b^2*c*e
^2*x^(4*n) + a^2*c*d^2 + 2*(b^2*c*d*e + a*b*c*e^2)*x^(3*n) + (b^2*c*d^2 + 4
*a*b*c*d*e + a^2*c*e^2)*x^(2*n) + 2*(a*b*c*d^2 + a^2*c*d*e)*x^n)*x^(2*n) +
(3*a*b^2*d^2 + 6*a^2*b*d*e + a^3*e^2)*x^(2*n) + (3*a^2*b*d^2 + 2*a^3*d*e)*x
^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
[Out] (c*d^2*e^6*(7*n - 1) - b*d*e^7*(4*n - 1) + a*e^8*(n - 1))*integrate(1/(c^4*
d^10*n - 4*b*c^3*d^9*e*n + 6*b^2*c^2*d^8*e^2*n - 4*b^3*c*d^7*e^3*n + b^4*d^
6*e^4*n + a^4*d^2*e^8*n + 4*(c*d^4*e^6*n - b*d^3*e^7*n)*a^3 + 6*(c^2*d^6*e^
4*n - 2*b*c*d^5*e^5*n + b^2*d^4*e^6*n)*a^2 + 4*(c^3*d^8*e^2*n - 3*b*c^2*d^7
*e^3*n + 3*b^2*c*d^6*e^4*n - b^3*d^5*e^5*n)*a + (c^4*d^9*e*n - 4*b*c^3*d^8*
e^2*n + 6*b^2*c^2*d^7*e^3*n - 4*b^3*c*d^6*e^4*n + b^4*d^5*e^5*n + a^4*d*e^9
*n + 4*(c*d^3*e^7*n - b*d^2*e^8*n)*a^3 + 6*(c^2*d^5*e^5*n - 2*b*c*d^4*e^6*n
 + b^2*d^3*e^7*n)*a^2 + 4*(c^3*d^7*e^3*n - 3*b*c^2*d^6*e^4*n + 3*b^2*c*d^5*
e^5*n - b^3*d^4*e^6*n)*a)*x^n), x) + 1/2*((b^3*c^5*d^5*e*(2*n - 1) - 3*b^4*
c^4*d^4*e^2*(2*n - 1) + 3*b^5*c^3*d^3*e^3*(2*n - 1) - b^6*c^2*d^2*e^4*(2*n
- 1) + 32*a^4*c^4*e^6*n + 2*(b*c^4*d*e^5*(33*n - 4) - 4*c^5*d^2*e^4*(11*n -
 1) - 8*b^2*c^3*e^6*n)*a^3 + 2*(b^2*c^4*d^2*e^4*(29*n - 1) - 3*b^3*c^3*d*e^
5*(7*n - 1) - 4*c^6*d^4*e^2*(3*n - 1) + 6*b*c^5*d^3*e^3*(n - 1) + b^4*c^2*e
^6*n)*a^2 - (3*b^3*c^4*d^3*e^3*(12*n - 5) + 2*b*c^6*d^5*e*(7*n - 2) - b^5*c
^2*d*e^5*(6*n - 1) - 14*b^2*c^5*d^4*e^2*(3*n - 1) - 2*b^4*c^3*d^2*e^4*(n -
2))*a)*x*x^(4*n) + (b^3*c^5*d^6*(2*n - 1) - b^4*c^4*d^5*e*(2*n - 1) - 3*b^5
*c^3*d^4*e^2*(2*n - 1) + 5*b^6*c^2*d^3*e^3*(2*n - 1) - 2*b^7*c*d^2*e^4*(2*n
 - 1) - 4*(c^4*d*e^5*(8*n - 1) - 16*b*c^3*e^6*n)*a^4 + (b^2*c^3*d*e^5*(163*
n - 21) - 6*b*c^4*d^2*e^4*(27*n - 2) - 8*c^5*d^3*e^3*(5*n - 1) - 32*b^3*c^2
*e^6*n)*a^3 - (b^4*c^2*d*e^5*(89*n - 13) - b^3*c^3*d^2*e^4*(77*n + 5) - 2*b
^2*c^4*d^3*e^3*(50*n - 19) + 8*b*c^5*d^4*e^2*(9*n - 2) + 4*c^6*d^5*e*(2*n -
 1) - 4*b^5*c*e^6*n)*a^2 - (b^4*c^3*d^3*e^3*(73*n - 29) - b^3*c^4*d^4*e^2*(
51*n - 16) - b^2*c^5*d^5*e*(13*n - 5) - b^5*c^2*d^2*e^4*(11*n - 10) + 2*b*c
^6*d^6*(7*n - 2) - 2*b^6*c*d*e^5*(6*n - 1))*a)*x*x^(3*n) + (2*b^4*c^4*d^6*(
2*n - 1) - 5*b^5*c^3*d^5*e*(2*n - 1) + 3*b^6*c^2*d^4*e^2*(2*n - 1) + b^7*c*
d^3*e^3*(2*n - 1) - b^8*d^2*e^4*(2*n - 1) + 64*a^5*c^3*e^6*n - 2*(2*c^4*d^2
*e^4*(34*n - 3) - b*c^3*d*e^5*(23*n - 2))*a^4 + (b^2*c^3*d^2*e^4*(81*n - 11
```

) + b^3*c^2*d*e^5*(48*n - 7) - 8*b*c^4*d^3*e^3*(18*n - 1) + 8*c^5*d^4*e^2*(
n + 1) - 12*b^4*c*e^6*n)*a^3 - (2*b*c^5*d^5*e*(43*n - 14) + b^4*c^2*d^2*e^4
*(21*n - 10) + 2*b^5*c*d*e^5*(20*n - 3) - 5*b^3*c^3*d^3*e^3*(19*n - 2) - 4*
c^6*d^6*(4*n - 1) - 10*b^2*c^4*d^4*e^2*(4*n - 3) - 2*b^6*e^6*n)*a^2 - (b^4*
c^3*d^4*e^2*(39*n - 19) + b^2*c^5*d^6*(29*n - 9) + b^5*c^2*d^3*e^3*(25*n -
6) - 3*b^3*c^4*d^5*e*(25*n - 9) - b^7*d*e^5*(6*n - 1) - 6*b^6*c*d^2*e^4*(2*
n - 1))*a)*x*x^(2*n) + (b^5*c^3*d^6*(2*n - 1) - 3*b^6*c^2*d^5*e*(2*n - 1) +
3*b^7*c*d^4*e^2*(2*n - 1) - b^8*d^3*e^3*(2*n - 1) - 4*(c^3*d*e^5*(10*n - 1
) - 16*b*c^2*e^6*n)*a^5 + (b^2*c^2*d*e^5*(115*n - 13) - 2*b*c^3*d^2*e^4*(55
*n - 4) - 8*c^4*d^3*e^3*(7*n - 1) - 32*b^3*c*e^6*n)*a^4 - (b^4*c*d*e^5*(55*
n - 7) - 3*b^3*c^2*d^2*e^4*(35*n - 2) + 2*b^2*c^3*d^3*e^3*(8*n + 7) + 4*c^5
*d^5*e*(4*n - 1) + 8*b*c^4*d^4*e^2*(n - 1) - 4*b^5*e^6*n)*a^3 + (b^3*c^3*d^
4*e^2*(41*n - 26) - b^5*c*d^2*e^4*(31*n - 1) - b^2*c^4*d^5*e*(23*n - 11) +
b^4*c^2*d^3*e^3*(8*n + 15) + b^6*d*e^5*(7*n - 1) - 2*b*c^5*d^6*n)*a^2 + (3*
b^4*c^3*d^5*e*(13*n - 5) - 3*b^5*c^2*d^4*e^2*(13*n - 6) + b^6*c*d^3*e^3*(9*
n - 7) - 4*b^3*c^4*d^6*(3*n - 1) + 3*b^7*d^2*e^4*n)*a)*x*x^n + (32*a^6*c^2*
e^6*n - 4*(c^3*d^2*e^4*(10*n - 1) + 4*b^2*c*e^6*n)*a^5 + (b^2*c^2*d^2*e^4*(
115*n - 13) - 12*b*c^3*d^3*e^3*(13*n - 1) + 48*c^4*d^4*e^2*n + 2*b^4*e^6*n)
*a^4 + (b^3*c^2*d^3*e^3*(57*n + 1) - b^4*c*d^2*e^4*(55*n - 7) - 4*b*c^4*d^5
e(23*n - 5) + 6*b^2*c^3*d^4*e^2*(11*n - 4) + 4*c^5*d^6*(6*n - 1))*a^3 + (
b^3*c^3*d^5*e*(65*n - 17) - b^2*c^4*d^6*(21*n - 5) - 6*b^4*c^2*d^4*e^2*(10*
n - 3) + b^5*c*d^3*e^3*(9*n - 5) + b^6*d^2*e^4*(7*n - 1))*a^2 + (b^4*c^3*d^
6*(3*n - 1) - 3*b^5*c^2*d^5*e*(3*n - 1) + 3*b^6*c*d^4*e^2*(3*n - 1) - b^7*d
^3*e^3*(3*n - 1))*a)*x)/(16*a^9*c^2*d^2*e^6*n^2 + 8*(6*c^3*d^4*e^4*n^2 - 6*
b*c^2*d^3*e^5*n^2 - b^2*c*d^2*e^6*n^2)*a^8 + (48*c^4*d^6*e^2*n^2 - 96*b*c^3
*d^5*e^3*n^2 + 24*b^2*c^2*d^4*e^4*n^2 + 24*b^3*c*d^3*e^5*n^2 + b^4*d^2*e^6*
n^2)*a^7 + (16*c^5*d^8*n^2 - 48*b*c^4*d^7*e*n^2 + 24*b^2*c^3*d^6*e^2*n^2 +
32*b^3*c^2*d^5*e^3*n^2 - 21*b^4*c*d^4*e^4*n^2 - 3*b^5*d^3*e^5*n^2)*a^6 - (8
*b^2*c^4*d^8*n^2 - 24*b^3*c^3*d^7*e*n^2 + 21*b^4*c^2*d^6*e^2*n^2 - 2*b^5*c*
d^5*e^3*n^2 - 3*b^6*d^4*e^4*n^2)*a^5 + (b^4*c^3*d^8*n^2 - 3*b^5*c^2*d^7*e*n
^2 + 3*b^6*c*d^6*e^2*n^2 - b^7*d^5*e^3*n^2)*a^4 + (16*a^7*c^4*d*e^7*n^2 + 8
*(6*c^5*d^3*e^5*n^2 - 6*b*c^4*d^2*e^6*n^2 - b^2*c^3*d*e^7*n^2)*a^6 + (48*c^
6*d^5*e^3*n^2 - 96*b*c^5*d^4*e^4*n^2 + 24*b^2*c^4*d^3*e^5*n^2 + 24*b^3*c^3*
d^2*e^6*n^2 + b^4*c^2*d*e^7*n^2)*a^5 + (16*c^7*d^7*e*n^2 - 48*b*c^6*d^6*e^2
*n^2 + 24*b^2*c^5*d^5*e^3*n^2 + 32*b^3*c^4*d^4*e^4*n^2 - 21*b^4*c^3*d^3*e^5
*n^2 - 3*b^5*c^2*d^2*e^6*n^2)*a^4 - (8*b^2*c^6*d^7*e*n^2 - 24*b^3*c^5*d^6*e
^2*n^2 + 21*b^4*c^4*d^5*e^3*n^2 - 2*b^5*c^3*d^4*e^4*n^2 - 3*b^6*c^2*d^3*e^5
*n^2)*a^3 + (b^4*c^5*d^7*e*n^2 - 3*b^5*c^4*d^6*e^2*n^2 + 3*b^6*c^3*d^5*e^3*
n^2 - b^7*c^2*d^4*e^4*n^2)*a^2)*x^(5*n) + (16*(c^4*d^2*e^6*n^2 + 2*b*c^3*d*
e^7*n^2)*a^7 + 8*(6*c^5*d^4*e^4*n^2 + 6*b*c^4*d^3*e^5*n^2 - 13*b^2*c^3*d^2*
e^6*n^2 - 2*b^3*c^2*d*e^7*n^2)*a^6 + (48*c^6*d^6*e^2*n^2 - 168*b^2*c^4*d^4*
e^4*n^2 + 72*b^3*c^3*d^3*e^5*n^2 + 49*b^4*c^2*d^2*e^6*n^2 + 2*b^5*c*d*e^7*n
^2)*a^5 + (16*c^7*d^8*n^2 - 16*b*c^6*d^7*e*n^2 - 72*b^2*c^5*d^6*e^2*n^2 + 8
0*b^3*c^4*d^5*e^3*n^2 + 43*b^4*c^3*d^4*e^4*n^2 - 45*b^5*c^2*d^3*e^5*n^2 - 6
*b^6*c*d^2*e^6*n^2)*a^4 - (8*b^2*c^6*d^8*n^2 - 8*b^3*c^5*d^7*e*n^2 - 27*b^4

*c^4*d^6*e^2*n^2 + 40*b^5*c^3*d^5*e^3*n^2 - 7*b^6*c^2*d^4*e^4*n^2 - 6*b^7*c
*d^3*e^5*n^2)*a^3 + (b^4*c^5*d^8*n^2 - b^5*c^4*d^7*e*n^2 - 3*b^6*c^3*d^6*e^
2*n^2 + 5*b^7*c^2*d^5*e^3*n^2 - 2*b^8*c*d^4*e^4*n^2)*a^2)*x^(4*n) + (32*a^8
*c^3*d*e^7*n^2 + 32*(3*c^4*d^3*e^5*n^2 - 2*b*c^3*d^2*e^6*n^2)*a^7 + 2*(48*c
^5*d^5*e^3*n^2 - 48*b*c^4*d^4*e^4*n^2 - 8*b^3*c^2*d^2*e^6*n^2 - 3*b^4*c*d*e
^7*n^2)*a^6 + (32*c^6*d^7*e*n^2 - 96*b^2*c^4*d^5*e^3*n^2 + 16*b^3*c^3*d^4*e
^4*n^2 + 30*b^4*c^2*d^3*e^5*n^2 + 20*b^5*c*d^2*e^6*n^2 + b^6*d*e^7*n^2)*a^5
 + (32*b*c^6*d^8*n^2 - 96*b^2*c^5*d^7*e*n^2 + 48*b^3*c^4*d^6*e^2*n^2 + 46*b
^4*c^3*d^5*e^3*n^2 - 6*b^5*c^2*d^4*e^4*n^2 - 21*b^6*c*d^3*e^5*n^2 - 3*b^7*d
^2*e^6*n^2)*a^4 - (16*b^3*c^5*d^8*n^2 - 42*b^4*c^4*d^7*e*n^2 + 24*b^5*c^3*d
^6*e^2*n^2 + 11*b^6*c^2*d^5*e^3*n^2 - 6*b^7*c*d^4*e^4*n^2 - 3*b^8*d^3*e^5*n
^2)*a^3 + (2*b^5*c^4*d^8*n^2 - 5*b^6*c^3*d^7*e*n^2 + 3*b^7*c^2*d^6*e^2*n^2
+ b^8*c*d^5*e^3*n^2 - b^9*d^4*e^4*n^2)*a^2)*x^(3*n) + (32*(c^3*d^2*e^6*n^2
+ b*c^2*d*e^7*n^2)*a^8 + 16*(6*c^4*d^4*e^4*n^2 - 6*b^2*c^2*d^2*e^6*n^2 - b^
3*c*d*e^7*n^2)*a^7 + 2*(48*c^5*d^6*e^2*n^2 - 48*b*c^4*d^5*e^3*n^2 - 48*b^2*
c^3*d^4*e^4*n^2 + 24*b^3*c^2*d^3*e^5*n^2 + 21*b^4*c*d^2*e^6*n^2 + b^5*d*e^7
*n^2)*a^6 + (32*c^6*d^8*n^2 - 64*b*c^5*d^7*e*n^2 + 16*b^3*c^3*d^5*e^3*n^2 +
 46*b^4*c^2*d^4*e^4*n^2 - 24*b^5*c*d^3*e^5*n^2 - 5*b^6*d^2*e^6*n^2)*a^5 - (
16*b^3*c^4*d^7*e*n^2 - 30*b^4*c^3*d^6*e^2*n^2 + 6*b^5*c^2*d^5*e^3*n^2 + 11*
b^6*c*d^4*e^4*n^2 - 3*b^7*d^3*e^5*n^2)*a^4 - (6*b^4*c^4*d^8*n^2 - 20*b^5*c^
3*d^7*e*n^2 + 21*b^6*c^2*d^6*e^2*n^2 - 6*b^7*c*d^5*e^3*n^2 - b^8*d^4*e^4*n^
2)*a^3 + (b^6*c^3*d^8*n^2 - 3*b^7*c^2*d^7*e*n^2 + 3*b^8*c*d^6*e^2*n^2 - b^9
*d^5*e^3*n^2)*a^2)*x^(2*n) + (16*a^9*c^2*d*e^7*n^2 + 8*(6*c^3*d^3*e^5*n^2 -
 2*b*c^2*d^2*e^6*n^2 - b^2*c*d*e^7*n^2)*a^8 + (48*c^4*d^5*e^3*n^2 - 72*b^2*
c^2*d^3*e^5*n^2 + 8*b^3*c*d^2*e^6*n^2 + b^4*d*e^7*n^2)*a^7 + (16*c^5*d^7*e*
n^2 + 48*b*c^4*d^6*e^2*n^2 - 168*b^2*c^3*d^5*e^3*n^2 + 80*b^3*c^2*d^4*e^4*n
^2 + 27*b^4*c*d^3*e^5*n^2 - b^5*d^2*e^6*n^2)*a^6 + (32*b*c^5*d^8*n^2 - 104*
b^2*c^4*d^7*e*n^2 + 72*b^3*c^3*d^6*e^2*n^2 + 43*b^4*c^2*d^5*e^3*n^2 - 40*b^
5*c*d^4*e^4*n^2 - 3*b^6*d^3*e^5*n^2)*a^5 - (16*b^3*c^4*d^8*n^2 - 49*b^4*c^3
*d^7*e*n^2 + 45*b^5*c^2*d^6*e^2*n^2 - 7*b^6*c*d^5*e^3*n^2 - 5*b^7*d^4*e^4*n
^2)*a^4 + 2*(b^5*c^3*d^8*n^2 - 3*b^6*c^2*d^7*e*n^2 + 3*b^7*c*d^6*e^2*n^2 -
b^8*d^5*e^3*n^2)*a^3)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*c^4*d^6 -
 4*(2*n^2 - 3*n + 1)*b^5*c^3*d^5*e + 6*(2*n^2 - 3*n + 1)*b^6*c^2*d^4*e^2 -
4*(2*n^2 - 3*n + 1)*b^7*c*d^3*e^3 + (2*n^2 - 3*n + 1)*b^8*d^2*e^4 - 4*(24*n
^2 - 10*n + 1)*a^5*c^3*e^6 + (4*(48*n^2 - 2*n - 1)*c^4*d^2*e^4 - 4*(96*n^2
- 29*n + 2)*b*c^3*d*e^5 + (240*n^2 - 115*n + 13)*b^2*c^2*e^6)*a^4 + (4*(32*
n^2 - 18*n + 1)*c^5*d^4*e^2 - 8*(48*n^2 - 37*n + 4)*b*c^4*d^3*e^3 + (288*n^
2 - 337*n + 49)*b^2*c^3*d^2*e^4 + 2*(32*n^2 + 29*n - 7)*b^3*c^2*d*e^5 - (10
2*n^2 - 55*n + 7)*b^4*c*e^6)*a^3 + (4*(8*n^2 - 6*n + 1)*c^6*d^6 - 4*(32*n^2
 - 29*n + 6)*b*c^5*d^5*e + (128*n^2 - 137*n + 39)*b^2*c^4*d^4*e^2 + 8*(8*n^
2 - 7*n - 1)*b^3*c^3*d^3*e^3 - 4*(37*n^2 - 43*n + 6)*b^4*c^2*d^2*e^4 + 4*(1
0*n^2 - 16*n + 3)*b^5*c*d*e^5 + (12*n^2 - 7*n + 1)*b^6*e^6)*a^2 - ((16*n^2
- 21*n + 5)*b^2*c^5*d^6 - 2*(32*n^2 - 43*n + 11)*b^3*c^4*d^5*e + 2*(44*n^2
- 61*n + 17)*b^4*c^3*d^4*e^2 - 20*(2*n^2 - 3*n + 1)*b^5*c^2*d^3*e^3 - (8*n^
2 - 7*n - 1)*b^6*c*d^2*e^4 + 2*(4*n^2 - 5*n + 1)*b^7*d*e^5)*a + ((2*n^2 - 3

```
*n + 1)*b^3*c^5*d^6 - 4*(2*n^2 - 3*n + 1)*b^4*c^4*d^5*e + 6*(2*n^2 - 3*n +
1)*b^5*c^3*d^4*e^2 - 4*(2*n^2 - 3*n + 1)*b^6*c^2*d^3*e^3 + (2*n^2 - 3*n + 1
)*b^7*c*d^2*e^4 - 2*(4*(35*n^2 - 12*n + 1)*c^4*d*e^5 - (81*n^2 - 37*n + 4)*
b*c^3*e^6)*a^4 - 2*(8*(7*n^2 - 8*n + 1)*c^5*d^3*e^3 - (83*n^2 - 97*n + 14)*
b*c^4*d^2*e^4 - (44*n^2 + 7*n - 3)*b^2*c^3*d*e^5 + 3*(15*n^2 - 8*n + 1)*b^3
*c^2*e^6)*a^3 - (8*(3*n^2 - 4*n + 1)*c^6*d^5*e - 2*(11*n^2 - 19*n + 8)*b*c^
5*d^4*e^2 - 4*(22*n^2 - 23*n + 1)*b^2*c^4*d^3*e^3 + (136*n^2 - 159*n + 23)*
b^3*c^3*d^2*e^4 - 2*(16*n^2 - 27*n + 5)*b^4*c^2*d*e^5 - (12*n^2 - 7*n + 1)*
b^5*c*e^6)*a^2 - 2*((7*n^2 - 9*n + 2)*b*c^6*d^6 - (28*n^2 - 37*n + 9)*b^2*c
^5*d^5*e + 2*(19*n^2 - 26*n + 7)*b^3*c^4*d^4*e^2 - 8*(2*n^2 - 3*n + 1)*b^4*
c^3*d^3*e^3 - 5*(n^2 - n)*b^5*c^2*d^2*e^4 + (4*n^2 - 5*n + 1)*b^6*c*d*e^5)*
a)*x^n)/(16*a^9*c^2*e^8*n^2 + 8*(8*c^3*d^2*e^6*n^2 - 8*b*c^2*d*e^7*n^2 - b^
2*c*e^8*n^2)*a^8 + (96*c^4*d^4*e^4*n^2 - 192*b*c^3*d^3*e^5*n^2 + 64*b^2*c^2
*d^2*e^6*n^2 + 32*b^3*c*d*e^7*n^2 + b^4*e^8*n^2)*a^7 + 4*(16*c^5*d^6*e^2*n^
2 - 48*b*c^4*d^5*e^3*n^2 + 36*b^2*c^3*d^4*e^4*n^2 + 8*b^3*c^2*d^3*e^5*n^2 -
 11*b^4*c*d^2*e^6*n^2 - b^5*d*e^7*n^2)*a^6 + 2*(8*c^6*d^8*n^2 - 32*b*c^5*d^
7*e*n^2 + 32*b^2*c^4*d^6*e^2*n^2 + 16*b^3*c^3*d^5*e^3*n^2 - 37*b^4*c^2*d^4*
e^4*n^2 + 10*b^5*c*d^3*e^5*n^2 + 3*b^6*d^2*e^6*n^2)*a^5 - 4*(2*b^2*c^5*d^8*
n^2 - 8*b^3*c^4*d^7*e*n^2 + 11*b^4*c^3*d^6*e^2*n^2 - 5*b^5*c^2*d^5*e^3*n^2
- b^6*c*d^4*e^4*n^2 + b^7*d^3*e^5*n^2)*a^4 + (b^4*c^4*d^8*n^2 - 4*b^5*c^3*d
^7*e*n^2 + 6*b^6*c^2*d^6*e^2*n^2 - 4*b^7*c*d^5*e^3*n^2 + b^8*d^4*e^4*n^2)*a
^3 + (16*a^8*c^3*e^8*n^2 + 8*(8*c^4*d^2*e^6*n^2 - 8*b*c^3*d*e^7*n^2 - b^2*c
^2*e^8*n^2)*a^7 + (96*c^5*d^4*e^4*n^2 - 192*b*c^4*d^3*e^5*n^2 + 64*b^2*c^3*
d^2*e^6*n^2 + 32*b^3*c^2*d*e^7*n^2 + b^4*c*e^8*n^2)*a^6 + 4*(16*c^6*d^6*e^2
*n^2 - 48*b*c^5*d^5*e^3*n^2 + 36*b^2*c^4*d^4*e^4*n^2 + 8*b^3*c^3*d^3*e^5*n^
2 - 11*b^4*c^2*d^2*e^6*n^2 - b^5*c*d*e^7*n^2)*a^5 + 2*(8*c^7*d^8*n^2 - 32*b
*c^6*d^7*e*n^2 + 32*b^2*c^5*d^6*e^2*n^2 + 16*b^3*c^4*d^5*e^3*n^2 - 37*b^4*c
^3*d^4*e^4*n^2 + 10*b^5*c^2*d^3*e^5*n^2 + 3*b^6*c*d^2*e^6*n^2)*a^4 - 4*(2*b
^2*c^6*d^8*n^2 - 8*b^3*c^5*d^7*e*n^2 + 11*b^4*c^4*d^6*e^2*n^2 - 5*b^5*c^3*d
^5*e^3*n^2 - b^6*c^2*d^4*e^4*n^2 + b^7*c*d^3*e^5*n^2)*a^3 + (b^4*c^5*d^8*n^
2 - 4*b^5*c^4*d^7*e*n^2 + 6*b^6*c^3*d^6*e^2*n^2 - 4*b^7*c^2*d^5*e^3*n^2 + b
^8*c*d^4*e^4*n^2)*a^2)*x^(2*n) + (16*a^8*b*c^2*e^8*n^2 + 8*(8*b*c^3*d^2*e^6
*n^2 - 8*b^2*c^2*d*e^7*n^2 - b^3*c*e^8*n^2)*a^7 + (96*b*c^4*d^4*e^4*n^2 - 1
92*b^2*c^3*d^3*e^5*n^2 + 64*b^3*c^2*d^2*e^6*n^2 + 32*b^4*c*d*e^7*n^2 + b^5*
e^8*n^2)*a^6 + 4*(16*b*c^5*d^6*e^2*n^2 - 48*b^2*c^4*d^5*e^3*n^2 + 36*b^3*c^
3*d^4*e^4*n^2 + 8*b^4*c^2*d^3*e^5*n^2 - 11*b^5*c*d^2*e^6*n^2 - b^6*d*e^7*n^
2)*a^5 + 2*(8*b*c^6*d^8*n^2 - 32*b^2*c^5*d^7*e*n^2 + 32*b^3*c^4*d^6*e^2*n^2
 + 16*b^4*c^3*d^5*e^3*n^2 - 37*b^5*c^2*d^4*e^4*n^2 + 10*b^6*c*d^3*e^5*n^2 +
 3*b^7*d^2*e^6*n^2)*a^4 - 4*(2*b^3*c^5*d^8*n^2 - 8*b^4*c^4*d^7*e*n^2 + 11*b
^5*c^3*d^6*e^2*n^2 - 5*b^6*c^2*d^5*e^3*n^2 - b^7*c*d^4*e^4*n^2 + b^8*d^3*e^
5*n^2)*a^3 + (b^5*c^4*d^8*n^2 - 4*b^6*c^3*d^7*e*n^2 + 6*b^7*c^2*d^6*e^2*n^2
 - 4*b^8*c*d^5*e^3*n^2 + b^9*d^4*e^4*n^2)*a^2)*x^n), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)^2}\,dx$$

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(d+e\,x^n)^2\,(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

```
[In] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x)
[Out] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3), x)
```

3.85 $\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 292

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{ex^{1+n}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{1}{2},-\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} + \frac{dx\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,-1/2,-1/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+n)/(1+2*c*x^n/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+d*x*A
ppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{1446, 1362, 440, 1399, 524\}$

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{dx\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}} + \frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{1}{2},-\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -1/2, -1/2, 2
 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (d*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1
[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d\sqrt{a+bx^n+cx^{2n}}+ex^n\sqrt{a+bx^n+cx^{2n}}\right)dx \\
&= d\int \sqrt{a+bx^n+cx^{2n}}\,dx+e\int x^n\sqrt{a+bx^n+cx^{2n}}\,dx \\
&= \frac{\left(d\sqrt{a+bx^n+cx^{2n}}\right)\int \sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} \\
&\quad+\frac{\left(e\sqrt{a+bx^n+cx^{2n}}\right)\int x^n\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} \\
&= \frac{ex^{1+n}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{1}{2},-\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} \\
&\quad+\frac{dx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.45

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{x\Big(-n(-4ace(1+n)+b^2e(2+n)-2bcd(1+2n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\Big(1+\frac{1}{n},\frac{1}{2}}{}$$

```
[In] Integrate[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(-(n*(-4*a*c*e*(1 + n) + b^2*e*(2 + n) - 2*b*c*d*(1 + 2*n))*x^n*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2,
 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])]) + 2*(1 + n)*((a + x^n*(b + c*x^n))*(b*e*n + 2*c*(d + 2*d*n + e*(
1 + n)*x^n)) + a*n*(-(b*e) + 2*c*(d + 2*d*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(4*(1 + n)^2
*(c + 2*c*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int (d+e\,x^n)\sqrt{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral((d + e*x**n)*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2n}+bx^n+a}(ex^n+d)\,dx$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)
```

Giac [F]

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2n}+bx^n+a}(ex^n+d)\,dx$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \int (d+e\,x^n)\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.86 $\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 294

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \frac{aex^{1+n}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{3}{2},-\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} + \frac{adx\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*e*x^(1+n)*AppellF1(1+1/n,-3/2,-3/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+n)/(1+2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+a*d
*x*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{1446, 1362, 440, 1399, 524\}$

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}dx = \frac{adx\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}} + \frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{3}{2},-\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -3/2, -3/2,
 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (a*d*x*Sqrt[a + b*x^n + c*x^(2*n)]*Appe
llF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d\left(a+bx^n+cx^{2n}\right)^{3/2}+ex^n\left(a+bx^n+cx^{2n}\right)^{3/2}\right)\,dx\\
&= d\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx+e\int x^n\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx\\
&= \frac{\left(ad\sqrt{a+bx^n+cx^{2n}}\right)\int \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{\left(ae\sqrt{a+bx^n+cx^{2n}}\right)\int x^n\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{aex^{1+n}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{adx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 690 vs. $2(294) = 588$.

Time = 3.26 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.35

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \frac{x\Big(3n^2(16a^2c^2e(1+4n+3n^2)+b^4e(4+8n+3n^2)-2b^3cd(2+9n+4n^2)-4ab^2ce(5+14n}{}$$

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(3*n^2*(16*a^2*c^2*e*(1 + 4*n + 3*n^2) + b^4*e*(4 + 8*n + 3*n^2) - 2*b^3
*c*d*(2 + 9*n + 4*n^2) - 4*a*b^2*c*e*(5 + 14*n + 6*n^2) + 8*a*b*c^2*d*(2 +
11*n + 12*n^2))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*((a + x^n*(b + c*x^n))*(-
3*b^3*e*n^2*(2 + 3*n) + 6*b^2*c*n^2*(d + 4*d*n + e*(1 + n)*x^n) + 8*c^3*(1
+ 3*n + 2*n^2)*x^(2*n)*(d + 4*d*n + e*(1 + 3*n)*x^n) + 4*b*c^2*(1 + n)*x^n*
(d*(2 + 15*n + 28*n^2) + e*(2 + 13*n + 18*n^2)*x^n) + 4*a*c*(3*b*e*n^2*(2 +
 5*n) + 2*c*(d*(1 + 2*n)*(1 + 4*n)^2 + e*(1 + 9*n + 23*n^2 + 15*n^3)*x^n)))
 + 3*a*n^2*(b^3*e*(2 + 3*n) - 2*b^2*c*d*(1 + 4*n) - 4*a*b*c*e*(2 + 5*n) + 8
*a*c^2*d*(1 + 6*n + 8*n^2))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(16*c^2*(1 + n)^2*(1 + 2*n)*(1
 + 3*n)*(1 + 4*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int (d+e\,x^n)\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((d + e*x**n)*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} (ex^n+d) dx$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d), x)
```

Giac [F]

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} (ex^n+d) dx$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int (d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.87 $\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 292

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{a+bx^n+cx^{2n}}} + \frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,1/2,1/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*x*App
ellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

= {1446, 1362, 440, 1399, 524}

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$
$$= \frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\mathrm{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$
$$+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\mathrm{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)
*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2
, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/Sqrt[a + b*x^n + c*x^(2*n)]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d}{\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^n}{\sqrt{a+bx^n+cx^{2n}}}\right) dx \\
&= d\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx+e\int \frac{x^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx \\
&= \frac{\left(d\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}} \\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^n}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}} \\
&= \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{a+bx^n+cx^{2n}}} \\
&\quad+\frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.84

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

$$= \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\left(ex^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+d(1+n)\,A\right.}{(1+n)\sqrt{a+x^n\left(b+cx^n\right)}}$$

```
[In] Integrate[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*(e*x^n*AppellF1[1 +
 n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])] + d*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
))/((1 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{d+e\,x^{n}}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d + e*x**n)/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{d + ex^n}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{ex^n + d}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{d + ex^n}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{ex^n + d}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d + ex^n}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{d + e\,x^n}{\sqrt{a + b\,x^n + c\,x^{2\,n}}}\,dx$$

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.88 $\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{3}{2},\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{a(1+n)\sqrt{a+bx^n+cx^{2n}}}$$
$$+\frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,3/2,3/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*x*A
ppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1446, 1362, 440, 1399, 524}

$$\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b^2}}\right.}{a\sqrt{a+bx^n+cx^{2n}}}$$
$$+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{3}{2},\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 3/2, 3/2, 2 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(1 +
n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3
/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d}{(a+bx^n+cx^{2n})^{3/2}}+\frac{ex^n}{(a+bx^n+cx^{2n})^{3/2}}\right)dx\\
&= d\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx+e\int \frac{x^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx\\
&= \frac{\left(d\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^n}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\
&= \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{3}{2},\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(1+n)\sqrt{a+bx^n+cx^{2n}}}\\
&\quad+\frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.39

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\frac{x\left(2c(bd-2ae)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\right.\right.}{}$$

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(2*c*(b*d - 2*a*e)*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2*d + b*(-(a*e)
 + c*d*x^n) - 2*a*c*(d + e*x^n)) + (2*a*b*e + b^2*d*(-2 + n) - 4*a*c*d*(-1
+ n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1),
 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1 + n)*Sqrt[a + x^n*(b + c*x^n)
])
```

Maple [F]

$$\int \frac{d+e\,x^{n}}{\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)

[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Timed out}$$

[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{ex^n+d}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)

Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.89 $\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^{5/2}}\,dx = \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{5}{2},\frac{5}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{a^2(1+n)\sqrt{a+bx^n+cx^{2n}}} + \frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{5}{2},\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,5/2,5/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*x
*AppellF1(1/n,5/2,5/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))^(1/2)/a^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1446, 1362, 440, 1399, 524}

$$\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^{5/2}}\,dx = \frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{5}{2},\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-\right.}}{a^2\sqrt{a+bx^n+cx^{2n}}} + \frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{5}{2},\frac{5}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 5/2, 5/2, 2 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a^2*(1
+ n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 5/2,
 5/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(a^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d}{(a+bx^n+cx^{2n})^{5/2}}+\frac{ex^n}{(a+bx^n+cx^{2n})^{5/2}}\right)dx \\
&= d\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx+e\int \frac{x^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx \\
&= \frac{\left(d\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{5/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^n+cx^{2n}}} \\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^n}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{5/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^n+cx^{2n}}} \\
&= \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{5}{2},\frac{5}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(1+n)\sqrt{a+bx^n+cx^{2n}}} \\
&\quad+\frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{5}{2},\frac{5}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 701 vs. $2(298) = 596$.

Time = 5.36 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.35

$$
\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \frac{x\left(-2c(2ab^2e+4abcd(2-5n)+8a^2ce(-1+2n)+b^3d(-2+3n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-}}}\right.}{}
$$

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

```
[Out] (x*(-2*c*(2*a*b^2*e + 4*a*b*c*d*(2 - 5*n) + 8*a^2*c*e*(-1 + 2*n) + b^3*d*(-
2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*(a + x^n
*(b + c*x^n))*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + n)*(2*(b^3*d*(
-2 + 3*n)*x^n*(b + c*x^n)^2 + 4*a^3*c*(b*e*(-2 + 3*n) + c*d*(-2 + 8*n) + 2*
c*e*(-1 + 3*n)*x^n) + 2*a*b*(b + c*x^n)*(-2*c^2*d*(-2 + 5*n)*x^(2*n) + b*c*
x^n*(d*(5 - 11*n) + e*x^n) + b^2*(d*(-1 + 2*n) + e*x^n)) + a^2*(-(b^3*e*(-2
```

```
+ n)) + 8*b*c^2*e*(-2 + 3*n)*x^(2*n) - 2*b^2*c*(d*(-5 + 14*n) - 3*e*(-1 +
n)*x^n) + 8*c^3*x^(2*n)*(d*(-1 + 3*n) + e*(-1 + 2*n)*x^n))) + (2*a*b^3*e*(-
2 + n) - 8*a^2*b*c*e*(-2 + 3*n) + b^4*d*(4 - 8*n + 3*n^2) + 16*a^2*c^2*d*(1
 - 4*n + 3*n^2) - 4*a*b^2*c*d*(5 - 14*n + 6*n^2))*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*(a + x^n*(b + c*x^n))*AppellF1[n^(-1), 1/2, 1/2
, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])))/(3*a^2*(b^2 - 4*a*c)^2*n^2*(1 + n)*(a + x^n*(b + c*x^n))^(3/2
))
```

Maple [F]

$$\int \frac{d+e\,x^{n}}{\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{5}{2}}}dx$$

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(5/2),x)
```

```
[Out] Integral((d + e*x**n)/(a + b*x**n + c*x**(2*n))**(5/2), x)
```

Maxima [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="giac")
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2\,n})^{5/2}}\,dx$$

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2), x)
```

3.90 $\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Int}\left((d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p, x\right)$$

```
[Out] Unintegrable((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$

```
[In] Int[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] Defer[Int][(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

Rubi steps

$$\text{integral} = \int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$

```
[In] Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
[Out] Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (d+e\,x^n)^q \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
[Out] int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (cx^{2\,n}+bx^n+a)^p(ex^n+d)^q \, dx$$

```
[In] integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**q*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (ex^n+d)^q \, dx$$

```
[In] integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (ex^n+d)^q \, dx$$

```
[In] integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

Mupad [N/A]

Not integrable

Time = 10.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d+e\,x^n)^q \left(a+b\,x^n+c\,x^{2\,n}\right)^p dx$$

```
[In] int((d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x)
```

3.91 $\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 606

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$= \frac{3d^2ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.}{1+n}$$

$$+\frac{3de^2x^{1+2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2}{b-\sqrt{}}\right.}{1+2n}$$

$$+\frac{e^3x^{1+3n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2cx}{b-\sqrt{b^2}}\right.}{1+3n}$$

$$+d^3x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(\frac{1}{n},\right.$$

$$\left.-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] 3*d^2*e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+3*d*e^2*x^(
1+2*n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(2+1/n,-p,-p,3+1/n,-2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*n)/((1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e^3*x^(1+3*n)*(a+b
*x^n+c*x^(2*n))^p*AppellF1(3+1/n,-p,-p,4+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+3*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d^3*x*(a+b*x^n+c*x^(2*n))^p*A
ppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1450, 1362, 440, 1399, 524}

$$\int (d+ex^n)^3\left(a+bx^n+cx^{2n}\right)^p\,dx=d^3x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$
$$+\frac{3d^2ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{\ldots}{b-\ldots}\right.}{n+1}$$
$$+\frac{3de^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{\ldots}{b-\ldots}\right.}{2n+1}$$
$$+\frac{e^3x^{3n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2c\ldots}{b-\sqrt{\ldots}}\right.}{3n+1}$$

```
[In] Int[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (3*d^2*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2
 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p) + (3*d*e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*n))^
p*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^3*x^(1 +
 3*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[3 + n^(-1), -p, -p, 4 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1
+ 3*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]))^p) + (d^3*x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p
, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\text{integral} = \int \left(d^3\left(a+bx^n+cx^{2n}\right)^p+3d^2ex^n\left(a+bx^n+cx^{2n}\right)^p+3de^2x^{2n}\left(a+bx^n+cx^{2n}\right)^p+e^3x^{3n}\left(a+bx^n+cx^{2n}\right)^p\right)\,dx$$

$$= d^3\int \left(a+bx^n+cx^{2n}\right)^p\,dx+\left(3d^2e\right)\int x^n\left(a+bx^n+cx^{2n}\right)^p\,dx+\left(3de^2\right)\int x^{2n}\left(a+bx^n+cx^{2n}\right)^p\,dx+e^3\int x^{3n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$= \left(d^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\right)\int\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$+\left(3d^2e\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\right)\int x^n\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$+\left(3de^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\right)\int x^{2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$+\left(e^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\right)\int x^{3n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$= \frac{3d^2ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2c}{b-\sqrt{b}}\right.}{1+n}$$

$$+\frac{3de^2x^{1+2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{}{b-}\right.}{1+2n}$$

$$+\frac{e^3x^{1+3n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(3+\frac{1}{n};-p,-p;4+\frac{1}{n};-\frac{2}{b-\sqrt{}}\right.}{1+3n}$$

$$+d^3x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.72

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(3d^2e(1+5n+6n^2)\,x^n\,\text{AppellF1}\left(1+\frac{1}{n},-p,\right.\right.}{}$$

```
[In] Integrate[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + x^n*(b + c*x^n))^p*(3*d^2*e*(1 + 5*n + 6*n^2)*x^n*AppellF1[1 + n^(-
1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + (1 + n)*(3*d*e^2*(1 + 3*n)*x^(2*n)*AppellF1[2 + n^(-
1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + (1 + 2*n)*(e^3*x^(3*n)*AppellF1[3 + n^(-1), -p, -p,
4 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])] + d^3*(1 + 3*n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))))/((1 + n)*(1
+ 2*n)*(1 + 3*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int \left(d+e\,x^{n}\right)^{3} \left(a+b\,x^{n}+c\,x^{2n}\right)^{p} dx$$

```
[In] int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^3 \left(cx^{2\,n}+bx^n+a\right)^p dx$$

```
[In] integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) + b
*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**3*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \,dx = \int (ex^n+d)^3 \left(cx^{2n}+bx^n+a\right)^p \,dx$$

```
[In] integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{512,[1,0,7,4,9,5,1,8,0,3]%%%}+%%%{-3072,[1,0,7,4,9,5,0,9,1,2]
%%%}+%
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \,dx = \int {\left(d+e\,x^n\right)}^3\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^p \,dx$$

```
[In] int((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p, x)
```

3.92 $\int (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 447

$$\int (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$$
$$= \frac{2dex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{1+n}$$
$$+ \frac{e^2x^{1+2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{1+2n}$$
$$+ d^2x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n}, -p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] 2*d*e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(b-
(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e^2*x^(1+2*n)
*(a+b*x^n+c*x^(2*n))^p*AppellF1(2+1/n,-p,-p,3+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d^2*x*(a+b*x^n+c*x^(2*n)
)^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1450, 1362, 440, 1399, 524}

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = d^2x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right) + \frac{2dex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2}{b-\sqrt{}}\right.}{n+1} + \frac{e^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2c}{b-\sqrt{b}}\right.}{2n+1}$$

```
[In] Int[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (2*d*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]))^p) + (e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*n))^p*Appe
llF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
- 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d^2*x*(a + b*x^n
 + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^2\left(a+bx^n+cx^{2n}\right)^p+2dex^n\left(a+bx^n+cx^{2n}\right)^p+e^2x^{2n}\left(a+bx^n+cx^{2n}\right)^p\right)\,dx\\
&= d^2\int \left(a+bx^n+cx^{2n}\right)^p\,dx+(2de)\int x^n\left(a+bx^n+cx^{2n}\right)^p\,dx+e^2\int x^{2n}\left(a+bx^n+cx^{2n}\right)^p\,dx\\
&= \left(d^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p\,dx\\
&\quad+\left(2de\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int x^n\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p\,dx\\
&\quad+\left(e^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int x^{2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p\,dx
\end{aligned}$$

$$= \frac{2dex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx}{b-\sqrt{b^2}}\right.}{1+n}$$

$$+\frac{e^2x^{1+2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2}{b-\sqrt{}}\right.}{1+2n}$$

$$+d^2x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1}{n};\right.$$
$$\left.-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.76

$$\int (d+ex^n)^2\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$= \frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(2de(1+2n)x^n\,\text{AppellF1}\left(1+\frac{1}{n},-p,-p,2\right.\right.}{}$$

```
[In] Integrate[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + x^n*(b + c*x^n))^p*(2*d*e*(1 + 2*n)*x^n*AppellF1[1 + n^(-1), -p, -p
, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])] + (1 + n)*(e^2*x^(2*n)*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + d
^2*(1 + 2*n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + n)*(1 + 2*n)*((b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int (d+e\,x^n)^2\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^2 \left(cx^{2n}+bx^n+a\right)^p \, dx$$

```
[In] integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^2 \left(cx^{2n}+bx^n+a\right)^p \, dx$$

```
[In] integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-128,[1,0,5,3,6,4,1,6,0,2]%%%}+%%%{512,[1,0,5,3,6,4,0,7,1,1]%
%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d+e\,x^n)^2 \left(a+b\,x^n+c\,x^{2\,n}\right)^p \, dx$$

```
[In] int((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```

3.93 $\int (d+ex^n) (a+bx^n+cx^{2n})^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 288

$$\int (d+ex^n) (a+bx^n+cx^{2n})^p \, dx = \frac{ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{1+n} + dx\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d*x*(a+b*x^n+c*x^
(2*n))^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used

$= \{1446, 1362, 440, 1399, 524\}$

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p dx = dx\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right) + \frac{ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.}{n+1}$$

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2 + n^(
-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]))^p) + (d*x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p,
 -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
```

```
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d\left(a+bx^n+cx^{2n}\right)^p+ex^n\left(a+bx^n+cx^{2n}\right)^p\right)\,dx\\
&= d\int \left(a+bx^n+cx^{2n}\right)^p\,dx+e\int x^n\left(a+bx^n+cx^{2n}\right)^p\,dx\\
&= \left(d\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p\,dx\\
&\quad+\left(e\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int x^n\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p\,dx\\
&= \frac{ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{1+n}\\
&\quad+dx\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.84

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(ex^n\,\text{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2c}{b+\sqrt{b}}\right.\right.}{1+n}$$

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + x^n*(b + c*x^n))^p*(e*x^n*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + d
*(1 + n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 + n)*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int (d+e\,x^n)\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p dx$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\int (ex^n+d)\left(cx^{2\,n}+bx^n+a\right)^p\,dx$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Giac [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\int (ex^n+d)\left(cx^{2\,n}+bx^n+a\right)^p\,dx$$

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\int (d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)
```

3.94 $\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{d+ex^n},x\right)$$

```
[Out] Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]
[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]
[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{d+e\,x^n}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2\,n}+bx^n+a)^p}{ex^n+d}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{ex^n+d}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{ex^n+d}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

Mupad [N/A]

Not integrable

Time = 10.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{d+e\,x^n}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x)
[Out] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x)
```

3.95 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \mathrm{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2},x\right)$$

```
[Out] Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

```
[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx=\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]
[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{(d+e\,x^n)^2}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx=\int \frac{(cx^{2\,n}+bx^n+a)^p}{(ex^n+d)^2}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2\,n}+bx^n+a)^p}{(ex^n+d)^2}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2\,n}+bx^n+a)^p}{(ex^n+d)^2}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 11.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{(d+e\,x^n)^2}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x)
```

3.96 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3},x\right)$$

```
[Out] Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{(d+e\,x^n)^3}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(cx^{2\,n}+bx^n+a)^p}{(ex^n+d)^3}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e
*x^n + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^3}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^3}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3, x)
```

Mupad [N/A]

Not integrable

Time = 11.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx=\int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{(d+e\,x^n)^3}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x)
[Out] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
      m1 = max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):

   #print ("Enter grade_antiderivative for sagemath")
   #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

   leaf_count_result  = leaf_count(result)
   leaf_count_optimal = leaf_count(optimal)

   #print("leaf_count_result=",leaf_count_result)
   #print("leaf_count_optimal=",leaf_count_optimal)

   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)

   if str(result).find("Integral") != -1:
      grade = "F"
      grade_annotation =""
   else:
      if expnType_result <= expnType_optimal:
         if result.has(I):
            if optimal.has(I): #both result and optimal complex
               if leaf_count_result <= 2*leaf_count_optimal:
                  grade = "A"
                  grade_annotation =""
               else:
                  grade = "B"
                  grade_annotation ="Both result and optimal contain complex but leaf count of result is large
            else: #result contains complex but optimal is not
               grade = "C"
               grade_annotation ="Result contains complex when optimal does not."
         else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
               grade = "A"
               grade_annotation =""
            else:
               grade = "B"
               grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
      else:
         grade = "C"
         grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```